Fall '04

Exam II Review Phys 270

- Self Inductance RL Circuit

$$L = N \frac{d\phi}{dI} \quad [H]$$

ϕ = magnetic flux

N = # loops or turns

$$\mathcal{E} = -L \frac{dI}{dt}$$

induced or back emf in the circuit

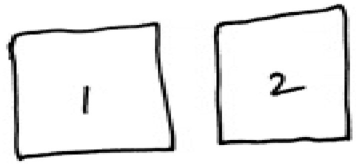
- when current ramps up from zero

$$I(t) = \frac{V}{R}\left(1 - e^{t/\tau}\right)$$

where V is the applied potential (the battery) and $\tau = \frac{L}{R}$, the inductive time constant.

- When current falls to zero $I(t) = I_0 e^{-t/\tau}$

- Mutual Inductance

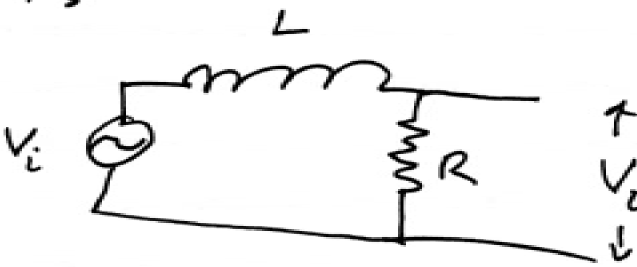


$$M = N_2 \frac{d\phi_2}{dI_1} = N_1 \frac{d\phi_1}{dI_2}$$

- In general $M \leq \sqrt{L_1 \cdot L_2}$ and will depend on geometry

- AC Circuits

RL



$$I(t) = I_p \sin \omega t$$

$$V_i(t) = I_p \sqrt{R^2 + (\omega L)^2} \sin(\omega t + \phi)$$

$$V_o(t) = R I_p \sin \omega t$$

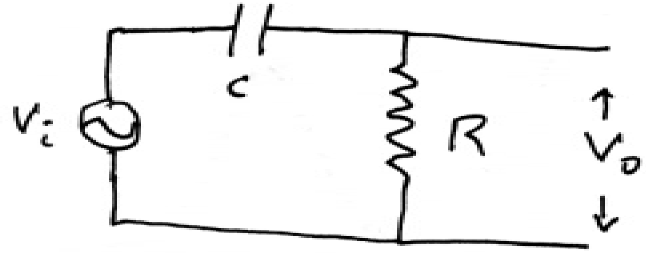
$$\phi = \tan^{-1}[\omega L / R]$$

Peak voltage drop across

R $\quad \Delta V_R = R I_p$

L $\quad \Delta V_L = \omega L I_p$

Rc



$I(t) = I_p \sin \omega t$

$V_i(t) = I_p \sqrt{R^2 + (1/\omega C)^2} \sin(\omega t + \phi)$

$V_o(t) = I_p R \sin \omega t$

$\phi = \tan^{-1} \dfrac{-\frac{1}{\omega C}}{R}$

Peak voltage drop

R: $\Delta V_R = I_p R$

C: $\Delta V_C = \dfrac{I_p}{\omega C}$

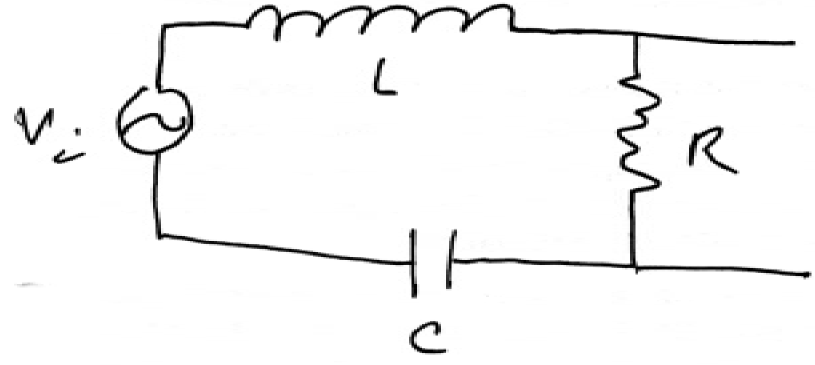
For LC $\phi \to \frac{\pi}{2}$ as $\omega \to \infty$

the V_i & V_o out of phase

For RC $\phi \to \frac{\pi}{2}$ as $\omega \to 0$

V_i & V_o out of phase

RLC



Resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

$\omega L - \frac{1}{\omega C} = 0$ circuit acts like

L & C not present.

For AC signals, we define

root-mean-square (rms)

$x_{rms} = \sqrt{\langle x^2 \rangle}$

$\langle \sin^2 \omega t \rangle = \langle \cos^2 \omega t \rangle = \frac{1}{2}$

$V_{rms} = \frac{V_p}{\sqrt{2}}$

Power flow in RLC @ ω_0

$$P = I^2_{rms} R$$

Quality factor $Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R}$

for $\omega \neq \omega_0$ power is reduced.

