

## Experiment

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# V

## Position, Velocity and Acceleration

### I. Purpose:

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To study the position, velocity, and acceleration of a body that is acted on by a constant force.

### II. References (Course Textbooks)

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Serway and Jewett, 6th Edition, Vol. 1, Chapter 2.

### III. Prelab Questions: (To be turned in at the start of lab)

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1. A cart of mass  $m$  rides on an air track which is tilted at an angle  $\theta$  from the horizontal:  
(a) What is the force on the cart? (b) What is the cart's acceleration ?
2. A sonic ranger measures positions  $x = 0.1$  m,  $0.15$  m,  $0.22$  m,  $0.28$  m at times  $t = 0$  sec,  $0.1$  sec,  $0.2$  sec, and  $0.3$  sec, respectively. (a) Find the average velocity between  $t = 0.1$  sec and  $t = 0.2$  sec. (b) Find the average acceleration at time  $t = 0.2$  seconds.

### IV. Equipment:

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Ultrasonic motion detector  
- with LabPro adapter wire  
Motion Detector software\*  
Motion Detector interface  
2-meter stick

Air track with blower  
Air track glider  
- with sonar reflector, 2-50 gm masses, rubber band  
air track step block  
Vernier calipers

\* File location: Motion Detector.mbl

### V. Introduction

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#### Uniformly Accelerated Motion

In this lab, you will use a tilted air track to study the motion of a mass which is acted on by a constant force. Newton's second law says that when a force  $F$  is applied to an object of mass  $M$ , the object obeys

$$F = Ma,$$

where the acceleration is

$$a = F/M.$$

In this lab, the accelerating mass will be a cart which rides on an air track. You will measure the mass with a simple balance. The force will be applied by tilting the air track, so that gravity pulls the mass down the track (see Fig. V.1). If the track is tilted at an angle  $\theta$  from the horizontal and the cart has a mass  $M$ , then the force which is pushing the cart down the track is

$$F = Mg \sin(\theta),$$

where  $g = 9.81 \text{ m/sec}^2$  is the acceleration due to gravity. The acceleration of the cart will thus be

$$a = F/M = M g \sin(\theta)/M = g \sin(\theta). \quad [\text{V.1}]$$

Notice this says that the acceleration of the cart is a constant, which is independent of the time, the mass of the cart, the position of the cart, the velocity of the cart or which way the cart is moving. By tilting the track by different amounts, you can change the force on the cart and its acceleration.

Equation V.1 can be rewritten by noting that

$$a = \frac{d^2x}{dt^2}, \quad [\text{V.2}]$$

where  $x$  is the position of the mass. Substituting these expressions into Equation V.1, one finds

$$m \frac{d^2x}{dt^2} = Mg \sin(\theta). \quad [\text{V.3}]$$

This is an example of a simple, second order, linear differential equation. This is easy to solve because the right hand side is a constant which is independent of  $x$  and  $t$ .

Integrating once with respect to time, one finds

$$\frac{dx}{dt} = g \cdot \sin(\theta) \cdot t + v_0, \quad [\text{V.4}]$$

where  $v_0$  is the velocity at time  $t = 0$ . The important thing to recognize about this equation is that  $dx/dt$  is just the velocity  $v$  of the object. The equation just says that the velocity is a linear function of the time; i.e., if you plot  $v$  versus time it will be a straight line.

Equation V.4 can be integrated once more to yield the position of the object as a function of time:

$$x = \frac{1}{2} g \cdot \sin(\theta) \cdot t^2 + v_0 t + x_0, \quad [\text{V.5}]$$

where  $x_0$  is the position of the object at time  $t=0$ . Notice that Equation V.5 just says that  $x$  is a quadratic function of time; i.e. if you plot the position  $x$  versus the time  $t$  it will be a parabola.

## Measuring the Position: The Sonic Ranger and the LabPro Interface Box.

To measure the position of the cart as it moves, you will use a device called a "sonic ranger", also called a motion detector. The sonic ranger works just like a sonar (see Figure V.1). A loudspeaker emits a brief pulse of sound that reflects off the cart. The reflected sound returns to the speaker, where it is detected by means of a microphone. The farther away the object is, the longer it takes for the sound to be reflected back to the microphone. If it takes  $t$  seconds for the sound to leave the speaker and return, then the object must be a distance of  $x=s/t$  where  $s=330\text{m/s}$  is the speed of sound. In the sonic ranger an electrical circuit measures the time  $t$  it takes for the pulse to return, and the computer then automatically computes the distance  $x$  to the object.

