Solutions to hw 12

1) According to the Bohr model for hydrogen, the wavelength emitted in a transition between levels n and m is given by

$$\lambda_{nm}^{-1} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \qquad R_H = 109.737 \text{ nm}^{-1}$$

$$E_{nm} = \frac{hc}{\lambda_{nm}} = E_0 \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \qquad E_0 = 13.6 \text{ eV}$$

The shortest wavelength and highest energy for a series with fixed n is then obtained with $m \to \infty$, such that

$$\min \lambda_n = \frac{n^2}{R_H}, \quad \max E_n = \frac{E_0}{n^2}$$

	n	$\min \lambda_n$	$\max E_n$
Lyman	1	91.1 nm	13.6 eV
Balmer	2	365	3.40
Paschen	3	820	1.51
Bracket	4	1458	0.85

2) $E_{nm} = E_0 \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$ emitted for $m \to n$ or absorbed for $n \to m$

A) $(m, n) = (3, 5) \Rightarrow E_{nm} = -0.071\, E_0$ gains the most energy

B) $(m, n) = (5, 3) \Rightarrow E_{nm} = +0.071\, E_0$ emits shortest wavelength

c) $(m, n) = (8, 4) \Rightarrow E_{nm} = +0.047\, E_0$

d) $(m, n) = (4, 8) \Rightarrow E_{nm} = -0.047\, E_0$

loses energy for B, C ; gains for A, D

3) The kinetic energy for each atom is $K = \frac{hc}{\lambda} = 10.2$ eV for $\lambda = 121.6$ nm. Because $K \ll mc^2$, we can use the nonrelativistic kinetic energy to find $\beta = (2K/mc^2)^{1/2} = 1.47 \times 10^{-4}$ where $mc^2 = 938$ MeV for hydrogen. $\therefore\ v = \beta c = 4.42 \times 10$ m/s.

4) The Bohr radius for a single-electron atom or ion is given by $a = a_0/Z$ where $a_0 = 0.0529\,nm$ and where Z is the nucleon charge in units of e.

a) $He^+ \Rightarrow Z=2 \Rightarrow a = 0.0265\,nm$

b) $Li^{2+} \Rightarrow Z=3 \Rightarrow a = 0.0176\,nm$

c) $Be^{3+} \Rightarrow Z=4 \Rightarrow a = 0.0132\,nm$

5) If an electron is confined to diameter D with $\lambda \le D$, it must have momentum $p = h/\lambda \ge h/D$. For this problem D is so small and p so large that we should use relativistic kinematics with $\gamma \gg 1 \Rightarrow K \approx pc \ge hc/D$. Thus, $D \sim 10^{-5}\,nm \Rightarrow K \gtrsim 1.2\times10^{8}\,eV$. The electric potential energy is of order $U \sim -\frac{Ze^2}{4\pi\varepsilon_0} D^{-1} \sim -0.14\,MeV$ for $Z=1$. Hence, $K \gg -U$ and we would expect the electron to escape almost immediately; it cannot be bound within a nucleus.

6) my numbers: $m = 60\,kg$, $w = 0.81\,m$ width, $d = 12\,cm$ thickness

a) $w < 10\lambda$, $\lambda = h/mv \Rightarrow v < 10\,h/mw = 1.36\times10^{-34}\,m/s$

b) $t = d/v = 8.8\times10^{32}\,s = 2.2\times10^{15}$ times age of Universe

c) not in your lifetime!

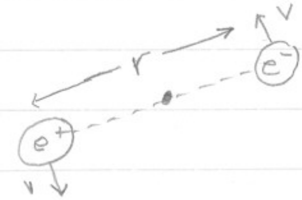
7) Assume that nonrelativistic kinematics is adequate, such that $K_{max} = (eBR)^2/2m_e$ and $\phi = K_{max} - hc/\lambda$.

my numbers: $\lambda = 328\,nm$, $B = 2.10\times10^{-5}\,T$, $R = 0.2\,m$

$K_{max} = 1.55\,eV$, $E_\gamma = 3.78\,eV \Longrightarrow \phi = 2.23\,eV$

Note that $K_{max} \ll m_e c^2 \Rightarrow$ nonrelativistic kinematics are fine.

67) Assume that nonrelativistic kinematics can be used.



energy: $E = 2\left(\frac{1}{2} m v^2\right) - \frac{k_E e^2}{r}$

angular momentum: $L = 2mv\left(\frac{r}{2}\right) = mvr = n\hbar$

circular motion: $\frac{mv^2}{r/2} = \frac{k_E e^2}{r^2} \Rightarrow mv^2 = \frac{1}{2}\frac{k_E e^2}{r} \Rightarrow E = -\frac{1}{2}\frac{k_E e^2}{r}$

$$m\left(\frac{n\hbar}{mr}\right)^2 = \frac{1}{2}\frac{k_E e^2}{r} \Rightarrow r = 2\frac{(n\hbar)^2}{k_E m e^2}$$

$$\therefore \quad E_n = -\frac{E_0}{n^2}, \quad E_0 = \frac{\alpha^2 m_e c^2}{4} = 6.80\,eV$$

$$r_n = n^2 a_0, \quad a_0 = 2\frac{\hbar c}{\alpha m_e c^2} = 0.106\text{ nm}$$

9) Bohr $\Rightarrow E_n = -E_0 n^{-2} \Rightarrow E_n - E_{n-1} = E_0 \frac{2n-1}{n^2(n-1)^2} = \Delta E_n$

$$n \to \infty \Rightarrow \Delta E_n \approx E_0 \frac{2n-1}{n^4} \approx 2E_0 n^{-3}$$

classically: $f = \frac{v}{2\pi r}$ with $v = \frac{n\hbar}{m_e r}$, $r = n^2 a_0 \Rightarrow f = \frac{\hbar}{2\pi m_e a_0^2} n^{-3}$

$E_0 = \frac{\alpha^2 m_e c^2}{2}$, $a_0 = \frac{\hbar c}{\alpha m_e c^2} \Rightarrow \Delta E_n = \alpha^2 m_e c^2 n^{-3}$, $f = h^{-1}\alpha^2 m_e c^2 n^{-3}$

(note $\alpha = k_E e^2/\hbar c$)

$$\therefore \Delta E_n = hf$$

The classical orbital frequency for large n agrees with quantum mechanical transition energy, divided by h, for adjacent orbits.

Can a free stationary particle absorb a photon without creating one or more additional particles in the final state? Energy and momentum conservation require

$$E' = E_\gamma + mc^2$$
$$p' = p_\gamma$$

but the invariance of rest mass requires

$$(mc^2)^2 = E'^2 - (p'c)^2 = (E_\gamma + mc^2)^2 - p_\gamma^2 c^2$$

Finally, using $E_\gamma = p_\gamma c$ we find that this equation requires $E_\gamma = 0$. Therefore, the process is impossible.