

Solutions to hw 6

$$n_1 = 1, \theta_1 = 44^\circ, \theta_2 = 19.24^\circ, \lambda_1 = 632.8 \text{ nm}$$

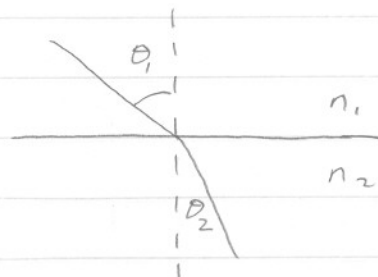
$$a) n_1 \sin \theta_1 = n_2 \sin \theta_2 \Rightarrow \frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2}$$

$$n_2 = 2.11$$

$$b) \lambda_2 = \frac{\lambda_1}{n_2} = 300 \text{ nm}$$

$$c) f = \frac{c}{\lambda} = 4.74 \times 10^{14} \text{ Hz} \quad \text{independent of } n$$

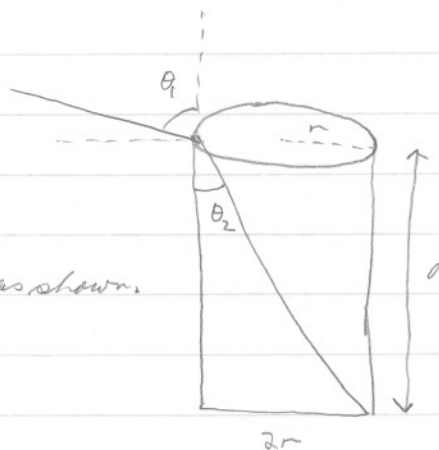
$$d) v_2 = \frac{c}{n_2} = 1.42 \times 10^8 \text{ m/s}$$



Light fails to illuminate bottom when a ray incident near the top rim is refracted to the diametrically opposite edge of bottom, as shown.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\tan \theta_2 = \frac{2r}{d}$$



$$\therefore d = 2r \cot \left(\sin^{-1} \left(\frac{\sin \theta_1}{n_2} \right) \right)$$

$$\text{my numbers: } 2r = 4.20 \text{ m}, \theta_1 = 90^\circ - 29^\circ = 61^\circ, n_2 = 1.333$$

$$\theta_2 = \sin^{-1} \left(\frac{\sin \theta_1}{n_2} \right) = 40.99^\circ \Rightarrow d = 4.83 \text{ m}$$

$$\text{Time delay: } \Delta t = \frac{d}{v} - \frac{d}{c} = \frac{d}{c}(n-1)$$

$$\text{my numbers: } d = 6.4 \text{ m}, n = 1.309 \Rightarrow \Delta t = 6.6 \text{ ns}$$

$$\text{my numbers: } \alpha = 60^\circ, \theta_1 = 64^\circ$$

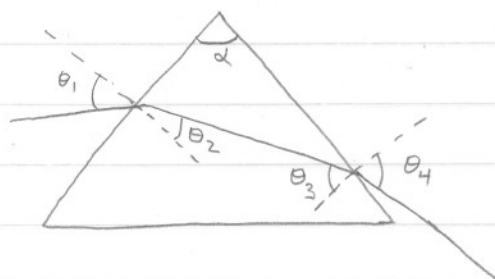
$$\lambda_1 = 700 \text{ nm} \Rightarrow n = 1.458 \text{ from graph}$$

$$a) \theta_2 = \sin^{-1} \left(\frac{\sin \theta_1}{n} \right) = 38.06^\circ$$

$$b) \alpha + \left(\frac{\pi}{2} - \theta_2 \right) + \left(\frac{\pi}{2} - \theta_3 \right) = \pi \Rightarrow \theta_3 = \alpha - \theta_2 = 21.94^\circ$$

$$c) \theta_4 = \sin^{-1} (n \sin \theta_3) = 33.0^\circ$$

$$d) \Delta \theta = \theta_1 + \theta_4 - \alpha = 37^\circ$$

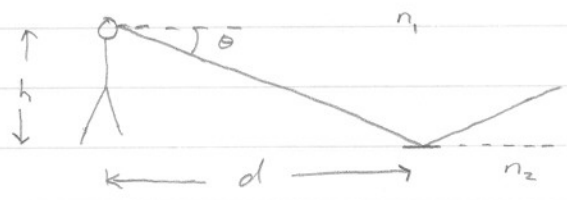


- 5) The critical angle for total internal reflection is
 $\sin \theta_c = \frac{n_2}{n_1} = \cos \theta$

where θ is wrt horizontal.

