Quiz 5

Sec 0101

Fig 1

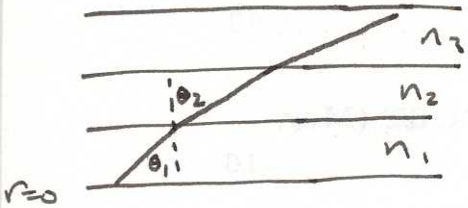

Fig 2

Imagine breaking the wire up into many interfaces with each layer having a different index of refraction. Taking the limit where the thickness of the regions goes to zero while the number of such regions goes to infinity will give a continuum result. (Figure 1).

Figure 2 is a blow-up of figure 1. We want a path like the one shown. Snell's law at the first interface says $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Since we need $\theta_2 > \theta_1$, which implies $\sin\theta_2 > \sin\theta_1$, it must be true that $n_2 < n_1$ in order for equality to hold. Thus $n(r)$ is a decreasing function of r.

Sec 0104

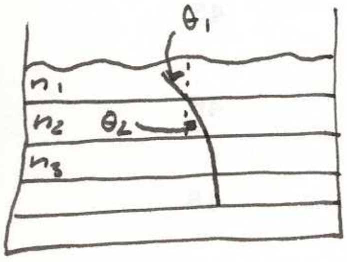

Fig 3

Approximate the solution with continuous index of refraction as many discrete layers of constant index of refraction. You were given that n increases as you go farther down in the tank. Snell's law says $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Since $n_2 > n_1$, it must be that $\sin\theta_2 < \sin\theta_1$, so $\theta_2 < \theta_1$. This is true at each interface, leading to the picture in figure 3.

Sec 0105

Normally the power of a sound wave drops off like $\frac{1}{r^2}$ where r is the distance from the source. This is true because the wave travels out isotropically. If the air over the lake is cooler than the surrounding air, it is more dense, and has a higher index of refraction. Approximate it as many discrete layers with different indices of refraction. At each interface Snell's law says $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (see Fig 4). Since $n_1 > n_2$, $\sin\theta_1 < \sin\theta_2$, so $\theta_1 < \theta_2$. This tends to bend waves toward propagating horizontally, so more power from the wave is diverted toward a listener on the edge of the lake.

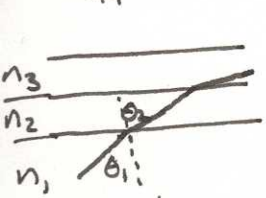

Fig 4