Solutions to hw 10

1) The interval between heartbeats is dilated by the factor $\gamma = (1-(\frac{v}{c})^2)^{-1/2}$ relative to the rest frame (propertime). Hence, the rate is multiplied by γ^{-1} where v is the relative velocity.

a) $v = 0.48c$, $R_0 = 74/\text{min} \Rightarrow R = 64.9/\text{min}$

b) $v = 0.98c \Rightarrow R = 14.7/\text{min}$

2) Let L_1 and L_2 represent proper lengths.

$$\frac{L_1}{\gamma_1} = \frac{L_2}{\gamma_2} \Rightarrow \left(1-\left(\frac{v_1}{c}\right)^2\right)L_1^2 = \left(1-\left(\frac{v_2}{c}\right)^2\right)L_2^2 \Rightarrow \left(\frac{v_2}{c}\right)^2 = 1 - \left(1-\left(\frac{v_1}{c}\right)^2\right)\left(\frac{L_1}{L_2}\right)^2$$

my numbers: $\frac{L_1}{L_2} = \frac{1}{4}$, $\frac{v_1}{c} = 0.55 \Rightarrow \frac{v_2}{c} = 0.978$

3) $d = v\Delta t = \gamma v \tau$ where τ is the proper lifetime and $\gamma = (1-(v/c)^2)^{-1/2}$ is the time dilation factor. Let $\beta = v/c$.

$$d^2 = \frac{(\beta c\tau)^2}{1-\beta^2} \Rightarrow \beta = \frac{d}{\sqrt{d^2+(c\tau)^2}}$$

my numbers: $d = 10\,m$, $\tau = 26\,ns \Rightarrow \beta = 0.789$

4) We must use relativistic velocity addition

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

where u_x is the velocity of the rocket relative to earth, v is the velocity of the spaceship relative to earth, and u_x' is the velocity of the rocket relative to the spaceship, all moving in the $\hat{x}$ direction.

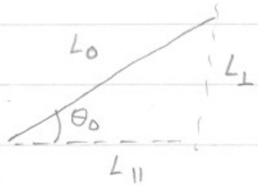
my numbers: $u_x = 0.95c$, $v = 0.6c \Rightarrow u_x' = 0.814c$

5) The parallel component of the rods length is contracted by a factor γ^{-1} relative to the rest frame, while the perpendicular component is unaffected.

$$L'_{\parallel} = L_{\parallel}/\gamma \,, \qquad L'_{\perp} = L_{\perp} = L_0 \sin\theta_0$$

$$\tan\theta' = \frac{L'_{\perp}}{L'_{\parallel}} = \gamma \frac{L_{\perp}}{L_{\parallel}} = \gamma \tan\theta_0$$

$$L_0 = L \frac{\sin\theta'}{\sin\theta_0}$$



my numbers: $\theta' = 30^\circ$, $\frac{v}{c} = 0.945 \Rightarrow \gamma = 3.057$, $L' = 2\,m$

$$\therefore \theta_0 = 10.7^\circ \,, \quad L_0 = 5.39\,m$$

6) We are given two events in S with spacetime coordinates (x_R, t_R) and (x_B, t_B) and are told that $x'_R = x'_B$ in S'. Note that the origins coincide at $t = t' = 0$. Then

$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

my numbers: $x_R = 3m$, $x_B = 4m$, $t_R = 1\,ns$, $t_B = 15\,ns$

a) $x'_R = x'_B \Rightarrow v = \frac{x_B - x_R}{t_B - t_R} = 7.14 \times 10^7\,m/s = 0.238c$, $\gamma = 1.03$

b) $x' = \gamma(x_R - vt_R) = 3.02\,m$

c) $t'_R = \gamma(t_R - vx_R/c^2) = -1.42\,ns$

8) Conservation of momentum requires $\vec{p}_2 = -\vec{P}_1$, where the magnitudes are $p_i = \gamma_i m_i v_i$ with $\gamma_i = (1-\beta_i^2)^{-1/2}$, $\beta_i = v_i/c$ for $i = 1, 2$.

my numbers: $m_1 = 3\times10^{-28}\,kg$, $\beta_1 = 0.853 \Rightarrow P_1 = 1.47\times10^{-19}\,kg\,m/s = P_2$

Solving for β_2^2 with $m_2 = 1.47\times10^{-27}\,kg$:

$$\beta_2^2 = \frac{P_2^2}{(m_2 c)^2 + P_2^2} \Rightarrow \beta_2 = 0.316$$

7) Relativistically, $p = \gamma m u \Rightarrow$ fractional error in nonrelativistic formula is $f = (\gamma - 1)/\gamma$ where $\gamma = \left(1 - \left(\frac{u}{c}\right)^2\right)^{-1/2}$. Solving for $\frac{u}{c}$ in terms of f we find

$$f = \frac{\gamma - 1}{\gamma} \Rightarrow \left(\frac{u}{c}\right)^2 = 1 - (1-f)^2$$

a) $f = 0.05 \Rightarrow \frac{u}{c} = 0.312$

b) $f = 0.19 \Rightarrow \frac{u}{c} = 0.586$

) The total energy is $E = K + mc^2$ where K is the kinetic energy and m is the rest mass. Alternatively, $E = \gamma mc^2$ where $\gamma = (1-\beta^2)^{-1/2}$ can be used to find $\beta = v/c$

a) $K = 3m_e c^2$, $m_e c^2 = 0.511$ MeV $\Rightarrow E = 4 m_e c^2 = 2.04$ MeV

b) $\gamma = 4 \Rightarrow \beta = 0.968$