Unlike a ruler, which must be read out by eye, the sonic ranger measures the distance from itself to an object and then transfers this information to a computer interface at a rate of up to several hundred times per second. The data transfer is handled by a small box called the LabPro Interface Box, which allows you to take data with a variety of sensors and transmit that information to almost any computer with a serial port. The LabPro takes data from probes such as the sonic ranger, force probes, temperature probes and photo gates.

To run the LabPro and take position measurements, you will use the software program called **Logger Pro**. The instructions for starting and running the program is given in the following sections.

## Finding the velocity and Acceleration

The output from the sonic ranger will be recorded by Logger Pro as a series of points which will allow you to find the position of the cart  $x_1, x_2, x_3, \dots, x_j \dots$  at times  $t_1, t_2, t_3, \dots, t_j$ .

From these measurements of the cart's position versus time, you can determine the speed and acceleration of the cart. The speed  $v_i$  of the cart at time  $t_i$  can be found from

$$v_i = \frac{x_i - x_{i-1}}{t_i - t_{i-1}}.$$

Notice that this is just the average speed of the cart between times  $t_i$  and  $t_{i-1}$ . Thus from these position measurements, we can obtain the velocity of the cart  $v_2, v_3, v_4, \dots, v_j \dots$  at times  $t_2, t_3, t_4, \dots, t_j$ . Notice that with this definition, we can't find the velocity  $v_1$  at time

$t_1$  (the first measurement time), because we haven't measured the position at times before  $t_1$ ; i.e., there is not a  $t_0$  or  $x_0$  measurement.

Once the velocity is known as a function of position, the acceleration can then be found from:

$$a_i = \frac{v_i - v_{i-1}}{t_i - t_{i-1}} .$$

## VI. Experiment

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### A. Setting up the LabPro

1. The LabPro is the dark green box sitting on your bench with three external wires. Examine the LabPro and ascertain that (a) the “Vernier AC Adapter Class 2 Transformer” is plugged into the power strip and that its output wire is connected to the LabPro’s power port (---); (b) a wire runs from the LabPro serial port 10101 to the computer serial port 10101 #1, and (c) a wire runs from the LabPro digital channel DIG/SONIC 2 to the Ultrasonic Motion Detector (which should already be mounted on the air track).
2. When the LabPro is first powered, all three of its lights (red, yellow and green) will flash once, and then it will proceed to power up. If this is successful (takes about 5 seconds), the LabPro box will produce a sequence of 6 beeps, as it cycles through the three lights. It will do this even if nothing but the power wire is connected. Lights: The yellow light will flash briefly just before the LabPro commences taking data, indicating it is ready to receive data. The green light will glow continuously while the instrument is collecting data. The red light will glow, if there is a malfunction. Pushbuttons: The three pushbuttons on the LabPro box are disabled for the current lab application – it will not improve the experiment to push them.
3. Now you should start the **Logger Pro**, software. (If it has been left running from the previous lab, shut down the Logger Pro program and restart the Logger Pro program.) On the computer desktop, click *Lab Templates*, then click *Distance-Time* MBL (accesses Logger Pro), click on *LabPro*, and then click on *sensor setup*. Check that dig/sonic 2 has been selected (default), and that this matches the actual connection to the LabPro interface, and click “ok.”
4. On the computer monitor you should now have a graph with the y-axis labeled as distance, and the x-axis labeled as time.
5. To set the scale of the axes, you should set the maximum time for the x-axis to be about 15 seconds. (This is done by clicking on the last number of the time axis and changing that number to 15.) In a similar fashion set the range along the y-axis to about 2 meters.

## **B. Setting up the Air track**

1. Turn on the air supply and set the power level between three and four. Check that there is nothing under any of the air track feet. Fine tune the air flow so that it is just enough to enable any glider movement to appear frictionless. **Run the air supply only while you are actually taking data. This will reduce the noise in the lab, will conserve energy and the environment, and also will reduce the build-up of dirt inside the air track which clogs the small holes and leads to degraded performance.**
2. Level the air track. With the air supply turned on, place a glider on the air track. Check that there is virtually no friction that would impede glider motion. Now, stop the glider from moving, then let go without imparting any impulse to the glider. If the glider moves, the air track is not level. Rotate the feet of the air track until it passes this leveling test.
3. Adjust the reflector flag to be aligned perpendicular to direction of glider travel. See that there is a bumper on the cart and that the cart bounces smoothly off the air track end stop.

