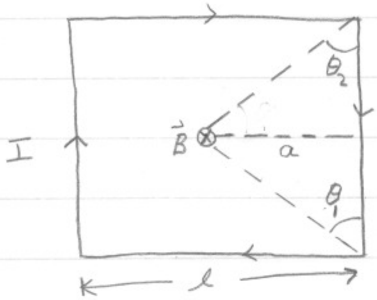
Solutions to hw 2

1a) At the center of the square each side makes an equal contribution to $\vec{B}$, which is into the page. The field for a finite line segment is



$$B = \frac{\mu_0 I}{4\pi a}\left(\cos\theta_1 - \cos\theta_2\right) \rightarrow \frac{\mu_0 I}{2\pi l} 2\cos 45^\circ \quad \text{per side}$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{l} 8\sqrt{2}$$

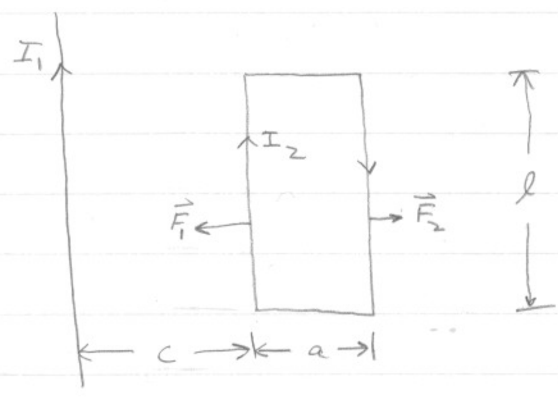
my numbers: $l = 0.42$ m, $I = 11$ A $\Rightarrow B = 29.6\,\mu T$ into page

b) At the center of a circular loop, $B = \frac{\mu_0 I}{2R}$ with $2\pi R = 4l$ for this length of wire

$$\therefore \quad B = \frac{\mu_0}{4\pi}\pi^2 \frac{I}{l}$$

This is smaller by a factor of $\frac{\pi^2}{8\sqrt{2}} = 0.872$.
my numbers $\Rightarrow B = 25.8\,\mu T$.

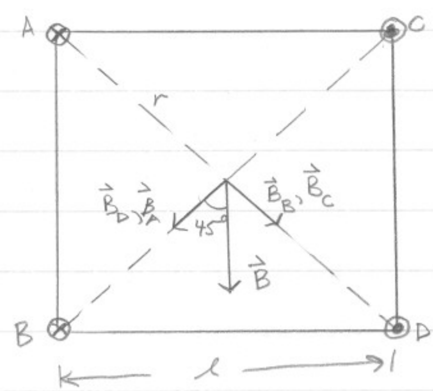
2) The force F_1 on the near side is toward left and is larger than force F_2 on the far side toward right. The net leftward force is then



$$F = \frac{\mu_0}{4\pi} 2 I_1 I_2 l \left(\frac{1}{c} - \frac{1}{c+a}\right)$$

my numbers: $I_1 = 8$A, $I_2 = 10$A, $l = 0.7$m, $c = 0.1$m, $a = 0.15$m $\Rightarrow F = 6.72 \times 10^{-5}$ N

3) With this geometry we find $\vec{B}_A = \vec{B}_D$, $\vec{B}_B = \vec{B}_C$ and all contributions of equal magnitude with directions along the diagonals as shown.



The net field is then

$$B = (4\cos 45^\circ)\ \frac{\mu_0}{4\pi}\ \frac{2I}{r}$$

with $\frac{l}{2} = r\cos 45^\circ$. Therefore, the net downward field is

$$B = \frac{\mu_0}{4\pi}\ \frac{8I}{l}$$

my numbers: $I = 4.5\ A$, $l = 0.2\ m$ $\Rightarrow B = 18\ \mu T$

4) The field within a tightly wound toroid is obtained using Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 N I = 2\pi r B \Rightarrow B = \frac{\mu_0 N I}{2\pi r}$.

