Solutions to hw 5

1) $E_y = E_0 \sin(kx - \omega t) \Rightarrow B_0 = c^{-1} E_0$, $\lambda = \frac{2\pi}{k}$, $f = \frac{\omega}{2\pi} = \frac{kc}{2\pi}$

my numbers: $E_0 = 102 \frac{V}{m}$, $k = 1.4 \times 10^{7} m^{-1} \Rightarrow$ a) $B_0 = 0.34 \mu T$ b) $\lambda = 0.449 \mu m$ c) $f = 6.68x$

2) $I = \frac{c}{2\mu_0} B_0^2$ where B_0 is plane-wave amplitude and $I = \frac{P}{4\pi r^2}$ is the intensity at distance r from a spherical source emitting power P at the appropriate frequency. Here $P = \varepsilon V_{rms}^2 / R$ where ε is the fraction of the total power at frequency f.

my numbers: $V_{rms} = 120 V$, $R = 106 \Omega$, $\varepsilon = 0.015$, $r = 1.5 m$

$\therefore I = 0.072 \frac{W}{m^2} \Rightarrow B_0 = 24.6 nT$

3) Let $\vec{S}_i$ represent the incident and $\vec{S}_r = -(1-f)\vec{S}_i$ represent the reflected power flux, where f is the fraction absorbed and when normal incidence is assumed. The energy absorbed in time t by area A is then $U = A S_i f t$. The momentum transferred to the surface is the change in momentum carried by radiation: $\vec{p} = c^{-1}(\vec{S}_i - \vec{S}_r) A t = \frac{2-f}{c} \vec{S}_i A t \Rightarrow p = \frac{2-f}{f} \frac{U}{c}$

my numbers: $f = 0.5$, $I = \langle S_i \rangle = 750 \frac{W}{m^2}$, $A = 0.52 m^2$, $t = 60 s$

a) $U = 11.7 kJ$ b) $p = 1.17 \times 10^{-4} kg\, m/s$

4) A particle in a circular orbit emits radiation at its cyclotron frequency: $m\omega^2 R = qvB \Rightarrow \omega = \frac{qB}{m}$. The wavelength is then $\lambda = \frac{2\pi c}{\omega} = 2\pi \frac{mc}{qB}$.

my numbers: $B = 0.5 T$, $m = 1.67 \times 10^{-27} kg$, $q = 1.6 \times 10^{-19} C \Rightarrow \lambda = 39.3 m$

5) my numbers: $E_{max} = 0.3\,\mu V/m$, diameter $d = 18\,m$

a) $B_{max} = c^{-1} E_{max} = 1.0 \times 10^{-15}\ T$

b) $I = E_{max}^2 / 2\mu_0 c = 1.19 \times 10^{-16}\ W/m^2$

c) $P = \pi r^2 I = 3.04 \times 10^{-14}\ W$

d) $F = \frac{P}{c} = 1.01 \times 10^{-22}\ N$

6) my numbers: $f = 18\ GHz$, $t = 1\,ns$, $P = 25\ kW$, $R = 3.5\ cm$

a) $\lambda = \frac{c}{f} = 1.67\ cm$

b) $U = Pt = 25\ \mu J$

c) average energy density $u = U/(\pi R^2 c t) = P/(\pi R^2 c) = 21.7\ mJ/m^3$

d) $I = E_{max}^2 / 2\mu_0 c = P/\pi R^2 \Rightarrow E_{max} = \left(\frac{2\mu_0 c}{\pi R^2} P\right)^{1/2} = 70\ kV/m$

$B_{max} = c^{-1} E_{max} = 233\ \mu T$

e) $F = \frac{P}{c} = 83.3\ \mu N$

P34.52) Consider a small spherical particle of mass m, radius r with uniform density and absorptivity at distance R from the sun. The gravitational attraction $F_{grav} = GMm/R^2 = \frac{4\pi}{3}\rho GM r^3/R^2$ is proportional to volume, hence r^3, while the force $F_{rad} = \pi r^2 \frac{I}{c} a$ due to radiation pressure is proportional to area, hence r^2, where here I is intensity and a is a factor related to the fraction absorbed and is constant. Using $I = P/4\pi R^2$, we find

$$\frac{F_{rad}}{F_{grav}} = \frac{r^2}{4R^2}\frac{Pa}{c}\frac{3R^2}{4\pi\rho GM r^3} = \frac{r_0}{r}$$

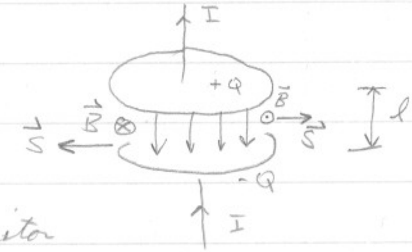
where

$$r_0 = \frac{3}{16\pi}\frac{aP}{GM\rho c} = \frac{3}{4}\frac{aIR^2}{GM\rho c}$$

is the radius for a particle for which these forces balance.

