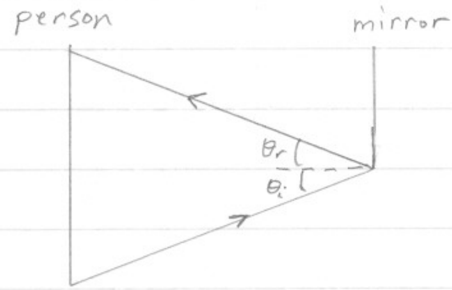
Solutions to hw 7

1) Ignoring the small distance between the eyes and the top of the head, the diagram shows that a ray originating from the feet and reflected from the bottom of the mirror will reach the eyes if the mirror is half the height of the person and is mounted with its top at the height of the person.



2) concave spherical mirror of radius $R \Rightarrow f = +R/2$.
Use $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$ and $M = -s_i/s_o$. My numbers: $R = 19.2$ cm.

a) $s_o = 40$ cm $\Rightarrow$ $s_i = 12.6$ cm, $M = -0.316$ $\Rightarrow$ real inverted image

b) $s_o = 19.0$ cm $\Rightarrow$ $s_i = 19.4$ cm, $M = -1.02$ $\Rightarrow$ real inverted image

c) $s_o = 9.6$ cm $\Rightarrow$ $s_i = \infty$, $M = \infty$ $\Rightarrow$ no image

3) Both images are behind the mirror, so that the image distances s_1 and s_2 are both negative. The closer image must correspond to the convex orientation. Thus,

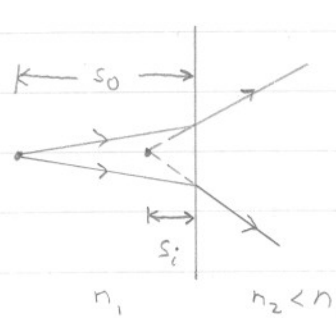
$$\left.\begin{aligned} \frac{1}{s_o} + \frac{1}{s_1} &= +\frac{2}{R} \\ \frac{1}{s_o} + \frac{1}{s_2} &= -\frac{2}{R} \end{aligned}\right\} \qquad s_o = -2\,\frac{s_1 s_2}{s_1 + s_2} \qquad R = 4\,\frac{s_1 s_2}{s_2 - s_1}$$

my numbers: $s_1 = -26$ cm, $s_2 = -10$ cm $\Rightarrow$ $s_o = 14.4$ cm, $R = 65$ cm

4) $\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} \rightarrow 0 \Rightarrow s_i = -\frac{n_2}{n_1} s_o$

$n_1 = 1.33$, $n_2 = 1$ $\Rightarrow$ $s_i = -0.752\, s_o$

The distance from the wall is reduced by the factor $1/n_1$, as is the apparent speed.



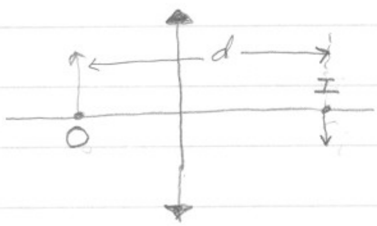
5) converging lens with $f = +19.2$ cm. Use $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$, $M = -s_i/s_o$.

a) $s_o = 40.6$ cm $\Rightarrow$ $s_i = 36.4$ cm, $M = -0.897$ $\Rightarrow$ real, inverted, opposite side

b) $s_o = 19.2$ cm $\Rightarrow$ $s_i = \infty$, $M = \infty$ $\Rightarrow$ no image

c) $s_o = 10.0$ cm $\Rightarrow$ $s_i = -20.9$ cm, $M = 2.09$ $\Rightarrow$ upright, virtual, same side

6) First consider the case of a real image behind a converging lens with focal length f at a distance $d = s_o + s_i$ from the object, as sketched.



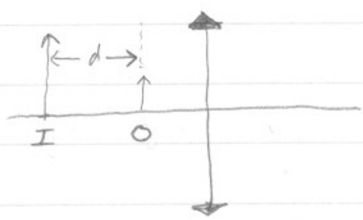
$$\left.\begin{array}{l} s_o + s_i = d \\ \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \end{array}\right\} \quad \begin{array}{l} s_i = d - s_o \\ s_i s_o = f d \end{array}$$

