

**Physics 263: Electromagnetism and Modern Physics**

Sections 0101- 0105

**Exam 3**

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**Instructions:**

- This is a closed book, closed notes exam to be completed in 50 minutes. You may use a basic scientific calculator, but no other aids are permitted. The final page provides a brief compilation of *possibly useful information*, including relevant physical constants.
- Work each problem in the space provided. If additional space is needed, use the back of the *previous* page and indicate that you have done so.
- Please write your name on each page, including this one. Do not use red ink.
- **Choose 4 out of 5 problems.** Use an X in the table below to indicate which problem to omit. If there is no X, we will grade the first four.
- **Explain your reasoning and show your work.** Partial credit will be awarded when the relevant physical principles are applied even if mistakes are made in execution of the steps. However, correct guesses without any explanation may be penalized.
- **Algebraic answers must have consistent dimensions and numerical answers must have consistent units.**
- **Honor pledge:** please copy and sign "I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

**Pledge:**

Signature: \_\_\_\_\_

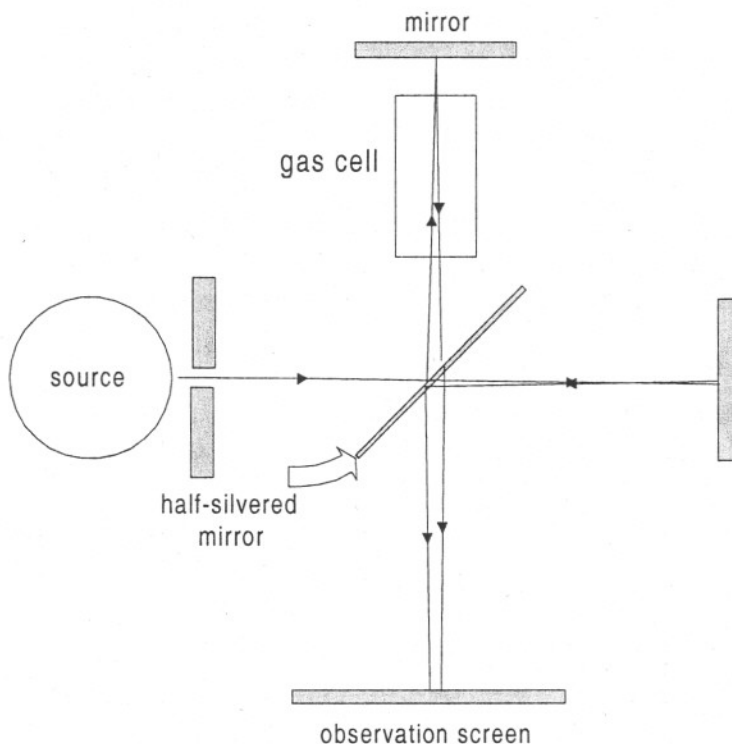
Student ID number: \_\_\_\_\_

Printed name: Solutions

Section: \_\_\_\_\_

| Problem | Score |
|---------|-------|
| 1       |       |
| 2       |       |
| 3       |       |
| 4       |       |
| 5       |       |
| total   |       |

1. A Michelson interferometer contains an evacuated cell of length  $\ell$  in one arm, as shown. A beam of monochromatic light with wavelength  $\lambda_0$  in vacuum is split by the half-silvered mirror, such that one beam passes through the cell twice before being recombined with a second beam that traverses a similar path in air. If a gas is slowly admitted into the cell,  $N$  fringes are observed to move past a fixed point on the observation screen. Find an expression for the index of refraction of the gas,  $n_{\text{gas}}$ , at its final pressure. Explain your reasoning. (25 pts)



We only need to consider how much the phase difference for the portion of the path within the cell changes when gas is introduced.

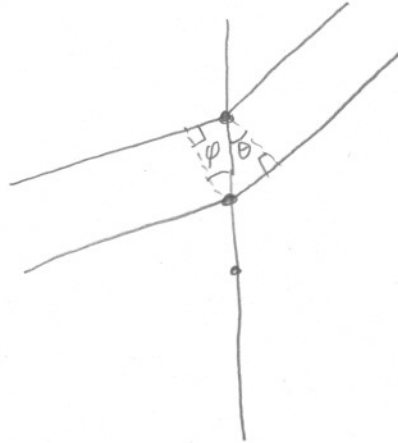
$$\frac{2\pi}{\lambda_0} (n-1) 2\ell = 2\pi N \Rightarrow n = 1 + \frac{N \lambda_0}{2\ell}$$

2a) Light with wavelength  $\lambda$  strikes a diffraction grating with spacing  $a$  at an angle  $\phi$  from perpendicular. Find an expression for the angle  $\theta$  for constructive interference of order  $m$ .

