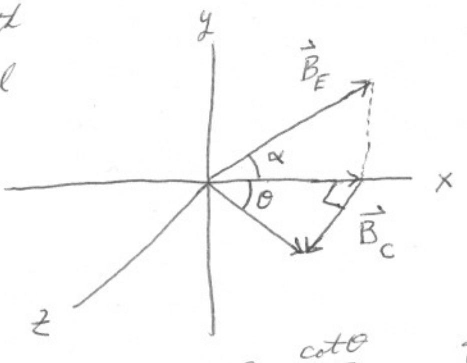
1. Using a *dip needle*, it is determined that the Earth's magnetic field makes an angle $\alpha$ with respect to the horizontal plane. Then using a horizontal compass, a circular coil with $N$ turns and radius $R$ is aligned with its plane vertical and its axis perpendicular to the horizontal component of the Earth's magnetic field. The compass is placed at the center of the coil. Passing a current $I$ through the coil produces a deflection $\theta$ in the direction of the compass needle.

a) Express the strength $B_E$ of the Earth's magnetic field in terms of the current $I$, the angles $\alpha$ and $\theta$, and the properties of the coil. *(10 pts)*

Choose a coordinate system with $\vec{B}_E = B_x \hat{x} + B_y \hat{y}$ where the normal to the coil is in the $\hat{z}$ direction.
Using the dip needle $\Rightarrow \tan\alpha = B_y/B_x$.
The coil adds $\vec{B}_C = B_z \hat{z}$ and deflects compass needle $\Rightarrow \tan\theta = B_z/B_x$.




Then using $B_x = B_E \cos\alpha$, we find $B_E = B_z \frac{\cot\theta}{\cos\alpha}$. Using Biot-Savart, the field at the center of a circular coil is

$$B_z = \frac{\mu_0}{4\pi} NI \oint \frac{R\,d\theta}{R^2} = \frac{\mu_0}{2}\frac{NI}{R}$$

$$\therefore \boxed{B_E = \frac{NI\mu_0}{2R}\frac{\cot\theta}{\cos\alpha}}$$

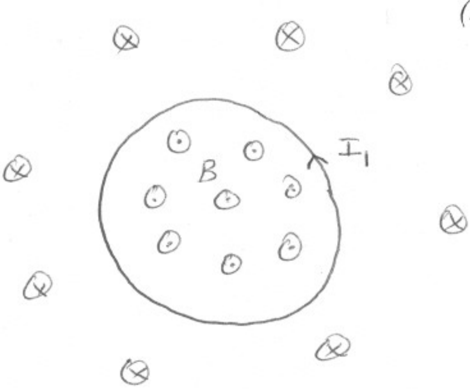
b) Compute $B_E$ assuming $N = 10$, $R = 10\ cm$, $I = 0.3\ A$, $\theta = 45^\circ$, $\alpha = 60^\circ$ *(10 pts)*

$B_E = 0.377\ G$ (note $1\ G = 10^{-4}\ T$)

2. Recall the *can-crusher* demonstration in which the charge stored by a large capacitor is rapidly discharged through a conducting band around the middle of a soda can. Suppose that the current flows in a counterclockwise direction within the plane of this page.

a) Sketch the discharge current and indicate the direction of the magnetic field it produces. *(5 pts)*

The interior field $\vec{B}_1$ produced by discharge current $I_1$ is stronger than the exterior return field. If $I_1$ is ccw, the interior field is out of the page.

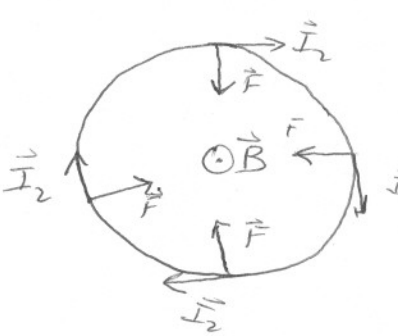



b) In what direction is the current induced within the can? *(5 pts)*

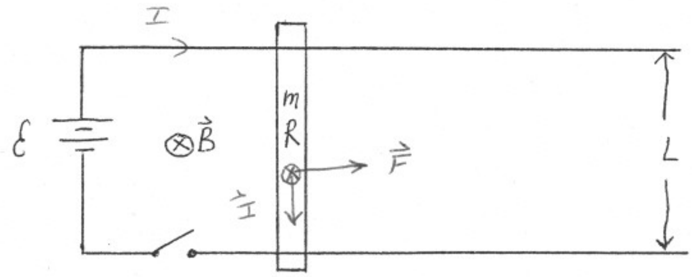
According to Lenz's law, the current $I_2$ induced in the can opposes the growth of $\vec{B}_1$ and hence is clockwise, opposite to $I_1$.

