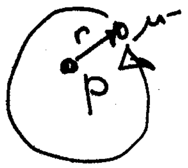
Quiz 10

Sec 0101 & 0104

Here's a full derivation. The proton can be taken to be at rest. (This is a bit of a bad approximation, but good enough for the quiz.)



Then angular momentum is $L_n = m_\mu v r = n\hbar$. (1)

The second equality comes from the demand of quantization suggested by Bohr.

The force on the muon is $\frac{-ke^2}{r^2}$, which should balance against the centripetal force:

$$\frac{ke^2}{r^2} = \frac{m v^2}{r}. \qquad (2)$$

From (1) comes
$$v = \frac{n\hbar}{m_\mu r} \qquad (3)$$

Subs (3) → (2)
$$\frac{ke^2}{r^2} = \frac{n^2\hbar^2}{m_\mu r^3} \qquad (4)$$

From (4) comes
$$r_n = \frac{n^2\hbar^2}{m_\mu k e^2} = \frac{a_0 n^2}{206} \qquad (*)$$

since $m_\mu = 206 m_e$.

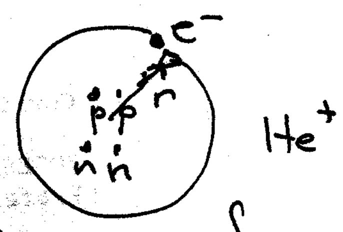
The energy expression is
$$E = K + U = \frac{1}{2} m_\mu v^2 - \frac{ke^2}{r} \qquad (5)$$

From (2)
$$m_\mu v^2 = \frac{ke^2}{r} \qquad (6)$$

(6) → (5), (*)
$$E = \frac{-ke^2}{2r} = \frac{-103 ke^2}{a_0 n^2}$$

As usual, $a_0 = \frac{\hbar^2}{mke^2}$ is the Bohr radius.

Sec 0105



The full derivation goes like this:

Angular momentum of the electron goes like $L_n = mvr = n\hbar$. (1) The second equality comes from the quantization demand of Bohr. The forces on the electron are the Coulomb force balanced by the centripetal force of the circular motion:

$$\frac{Zke^2}{r^2} = \frac{mv^2}{r} \qquad (2).$$

From (1) get $$v = \frac{n\hbar}{mr} \qquad (3)$$

Subs (3) → (2) $$\frac{Zke^2}{r^2} = \frac{n^2\hbar^2}{mr^3} \qquad (4)$$

From (4) get $$r_n = \frac{n^2\hbar^2}{Zmke^2} = \frac{a_0 n^2}{2} \qquad (*)$$

The energy of the system is
$$E = K + U = \frac{1}{2}mv^2 - \frac{Zke^2}{r} \qquad (5)$$

From (2) get $$mv^2 = \frac{Zke^2}{r} \qquad (6)$$

Subs (6) → (5) $$E = \frac{ke^2}{r} - \frac{Zke^2}{r} = \frac{-ke^2}{r} \qquad (7)$$

(*) → (7) $$E_n = \frac{-Zke^2}{n^2 a_0}.$$