Evaluate pathlength differences before and after grating: (15 pts)

$$a \sin \phi - a \sin \theta = m \lambda$$

$$\therefore \sin \theta = \frac{m \lambda}{a} - \sin \phi$$



2b) An American standard television picture (before HDTV) contains approximately 485 horizontal lines of varying intensity. Estimate the ratio between minimum viewing distance and vertical picture size for which these lines cannot be resolved by a typical viewer. Explain your reasoning. [Hint: estimate the pupil diameter and use  $\lambda \sim 550 \text{ nm}$  for visible light.] (10 pts)

Let  $a = h/N$  be spacing between lines for picture height  $h$  and let  $L$  be viewing distance.

Estimate  $d \sim 3 \text{ mm}$  as pupil diameter.

diffraction  $\Rightarrow \Delta \theta \sim \frac{\lambda}{d}$  is resolution

angular separation between lines  $\Rightarrow \Delta \theta \sim \frac{a}{L} = \frac{h}{NL}$

$$\therefore \frac{L}{h} = \frac{d}{N \lambda} \sim 11$$

3. A negative pion at rest decays to a muon and a muonic antineutrino; the reaction is written as  $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ . Compute the ~~pion~~ <sup>muon</sup> and antineutrino kinetic energies in MeV given  $m_\pi = 270m_e$ ,  $m_\mu = 206m_e$ , and  $m_\nu \approx 0$ . (25 pts)

Conservation of energy + momentum:  $E_\mu + E_\nu = m_\pi c^2$   
 $p_\mu = p_\nu = E_\nu / c$

invariant mass of muon:  $(p_\mu c)^2 = E_\mu^2 - (m_\mu c^2)^2$

substitute:  $(m_\pi c^2 - E_\mu)^2 = E_\mu^2 - (m_\mu c^2)^2$

$$\therefore E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} c^2 = 213.59 m_e c^2$$

$$K_\mu = E_\mu - m_\mu c^2 = 7.59 m_e c^2 = 3.88 \text{ MeV}$$

$$E_\nu = p_\nu c = p_\mu c = 5.64 m_e c^2 = 28.8 \text{ MeV}$$

Notice that  $\bar{\nu}$  carries most of the kinetic energy — zero mass does not imply zero energy.

4. By analyzing the scattering of sunlight, Thomson estimated the classical radius of the electron to be 2.82 fm where 1 femtometer is  $10^{-15}$  m. Suppose that an electron is illuminated by sunlight with an intensity of  $500 \text{ W/m}^2$  and that all light within a disk of this radius is completely absorbed.

a) How long would it take for the electron to acquire an energy of 1 eV? How does this compare with the observation that photoelectrons are emitted promptly when light of sufficiently short wavelength illuminates a metal, even if the intensity is small? (15 pts)

$$\frac{\Delta E}{\Delta t} = I \pi r^2 \Rightarrow \Delta t = \frac{\Delta E}{I \pi r^2} = 1.28 \times 10^7 \text{ s} = 148 \text{ days}$$

The classical model offers an absurdly long estimate of the time delay for the photoelectric effect.

b) How does the photon hypothesis solve this problem? (10 pts)

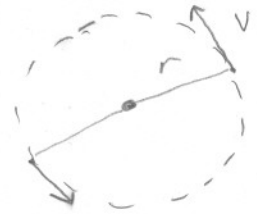
If the frequency is high enough, a single photon carries sufficient energy to eject an electron, which absorbs the entire photon and emerges promptly. The actual delay is probabilistic, depending upon the photon flux, electron density, and the absorption probability (effective cross section).

5. Positronium consists of an ordinary (negatively charged) electron plus a positron (the positively-charged antiparticle of the electron) orbiting about their center of mass as a bound system similar to an atom. Use the Bohr model to deduce the binding energy of positronium. Express your final answer in electron volts. [Hint: the electron and positron have equal masses. In the Bohr model these two particles would be on opposite sides of a diameter through the center of their common circular orbit and the total orbital angular momentum is quantized.]

(25 pts)

$$E = 2\left(\frac{1}{2}m_e v^2\right) - \frac{k_E e^2}{2r}$$

$$\left. \begin{aligned} \frac{m_e v^2}{r} &= \frac{k_E e^2}{(2r)^2} \\ 2m_e v r &= n\hbar \end{aligned} \right\} \begin{aligned} E &= -\frac{k_E e^2}{4r} \\ r &= \frac{(n\hbar)^2}{m k_E e^2} \end{aligned}$$



$$\therefore E_n = -\frac{1}{4} \frac{m_e (k_E e^2)^2}{(n\hbar)^2} = -\frac{1}{4} m_e c^2 \frac{\alpha^2}{n^2} \Rightarrow E_B = 6.8 \text{ eV}$$

Note that  $E_B = -E_1$  is the energy needed to separate the particles from the ground state to infinite distance without kinetic energy.