## **C. Setting up and Calibrating the Ultrasonic Motion Detector**

1. Check that the Ultrasonic Motion Detector (Sonic Ranger) is mounted at the end of the air track with only one foot, **that it is mounted with speaker aperture up (to avoid some spurious sound reflections)**, and that this end of the air track is in front of the computer monitor.
2. Take some preliminary position measurements. Find the **COLLECT** button on the toolbar and click on it. After a delay of 1-2 seconds, you should hear the sonic ranger start sending out clicks, and you should see the results being plotted on your screen. To stop collecting data, just click on the **STOP** button (it replaces the **COLLECT** button while data is being collected.)
3. Now install the step block and send a cart down the track. Check that the cart bounces elastically off the end stop, and that the detector is faithfully recording the cart's position. You may find that the sonic ranger loses the cart when it gets too far away or too close. If this happens:
  - a. Unintended reflections: Sometimes merely making no change and proceeding with another run will inexplicably produce good data. The problems with the previous run may simply have been due to motion of someone or something within the cone of the sonic emitter.
  - b. Operating distance: Observe that the motion detector will not operate at distances of less than about  $\frac{1}{2}$  meter.
  - c. Aiming: Check that the speaker is at the same height as the reflector. Check that the speaker is pointing straight down the air track. With the program running and the cart in motion, try turning the speaker a little bit from one side to the other and see if the range plot improves. Also, tilting the

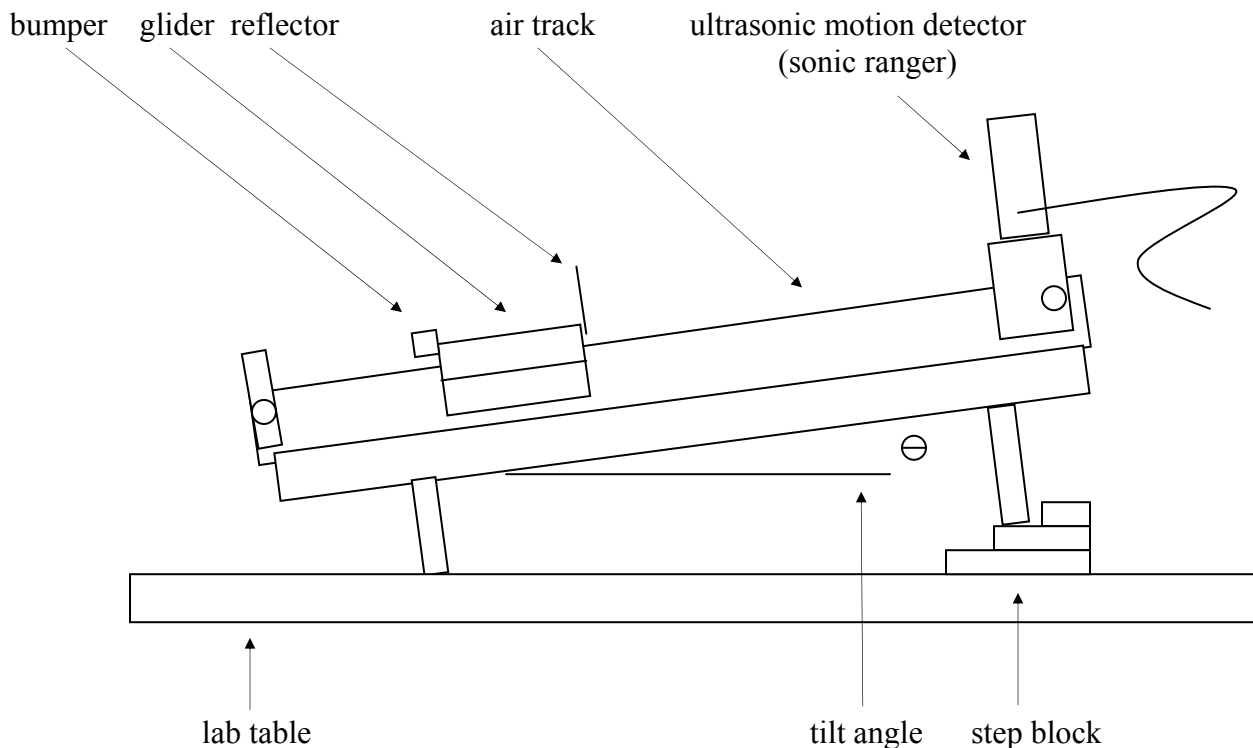
speaker up slightly by placing a penny under the front of the housing may improve performance at greater distances.