Generalized Ohm's Law

$$V = I Z$$

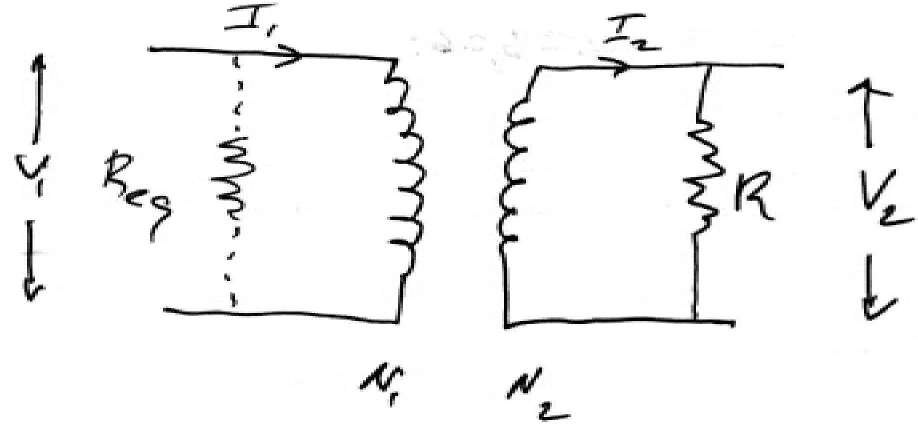
$$Z_R = R; \; Z_L = i\omega L; \; Z_C = -\frac{i}{\omega C}$$

$$|Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$I_{rms} = \frac{V_{rms}}{|Z|}$$

RCL @ ω_0 $\quad Z = R$!

- Transformer



$$I_1 = \frac{V_1}{R_{eq}} \qquad I_2 = \frac{V_2}{R}$$

R_{eq} is the apparent resistance seen on the primary side

$$R_{eq} = \left(\frac{N_2}{N_1}\right)^2 R$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \; ; \quad N_1 I_1 = N_2 I_2$$

- Maxwell's Equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$\vec{\nabla} \cdot$ divergence - measures the flow of field lines

$\vec{\nabla} \times$ curl - measures the rotation

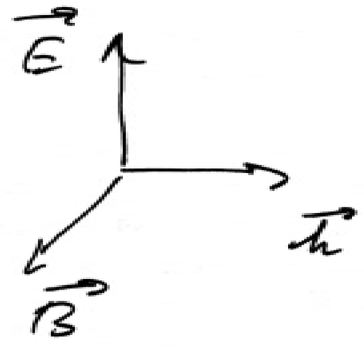
- Electromagnetic Waves

$$\underbrace{\frac{\partial^2 E}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}}$$

Wave equation also holds for B.

$\epsilon_0 \mu_0 = \frac{1}{c^2}$ where c = speed of light in vacuum

$$|\vec{E}| = c|\vec{B}|$$



right-handed coordinate system in vacuum $\vec{k} \perp$ to plane (surface) containing $\vec{E}$ & $\vec{B}$. $\vec{E} \perp \vec{B}$

- plane waves

$$\vec{E} = \hat{\epsilon}_1 E_0 \cos(\vec{k}\cdot\vec{r} - \omega t)$$

$$\vec{B} = \hat{\epsilon}_2 B_0 \cos(\vec{k}\cdot\vec{r} - \omega t)$$

- Intensity

$$I = \langle|\vec{S}|\rangle = \frac{1}{\mu_0}\langle|\vec{E}\times\vec{B}|\rangle$$

$$= \frac{1}{2}\epsilon_0 c E_0^2 = \frac{E_0^2}{2\mu_0 c} \quad W/m^2$$

- Energy density

$$u_E = \underbrace{\frac{1}{2}\epsilon_0 E^2} \qquad u_B = \underbrace{\frac{B^2}{2\mu_0}}$$

Instantaneous!

Total instantaneous field

$$u = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$\langle u \rangle = \frac{1}{2} \epsilon_0 E^2$$

$$\therefore \quad I = c \langle u \rangle$$ flow of energy in direction $\vec{k}$

- Momentum

$$p_{EM} = \frac{u}{c}$$

- Light pressure

Absorber $P = \frac{I}{c}$

Reflector $= \frac{2I}{c}$

- Geometrical Optics (Ray Optics)

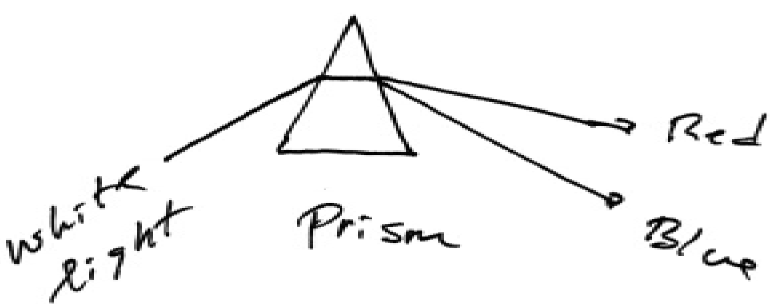
  - reflection $\theta_i = \theta_r$

  - refraction $\underbrace{n_1 \sin\theta_1 = n_2 \sin\theta_2}_{\text{Snell's Law}}$

  Angles measured from $\perp$ to interface surface

  - n = index of refraction

Dispersion Curve



$$\nu = \frac{c}{\lambda} \qquad \omega = 2\pi\nu \qquad |\vec{k}| = \frac{2\pi}{\lambda}$$

in medium $v = \dfrac{c}{n}$

$$\lambda = \frac{\lambda_{vac}}{n}$$

- Optical Path Length

$$OPL = \int n(l)\,dl$$

When n constant along l

$$OPL = n \cdot l$$

- Fermat's Principle

Huygens' Principle

- Ray Optics

Paraxial rays approximation

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

p = object location

q = image location

Thin lens in vaccum

$$\frac{1}{p} + \frac{1}{q} = n-1\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

- should know when p, q, R_1 & R_2 are positive or negative and how to trace rays.