Possibly useful information

$$\oint d\vec{a} \cdot \vec{E} = \frac{Q}{\epsilon_0}$$

$$\oint d\vec{s} \cdot \vec{E} = -\frac{d\Phi_B}{dt}$$

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

$$\text{static: } \vec{E} = \frac{1}{4\pi\epsilon_0} \int dq \frac{\hat{r}}{r^2}$$

$$u = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\text{single slit: } I(\theta) = I_0 \left( \frac{\sin \alpha}{\alpha} \right)^2$$

$$\alpha = \frac{\pi a}{\lambda} \sin \theta$$

$$x' = \gamma(x - ut)$$

$$t' = \gamma\left(t - \frac{ux}{c^2}\right)$$

$$\gamma = \left(1 - \left(\frac{u}{c}\right)^2\right)^{-1/2}$$

$$-\frac{\hbar^2}{2m} \psi''(x) + V(x)\psi(x) = E\psi(x)$$

$$\oint d\vec{a} \cdot \vec{B} = 0$$

$$\oint d\vec{s} \cdot \vec{B} = \mu_0 \left( I + \epsilon_0 \frac{d\Phi_E}{dt} \right)$$

$$\vec{B} = \mu \vec{H} = \mu_0 (\vec{H} + \vec{M})$$

$$\text{static: } \vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{s} \times \hat{r}}{r^2}$$

$$\vec{S} = \vec{E} \times \vec{B} / \mu_0$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$\text{grating: } I(\theta) = I_1 \left( \frac{\sin N\beta}{\sin \beta} \right)^2$$

$$\beta = \frac{\pi d}{\lambda} \sin \theta$$

$$E = \gamma mc^2$$

$$\vec{p} = \gamma m \vec{v}$$

$$(mc^2)^2 = E^2 - (pc)^2$$

$$c^2 = \frac{1}{\epsilon_0 \mu_0}$$

$$\epsilon_0 = 8.8542 \times 10^{-12} \frac{F}{m}$$

$$e = 1.6022 \times 10^{-19} C$$

$$m_e = 9.109 \times 10^{-31} kg = 0.511 MeV / c^2$$

$$\mu_B = 9.27 \times 10^{-24} \frac{J}{T}$$

$$h = 6.626 \times 10^{-34} J \cdot s$$

$$\hbar c = 197.3 eV \cdot nm$$

$$c = 2.9979 \times 10^8 \frac{m}{s}$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$$

$$N_A = 6.022 \times 10^{23} mole^{-1}$$

$$m_p = 1.672 \times 10^{-27} kg = 938.27 MeV / c^2$$

$$\mu_N = 5.05 \times 10^{-27} \frac{J}{T}$$

$$\hbar = 1.055 \times 10^{-34} J \cdot s$$

$$\frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}$$

### Dimensions and SI Units for Basic Electromagnetic Quantities

| Quantity           | Unit       | Abbreviation | Conversions    | Dimensions           |
|--------------------|------------|--------------|----------------|----------------------|
| Charge             | Coulomb    | <i>C</i>     |                | <i>Q</i>             |
| Electric potential | Volt       | <i>V</i>     | <i>J/C</i>     | $ML^2T^{-2}Q^{-1}$   |
| Electric field     | Volt/meter | <i>V/m</i>   | <i>N/C</i>     | $MLT^{-2}Q^{-1}$     |
| Capacitance        | Farad      | <i>F</i>     | <i>C/V</i>     | $M^{-1}L^{-2}T^2Q^2$ |
| Current            | Ampere     | <i>A</i>     | <i>C/s</i>     | $QT^{-1}$            |
| Magnetic field     | Tesla      | <i>T</i>     | <i>N/(A m)</i> | $MT^{-1}Q^{-1}$      |
| Magnetic flux      | Weber      | <i>Wb</i>    | <i>J/A</i>     | $ML^2T^{-1}Q^{-1}$   |
| Inductance         | Henry      | <i>H</i>     | <i>J/A^2</i>   | $ML^2Q^{-2}$         |
| Resistance         | ohm        | $\Omega$     | <i>V/A</i>     | $ML^2T^{-1}Q^{-2}$   |
|                    |            |              |                |                      |