4. When you are feeling comfortable operating the apparatus, you can adjust the data sampling rate. To do so, click on *setup*, click on *data collection*, and click on *sampling*. Then observe and record the default setting in the *sampling speed* box. For fun, set this sampling speed at 1 sample/second and observe the results as you take some more preliminary measurements. Try the same thing with settings of 2, 5, and 10 samples/second, while observing the smoothness of the graph obtained. Also listen to the ultrasonic motion detector – can you hear the sampling rate? Which of these plots looks most like the plot you would make if you were doing this experiment without the benefit of the computer? Leave the sampling rate at 10 samples/second unless you TA “dictates” otherwise.
5. At this point, you should check the **accuracy** of the Sonic Ranger (i.e., how close is the cart’s true position to the position which the ranger reports.) To do this, turn off the air supply and set the cart at various precise locations on the track.
  - (a) **Manufacturer’s calibration procedure**: Investigate the automatic (default) system start-up settings. Turn off the Logger Pro program and restart the Logger Pro program. Observe the default settings on the axes, then set the time axis to about 10 seconds and the distance axis to a range of  $-0.5$  m to  $+2.0$  m. Hit collect and move the glider with the blower turned off to a position where the trace exactly follows the zero distance line on the graph. Using the meter stick measure and record the exact distance between the forward-most part of the glider and the front face of the motion detector. Do another run, but this time slowly move the glider toward the motion detector until it no longer records a change in position. Also measure this distance with the meter stick and record it. These tests will reveal to you the minimum distance at which the sonic ranger will operate, and the “safety margin” between that distance and the zero distance point that the manufacturer has used.
  - (b) **Your calibration**: In your experiment you will be interested in distance between various positions of the glider and your chosen zero point on the air track – not the absolute distances between the glider and the sonic ranger. Using the scale on the track, set the leading edge of the cart at the 1 meter mark and then hit the **ZERO button**. This sets the system so that any graph that is plotted will show zero distance at any time that the cart is at the 1 meter position on the air track. Now set the maximum graph time value to 1 second and hit the **collect** button. The graph should be a straight, horizontal line indicating an unchanging, exact zero distance from the zero point that you just established. Look at the columns of times and distances on the screen. The distances should read zero for all times listed. Next, move the cart to the 1.2 meter position on the air track and hit **collect**. Check that the line appears to have been plotted in the proper place on the graph, and check that the column of distances reads, within about 2 %, a constant distance of  $-0.2$  m. Repeat this for 5 additional distances

spanning the entire air track, but keep the cart more than 0.5 m from the ultrasonic motion detector.

### **D. Taking the Data**

1. Now use the vernier caliper to measure the step block provided. Insert the step block under the leg closest to the sonic ranger (single foot). Measure the distance between the center of the single foot and a line between the centers of the two feet on the other end of the air track. Record this data on paper. Ask your TA which step(s) to use when you are taking data.
2. Since you will now be taking final data, it would be helpful to make the graph more intuitive; i.e., as the glider goes uphill against gravity, the graph should be recording increasing, positive values. To do this you will need to access the **reverse direction** command, which will change the calibration so that, as the glider recedes from the motion detector, decreasing distances will be recorded on the graph. To do so, click on *setup*, click on *sensors*, click on *details*, and click on *reverse direction*. (Note: If you try to take data now, you will get no trace independent of what you do with the glider. Why is this? See instructions in paragraph 3 to fix this.)



**Figure V.1. Arrangement for tilted air track and sonic ranger.** The tilt angle,  $\Theta$  in radians is just the height of the step block divided by the distance between the support legs of the air track.