my numbers: $N = 950$, $I = 16\ kA$

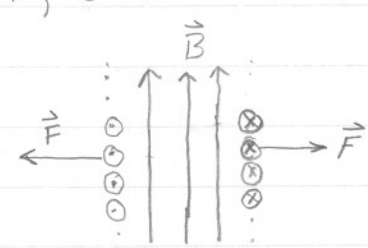
a) $r_1 = 0.7\,m \Rightarrow B = 4.34\,T$

b) $r_2 = 1.3\,m \Rightarrow B = 2.34\,T$

5) The field within a tightly-wound solenoid is simply $B = \mu_0 n I$ where n is the number of turns per unit length

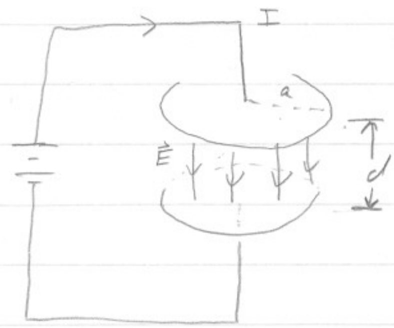
a) $I = \frac{B}{\mu_0 n}$ my numbers: $n = 2090\ m^{-1}$, $B = 15\,T \Rightarrow I = 5.71\ kA$

b) The outward magnetic force/length is then rather large:



$$\frac{F}{l} = IB = 8.57\times 10^4\ \frac{N}{m}$$

6) $E = \frac{Q}{\varepsilon_0 A} \Rightarrow \frac{dE}{dt} = \frac{I}{\varepsilon_0 A}$



Ampere-Maxwell $\Rightarrow 2\pi r B = \mu_0 I \left(\frac{r}{a}\right)^2$

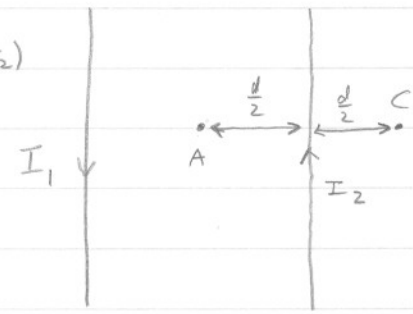
my numbers: $I = 0.6\,A$, $a = 0.14\,m$, $d = 0.004\,m$

a) $\frac{dE}{dt} = 1.10 \times 10^{12} \frac{V}{m\cdot s}$

b) $r = 5\,cm \Rightarrow B = 2.67 \times 10^{-7}\,T$

7) $B_C = 0 \Rightarrow I_1 = 3 I_2$ (3x further than I_2)

Both contributions to field at A are out of page. The magnitude is

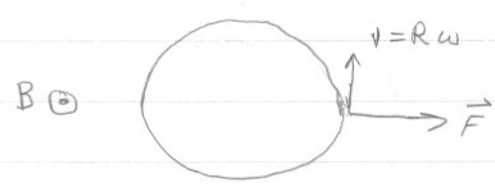


$$B = \frac{\mu_0}{4\pi}\left(\frac{2I_1}{d/2} + \frac{2I_2}{d/2}\right) = \frac{\mu_0}{4\pi}\frac{16 I_2}{d}$$

my numbers: $I_2 = 10\,A$, $d = 28\,cm \Rightarrow$ a) $I_1 = 30\,A$

b) $B = 57.1\,\mu T$ out of page

8) The Lorentz force on charge q is radially outward with magnitude $F = qBR\omega$ at the edge of the disk.



Thus, the potential difference between the center and the edge is $\mathcal{E} = \frac{1}{2}BR^2\omega$. If the field is in the same direction as the angular velocity, the rim is at higher potential than the axis.

$$\left.\begin{aligned} q\,\mathcal{E} &= \int_0^R d\vec{r}\cdot\vec{F} \\ \vec{F} &= qB\omega\,\vec{r} \end{aligned}\right\} \quad \mathcal{E} = \frac{1}{2}BR^2\omega$$