combine eqs. to obtain

$$s_o^2 - s_o d + f d = 0 \Rightarrow s_o = \frac{d \pm \sqrt{d^2 - 4fd}}{2}$$

my numbers: $f = 2.36$ cm, $d = 13.2$ cm $\Rightarrow$ $s_o = 3.08$ or 10.12 cm

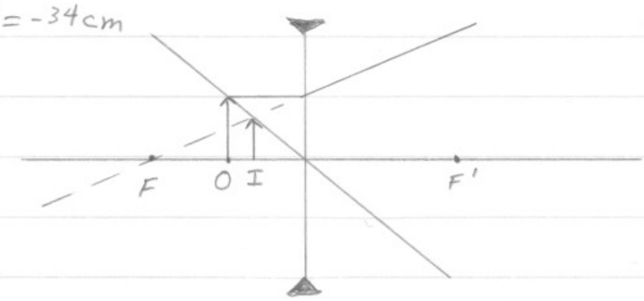
Next consider a virtual image behind the object as sketched. Recognizing that $s_i < 0$, $s_o > 0$ using the sign conventions for lens, we identify $d = -(s_o + s_i)$ and obtain



$$s_o = \frac{\sqrt{d^2 + 4fd} - d}{2}$$

by changing the sign of d in the previous solution. We must choose the positive sign for the radical to insure $s_o > 0$. Thus, the nearest lens position is $s_o = 2.04$ cm and there is no furthest distance.

my numbers: $s_0 = 15\text{ cm}$, $f = -34\text{ cm}$



$\frac{1}{s_0} + \frac{1}{s_i} = \frac{1}{f} \Rightarrow s_i = -10.4\text{ cm}$

$M = -\frac{s_i}{s_0} = 0.694$

To correct nearsightedness, we use a lens with $f = -s_{fp}$ where s_{fp} is the distance to the far point, the furthest distance the eye can accommodate. The closest object that can be seen clearly with this prescription is at a distance where the virtual image produced by the lens is located at the near point of the eye, such that $s_i = -s_{np}$. Then use $\frac{1}{s_0} + \frac{1}{s_i} = \frac{1}{f} \Rightarrow \frac{1}{s_0} - \frac{1}{s_{np}} = -\frac{1}{s_{fp}}$.

$$\therefore \quad s_0 = \frac{s_{np}\, s_{fp}}{s_{fp} - s_{np}}$$

my numbers: $s_{np} = 20\text{ cm}$, $s_{fp} = 82\text{ cm} \Rightarrow s_0 = 26.5\text{ cm}$

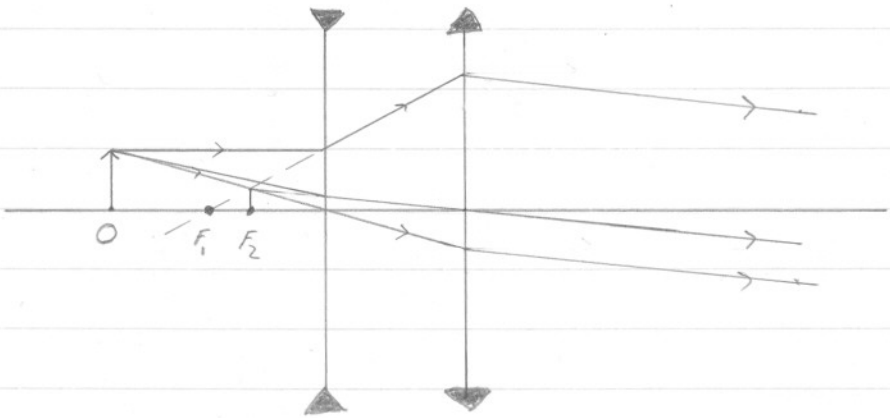
The object at distance s_0 from lens f_1 produces an image at s_{i1}. The object for lens f_2 at distance d to the right of the first lens is the first image and corresponds to an object distance $s_{o2} = d - s_{i1}$. Thus,

$$\frac{1}{s_0} + \frac{1}{s_{i1}} = \frac{1}{f_1} \qquad \frac{1}{d - s_{i1}} + \frac{1}{s_i} = \frac{1}{f_2}$$

To obtain a final image at ∞, we set $s_i \to \infty$ and solve for d:

$$s_i \to \infty \Rightarrow \quad d = f_2 + s_{i1} = f_2 + \frac{f_1 s_0}{s_0 - f_1}$$

my numbers: $s_0 = 28\text{ cm}$, $f_1 = -15\text{ cm}$, $f_2 = 28\text{ cm} \Rightarrow d = 18.2\text{ cm}$



The virtual image produced by the diverging lens is in the focal plane of the converging lens. The final bundle is parallel, at an angle to the optical axis determined using a ray through the center of lens 2.

10 a) To obtain good distance vision, the implanted lens must produce an image on the retina for an object at ∞. Hence, the power of the lens in diopters is the inverse of the lens to retina distance in meters. If this distance is 23.9 mm, the power is 41.8 diopter.

b) If the reading distance is 30 cm, reading glasses must have a power of 3.33 diopters to produce a virtual object at ∞ where the implant can focus without accommodation.