c) Explain why the can is crushed. *(10 pts)*

The net magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2$ at the surface of the can remains outward. The magnetic force $\vec{I}_2 \otimes \vec{B}\, l$ is then radially inward. Rapid $\dot{I}_1 \Rightarrow$ large $I_2 \Rightarrow$ strong force that pinches (crushes) the can under the band. Alternatively, currents $I_1$ and $I_2$ in opposite directions repel each other $\Rightarrow$ force on can is radially inward.



3. A thick metal bar of mass $m$, length $L$, and resistance $R$ slides without friction on a pair of parallel metal rails with negligible resistance. The rod is initially at rest and the switch is closed at $t=0$. The rails are connected to a battery that provides constant electromotive force $\mathcal{E}$. A uniform magnetic field $B$ is directed perpendicular to the plane of the circuit formed by the rod and rails. Show that the velocity of the rod as a function of time takes the form $v = v_\infty(1-e^{-t/\tau})$ and determine $v_\infty$ and $\tau$. *(20 pts)*




Here the magnetic force $F = IBL$ acts to the right. The induced electromotive force $vBL$ opposes the battery, such that

$$I = \frac{\mathcal{E} - vBL}{R}$$

$$m\frac{dv}{dt} = \frac{\mathcal{E} - vBL}{R}BL \Rightarrow \frac{dv}{dt} + \frac{B^2L^2}{mR}v = \frac{\mathcal{E}BL}{mR}$$

Proposed solution: $v = v_\infty(1-e^{-t/\tau})$

$$t\to\infty \Rightarrow v \to v_\infty,\ \frac{dv}{dt}\to 0 \Rightarrow v_\infty = \frac{\mathcal{E}}{BL}$$

$$t\to 0 \Rightarrow v\to 0,\ \frac{dv}{dt}\to\frac{v_\infty}{\tau} \Rightarrow \frac{v_\infty}{\tau} = \frac{\mathcal{E}BL}{mR}$$

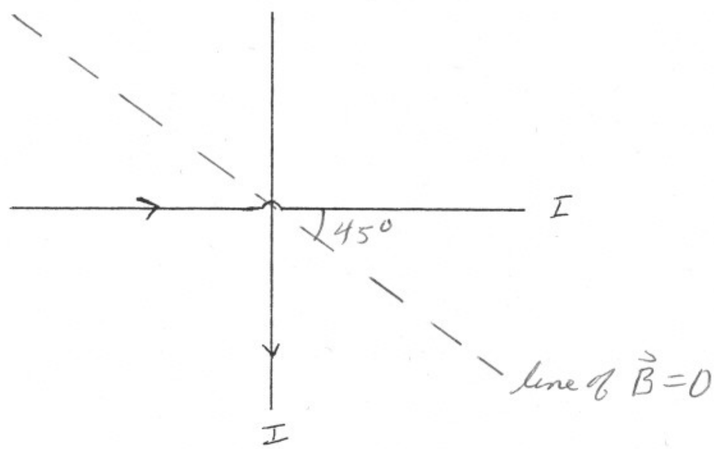
$$\therefore \boxed{v_\infty = \frac{\mathcal{E}}{BL} \qquad \tau = \frac{mR}{B^2L^2}}$$

To check, rewrite equation of motion as $\frac{dv}{dt} + \frac{v}{\tau} = \frac{v_\infty}{\tau}$ and substitute $v = v_\infty(1-e^{-t/\tau}) \Rightarrow \frac{dv}{dt} = \frac{v_\infty}{\tau}e^{-t/\tau}$ to obtain identity.

4a) Two long perpendicular wires carrying equal currents cross, as shown, but do not quite touch. Indicate on the diagram the locus of points at which the net magnetic field vanishes and explain your reasoning. *(10 pts)*

For the fields to cancel, the two contributions must have equal magnitude and opposite directions. If equal currents are ~~on~~ on $\hat{x}$ and $\hat{y}$ axes, equal magnitude $\Rightarrow x^2+z^2 = y^2+z^2 \Rightarrow y = \pm x$.

To have opposite directions, need $y=-x$, $z=0$ as shown.