3. Now use the sonic ranger to measure the position of the cart as it goes down and comes back up the inclined air track. For starters try settings of 20 seconds on the time axis, and  $-0.2$  m and 1.5 m on the ends of the distance axis. Set the **zero** position with the front of the glider **at about 30 cm from the ranger**. In order to get a reasonable measurement, start with the cart about 10 cm from the motion detector (Too close will not matter in this case.). Then hit **collect** and start the air supply at the same time. This will coordinate the glider launch and the graph trace to start at nearly the same time. Experiment with different axis values, zero points and launch timing until you get a graph that you like. You will use the data between bounces in your study, so make sure that a complete bounce or two are recorded in your graph.

## VII. Analysis

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1. After you have examined your data on the screen and made sure you have a nice looking parabola. **Export** the data **Text** to a file in the user directory on the **c drive**. (It's a good idea to use your name, or part of it, as the file name so you can find it later.)
2. Prepare your data for export to EXCEL:
  - a. Click *file* while the Logger-Pro screen is still displayed.
  - b. Click *export data*.
  - c. Enter a name for the file.
  - d. Save wherever you usually save (My Documents, P262a\_02, floppy disc, etc.)
  - e. Click *save*.
3. Import your data into Excel:
  - a. Bring up Excel
  - b. Click *file*.
  - c. Click *open*.
  - d. Double click the file name you entered. (This opens the "Text Import Wizard" the defaults of which will already be correct, but make sure that the file is entered as "delimited" and that the delimiting is entered as "tab".)
4. There will be two columns of data as follows:

Time (sec)	Distance (m)
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Insert labels for these columns at the top of your spreadsheet.
5. Use Excel to make a graph of the Position vs. Time. This plot should look almost the same as it did on the Logger Pro display.
6. Add titles and units to the x and y axes and then print out your graph.



7. **The original step 7. has been deleted.** Now compute the velocity at each point and add these values to your spreadsheet. Do this by taking the difference in the distance between steps and dividing by the time difference between steps; i.e.,

$$v_i = \frac{x_i - x_{i-1}}{t_i - t_{i-1}}.$$

8. Plot the velocity versus time. It should be a straight line except at the collisions.
9. Now look at your data and pick an interval of time between collisions where the cart is moving up and down but not hitting the end. Fit the data in this region. Find the slope of the curve of velocity versus time..

This is the acceleration,  $a_{\text{fit}}$ . The acceleration should be:  $a = g \sin(\theta)$ . Compare your results for the acceleration using the angle you compute from the measurement. Note any significant differences.

- 10.** For this same interval of time compute the predicted value of the velocity from the fit. Plot this as a line on your velocity graph. Print this graph out and save it. (See above). **Print precaution: To avoid printing many unwanted pages of numbers, insert a new worksheet and call it “analysis”, then copy all calculations and graphs onto this worksheet (omitting the original exported table from Logger Pro). Then, print this new worksheet.**

11. The slope of the velocity curve, from the fit in part 10 above, is the acceleration,  $a_{\text{fit}}$ . Use this and the fitted value for the velocity  $v_0$  at the beginning of the interval to find the predicted value of position from the formula

$$x = \frac{1}{2} a_{\text{fit}} \cdot (t - t_0)^2 + v_0(t - t_0) + x_0.$$

Note:  $g$  is negative (-9.8) and  $x_0$  and  $v_0$  are the position and velocity at time  $t_0$ . The time  $t_0$  is the earliest time you used when fitting the data.

12. Plot this parabola on your first graph of position. Add titles to the x and y axes and print it out. **See print precaution in #11.**
13. Compute the difference between each point  $x$  and the fitted value from the equation in step 12. Find the average of and standard deviation of this difference. The average difference is a systematic offset which comes from choosing a particular starting value of position  $x_0$  at the starting time  $t_0$ . The standard deviation is an estimate of the uncertainty in your measurement.
- 14.** Print out the top part of your spreadsheet (do not print all the data!). **See print precaution in #11.**

## **VIII. Final Questions**

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1. Explain why the slope of the velocity curve is the same when the cart is moving up and down. Is there any special feature in your velocity graph at the time when the cart is stopped at the top?
2. Compare the standard deviation you got in the last part of the analysis with the accuracy you measured for the sonic ranger using the ruler.
3. You plotted velocity versus time and found a straight line. Suppose you plotted velocity versus distance, what would the curve look like. Make a sketch or plot it out.