Using $a = 1$, $I = 214\ W/m^2$, $R = 3.75 \times 10^{11}\ m$, $\rho = 1.5\ g/cm^3$,
$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$, $M = 1.99 \times 10^{30}\ kg$, we find $r_0 = 0.38\ \mu m$ is the radius of a dust particle.

P34.54) Assuming that the field is uniform in the gap, we find $E = Q/\varepsilon_0 A$ downward as shown. The displacement current $I_d = \varepsilon_0 \dot{E} = I/A$ is upward while the capacitor is being discharged. At the surface of the cylindrical gap we use Ampere's law to find



$$2\pi r B_\phi = \mu_0 I \implies B_\phi = \frac{\mu_0 I}{2\pi r}$$

Finally, using $\vec{S} = \vec{E} \otimes \vec{B}/\mu_0$ we obtain

a) $S = \frac{IQ}{\varepsilon_0} (2\pi^2 r^3)^{-1}$ radially outward

b) $P = 2\pi r l S = \frac{IQ}{\varepsilon_0} \frac{l}{\pi r^2}$ drawn from capacitor

It is useful to compare this with the energy stored in the electric field of the capacitor:

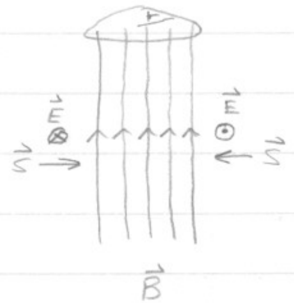
$$U = \varepsilon_0 \frac{E^2}{2} \pi r^2 l = \frac{Q^2}{2\varepsilon_0} \frac{l}{\pi r^2} \implies \dot{U} = \frac{IQ}{\varepsilon_0} \frac{l}{\pi r^2} = P$$

c) If the stored charge increases, I_d reverses direction. Then $\vec{B}$ and hence $\vec{S}$ reverse direction. Thus, $\vec{S}$ is radially inward at the surface when the stored energy is increasing.

P34.55) The magnetic field in the central section of a long solenoid is approximated as uniform, such that $B = n\mu_0 I$ where n is the number of turns per unit length. Using Faraday's law, the electric field at surface is

$$2\pi r E = \pi r^2 n \mu_0 \dot{I} \Rightarrow E = \tfrac{1}{2} n \mu_0 \dot{I} r$$

with the direction indicated for $\dot{I} > 0$.



$\dot{I} > 0$

a) $\vec{S} = \mu_0^{-1} \vec{E} \otimes \vec{B} \Rightarrow S = \mu_0 \frac{n^2}{2} I \dot{I} r$ radially inward for $\dot{I} > 0$.

b) $P = 2\pi r \ell S = \mu_0 n^2 I \dot{I} \pi r^2 \ell$

We compare this with the magnetic energy stored in this volume:

$U = \frac{B^2}{2\mu_0} \pi r^2 \ell = \mu_0 \frac{n^2}{2} I^2 \pi r^2 \ell \Rightarrow \dot{U} = \mu_0 n^2 I \dot{I} \pi r^2 \ell = P$

c) The voltage can be obtained by integrating the electric field around $n\ell$ loops, such that

$$\Delta V = n \ell \, 2\pi r E = \mu_0 n^2 \dot{I} \pi r^2 \ell \Rightarrow P = I \Delta V$$

as expected.

P34.61a) The force F on the astronaut due to the radiated power P is $F = P/c$. Using $s = \frac{1}{2} a t^2$ for uniform acceleration, we find

$$t = \sqrt{\frac{2s}{a}} = \left(\frac{2mcs}{P}\right)^{1/2}$$

$$m = 110\,\text{kg}, \quad s = 10\,\text{m}, \quad P = 100\,\text{W} \Rightarrow t = 8.12 \times 10^4\,\text{s} = 22.6\,\text{h}$$

b) Throwing a flashlight with mass m_1 at velocity v_1 produces relative velocity $v_r = v_1 - v_2$ where

$$\left.\begin{aligned} m_1 v_1 + m_2 v_2 &= 0 \\ v_1 - v_2 &= v_r \end{aligned}\right\} \quad v_2 = \frac{m_1 v_r}{m_1 + m_2} \Rightarrow t = \frac{s}{v_2}$$

$$m_1 = 3\,\text{kg}, \quad m_1 + m_2 = m = 110\,\text{kg}, \quad v_r = 12\,\text{m/s}, \quad s = 10\,\text{m} \Rightarrow t = 30.6\,\text{s}$$

Throwing the flashlight is obviously a better strategy (simple but strong).