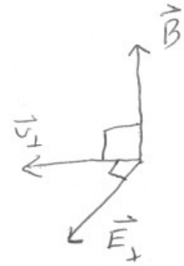



b) A particle with mass *m*, charge *q*, and velocity $\vec{v}$ is subject to uniform static electric and magnetic fields, $\vec{E}$ and $\vec{B}$. (i) Under what conditions, if any, will the particle remain stationary? (ii) Under what conditions will $\vec{v}$ remain constant? *(10 pts)*

$$\vec{F} = q(\vec{E} + \vec{v} \otimes \vec{B}) = 0 \Rightarrow \frac{d\vec{v}}{dt} = 0$$

Constant $\vec{v}$ requires $\vec{E} = -\vec{v} \otimes \vec{B}$. For the particle to remain stationary ($\vec{v}=0$) we must have $\vec{E}=0$, no $\vec{E}$ field. (or $q=0$)

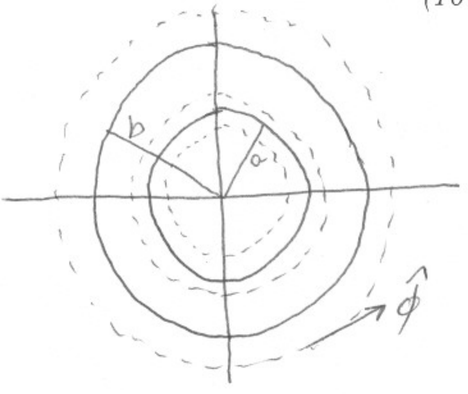
For a moving particle, let $\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$ where $\vec{v}_{\parallel}$ is parallel and $\vec{v}_{\perp}$ is perpendicular to $\vec{B}$. We require $\vec{E}_{\parallel} = 0$ and $\vec{E}_{\perp} = -\vec{v}_{\perp} \otimes \vec{B} \Rightarrow E_{\perp} = v_{\perp} B$ with the directions shown.



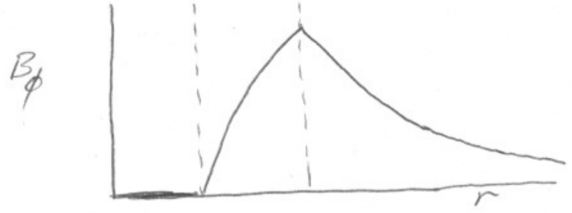
5. A long cylindrical shell with inner radius $a$ and outer radius $b$ carries total current $I$ along its length.

a) Find expressions for the magnetic field in the regions (i) $r < a$, (ii) $a < r < b$, and (iii) $r > b$. Sketch the function $B(r)$. *(10 pts)*

$2\pi r B_\phi = \mu_0 I \text{ (enclosed)}$




$$\frac{2\pi r}{\mu_0 I} B_\phi = \begin{cases} 0, & r < a \\ \dfrac{r^2 - a^2}{b^2 - a^2}, & a < r < b \\ 1, & b < r \end{cases}$$



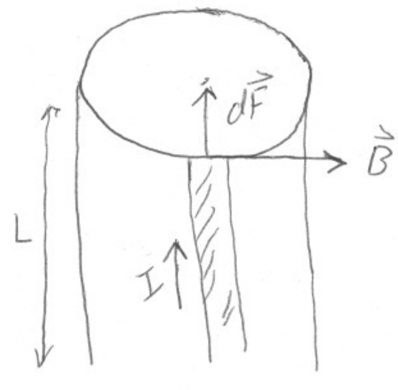

b) Evaluate the pressure assuming that the shell is thin, such that $a \approx b \approx R$. Is the pressure inward or outward? [Hint: consider the force upon a narrow strip of current running the length of the cylinder.] *(5 pts)*

$2\pi R B_\phi = \mu_0 I \Rightarrow B_\phi = \dfrac{\mu_0 I}{2\pi R}$

$dF = \dfrac{\mu_0 I}{2\pi R} I L \dfrac{d\theta}{2\pi} \Rightarrow F = \dfrac{\mu_0 I^2 L}{2\pi R}$

$A = 2\pi R L$ surface area

$\therefore$ inward pressure $P = \mu_0 \left(\dfrac{I}{2\pi R}\right)^2$




c) Compute the pressure for $R = 0.3\ cm$ and $I = 8\ A$. *(5 pts)*

$P = 0.226\ \frac{N}{m^2}$