Solutions to hw 11

1) One dimension of the cube appears to be Lorentz contracted.
my numbers: $V = 3\,cm^3$, $m = 10\,g$, $\beta = 0.95$
$\gamma = 3.2 \Rightarrow \rho = \gamma m / V = 10.7\ g/cm^3$

2) Proton with $K = 28\,GeV$, $mc^2 = 938.272\,MeV$
$(mc^2)^2 = E^2 - (pc)^2 = (K + mc^2)^2 - (pc)^2 \Rightarrow pc = \sqrt{K(K + 2mc^2)}$
$\therefore\ pc = 28.92\ GeV \Rightarrow p = 1.54 \times 10^{-17}\ kg\ m/s$
$\gamma = 1 + \frac{K}{mc^2} = 30.84 \Rightarrow \beta = 0.99947 \Rightarrow v = 2.9963 \times 10^8\ m/s$

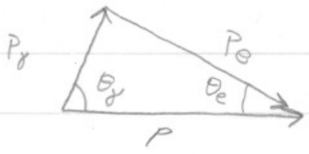
3) $\Delta mc^2 = E = Pt\varepsilon$ where P is rated power, ε = fraction of capacity, t = time of operation.
my numbers: $P = 1\ GW$, $\varepsilon = 0.74$, $t = 4\,yr \Rightarrow \Delta m = 1.04\ kg$

4) $n = P/h\nu$ where n = # photons/sec, P is power, $h\nu$ is energy per photon. my numbers: $P = 110\ kW$, $\nu = 95.5\ MHz \Rightarrow n = 1.74 \times 10^{30}\ s^{-1}$

5) $K_{max} = hf - \phi$ with $\phi = 2.30\,eV$ (Lithium), $3.90\,eV$ (beryllium), $4.50\,eV$ (mercury).
my numbers: $\lambda = 288\,nm \Rightarrow hf = 4.30\ eV$
lithium: $K_{max} = 2.0\,eV$
beryllium: $K_{max} = 0.40\,eV$
mercury: no photoelectric effect

6) Let ϕ_i for $i = 1, 2$ represent the work functions for two metals.
Given $\phi_1 = \phi_2 - a$, $\phi_1 = (1 - b)\phi_2$ we find
$\phi_2 = a/b$, $\phi_1 = \frac{a}{b} - a = a(1-b)/b$
my numbers: $a = 2.56\,eV$, $b = 0.545 \Rightarrow \phi_1 = 2.14\,eV$, $\phi_2 = 4.7\,eV$

7) Energy and momentum conservation are expressed by



$$p = p_\gamma \cos\theta_\gamma + p_e \cos\theta_e$$

$$p_\gamma \sin\theta_\gamma = p_e \sin\theta_e$$

$$p_e^2 = (p + \mu - p_\gamma)^2 - \mu^2$$

where it is convenient to define $\mu = m_e c$ and to express momenta in units of MeV/c. Note that it is important to include the mass of the electron in the energy balance for both initial and final states. Eliminating θ_e from (a) and (b), we obtain

$$p_e^2 = p^2 + p_\gamma^2 - 2pp_\gamma \cos\theta_\gamma$$

which could have been obtained directly from the momentum triangle using the law of cosines. Combining this with (c), we next obtain

$$\mu(p - p_\gamma) = pp_\gamma(1 - \cos\theta_\gamma) = 2pp_\gamma \sin^2\frac{\theta_\gamma}{2}$$

For this problem we are given the rather special condition

$$\theta_\gamma = 2\theta_e \implies \mu(p - p_\gamma) = 2pp_\gamma \sin^2\theta_e$$

$$2p_\gamma \cos\theta_e = p_e$$

where the second relationship is obtained using $\sin 2\theta_e = 2\sin\theta_e\cos\theta_e$ in (b). Thus, we can form two equations for $\sin^2\theta_e$

$$\sin^2\theta_e = 1 - \left(\frac{p_e}{2p_\gamma}\right)^2 = \frac{\mu(p-p_\gamma)}{2pp_\gamma}$$

such that

$$p_e^2 = 4p_\gamma^2\left(1+\frac{\mu}{2p}\right) - 2\mu p_\gamma$$

can be used with (c) to finally obtain

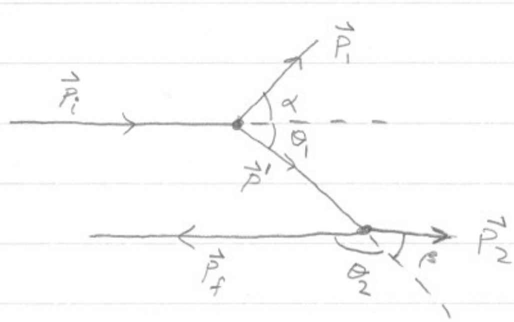
$$A p_\gamma^2 + B p_\gamma + C = 0 \quad \text{with} \quad A = 3 + \frac{2\mu}{p}, \; B = 2p, \; C = -p(p+2\mu)$$

Although one could easily write an algebraic solution for this equation, the condition $\theta_\gamma = 2\theta_e$ is too specific for such an expression to be of much interest in itself. Therefore, we settle for numerical evaluation.

$$\left.\begin{aligned} p_\gamma &= 0.76 \text{ MeV/c} \\ \mu &= 0.511 \text{ MeV/c} \end{aligned}\right\} \Rightarrow \quad A = 4.3447 \quad B = 1.52 \quad C = -1.3543$$

$$\begin{aligned}
\therefore\ p_\gamma &= 0.4102 \text{ MeV/c} \\
E_e &= (p+\mu-p_\gamma)c = 0.8609 \text{ MeV} \\
\gamma &= E_e/\mu c = 1.685 \\
\beta &= \left(1-\frac{1}{\gamma^2}\right)^{-1/2} = 0.805 \\
p_e &= 0.6928 \text{ MeV/c} \\
\theta_e &= \cos^{-1}(p_e/2p_\gamma) = 32.4^\circ
\end{aligned}$$

8) We apply the Compton formula twice, using $\theta_2 = \pi - \theta_1$ to obtain $\vec{p}_f$ antiparallel to $\vec{p}_i$. Thus, $\cos\theta_2 = -\cos\theta_1$



$$\lambda' - \lambda_i = \frac{h}{m_e c}(1-\cos\theta_1)$$

$$\lambda_f - \lambda' = \frac{h}{m_e c}(1+\cos\theta_1)$$

$$\lambda_f - \lambda_i = 2\frac{h}{m_e c} = 0.00485 \text{ nm}$$