$$\therefore n_2 = n_1 \cos \theta$$

my numbers: $n_1 = 1.0004$, $\theta = 1.25^\circ \Rightarrow n_2 = 1.00016$



- 6) my numbers: $v = 61.35 \text{ km/hr} = 17.04 \text{ m/s}$

a) $n = \frac{c}{v} = 1.76 \times 10^7$

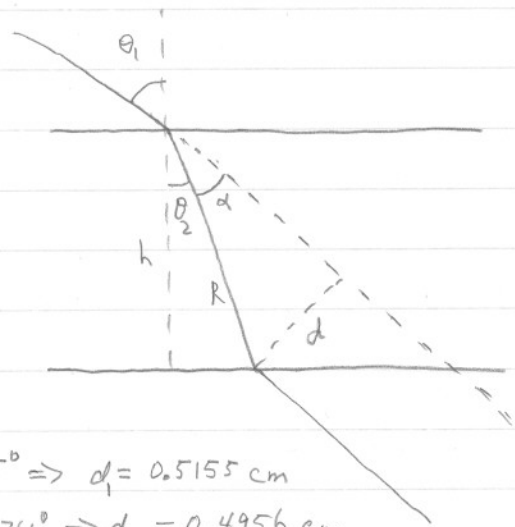
b) $\theta_c = \sin^{-1}(\frac{1}{n}) \approx \frac{1}{n} = 3.25 \times 10^{-6} \text{ deg.}$

- 7) $d = R \sin \alpha$ with $\alpha = \theta_1 - \theta_2$

$$R = \frac{h}{\cos \theta_2}$$

$$\therefore d = h \frac{\sin(\theta_1 - \theta_2)}{\cos \theta_2}$$

$$\sin \theta_2 = \frac{1}{n} \sin \theta_1$$



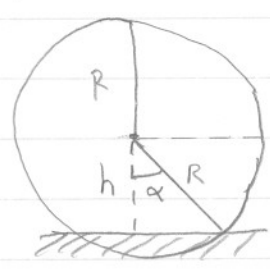
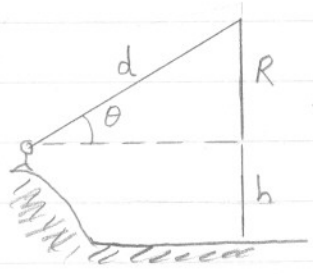
my numbers: $h = 3 \text{ cm}$, $\theta_1 = 23^\circ$

$$\lambda_1 = 400 \text{ nm} \Rightarrow n_1 = 1.689 \Rightarrow \theta_2 = 13.375^\circ \Rightarrow d_1 = 0.5155 \text{ cm}$$

$$\lambda_2 = 700 \text{ nm} \Rightarrow n_2 = 1.642 \Rightarrow \theta_2 = 13.766^\circ \Rightarrow d_2 = 0.4956 \text{ cm}$$

$$\therefore \Delta d = d_1 - d_2 = 0.199 \text{ mm}$$

- 8)



$$R = d \sin \theta, \cos \alpha = \frac{h}{R} \Rightarrow \alpha = \cos^{-1}(\frac{h}{d \sin \theta})$$

my numbers: $\theta = 42^\circ$, $d = 6 \text{ km}$, $h = 2 \text{ km} \Rightarrow \alpha = 60.1^\circ$

fraction of arc is $1 - \frac{\alpha}{180^\circ} = 0.666$