19.53. Model: The heat engine follows a closed cycle. For a diatomic gas, $C_V = \frac{5}{2}R$ and $C_P = \frac{7}{2}R$.

Visualize: Please refer to Figure P19.53.

Solve: **(a)** Since T_1 = 293 K, the number of moles of the gas is

$$n = \frac{p_1V_1}{RT_1} = \frac{(0.5\times1.013\times10^5 \text{ Pa})(10\times10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol K})(293 \text{ K})} = 2.08\times10^{-4} \text{ mol}$$

At point 2, $V_2 = 4V_1$ and $p_2 = 3p_1$. The temperature is calculated as follows:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \Rightarrow T_2 = \frac{p_2}{p_1}\frac{V_2}{V_1}T_1 = (3)(4)(293 \text{ K}) = 3516 \text{ K}$$

At point 3, $V_3 = V_2 = 4V_1$ and $p_3 = p_1$. The temperature is calculated as before:

$$T_3 = \frac{p_3}{p_1}\frac{V_3}{V_1}T_1 = (1)(4)(293 \text{ K}) = 1172 \text{ K}$$

For process 1 → 2, the work done is the area under the *p*-versus-*V* curve. That is,

$$W_s = (0.5 \text{ atm})(40 \text{ cm}^3 - 10 \text{ cm}^3) + \tfrac{1}{2}(1.5 \text{ atm} - 0.5 \text{ atm})(40 \text{ cm}^3 - 10 \text{ cm}^3)$$
$$= (30\times10^{-6} \text{ m}^3)(1 \text{ atm})\left(\frac{1.013\times10^5 \text{ Pa}}{1 \text{ atm}}\right) = 3.04 \text{ J}$$

The change in the thermal energy is

$$\Delta E_{th} = nC_V\Delta T = (2.08\times10^{-4} \text{ mol})\tfrac{5}{2}(8.31 \text{ J/mol K})(3516 \text{ K} - 293 \text{ K}) = 13.93 \text{ J}$$

The heat is $Q = W_s + \Delta E_{th} = 16.97$ J. For process 2 → 3, the work done is $W_s = 0$ J and

$$Q = \Delta E_{th} = nC_V\Delta T = n\left(\tfrac{5}{2}R\right)(T_3 - T_2)$$
$$= (2.08\times10^{-4} \text{ mol})\tfrac{5}{2}(8.31 \text{ J/mol K})(1172 \text{ K} - 3516 \text{ K}) = -10.13 \text{ J}$$

For process 3 → 1,

$$W_s = (0.5 \text{ atm})(10 \text{ cm}^3 - 40 \text{ cm}^3) = (0.5\times1.013\times10^5 \text{ Pa})(-30\times10^{-6} \text{ m}^3) = -1.52 \text{ J}$$
$$\Delta E_{th} = nC_V\Delta T = (2.08\times10^{-4} \text{ mol})\tfrac{5}{2}(8.31 \text{ J/mol K})(293 \text{ K} - 1172 \text{ K}) = -3.80 \text{ J}$$

The heat is $Q = \Delta E_{th} + W_s = -5.32$ J.

	W_s (J)	Q (J)	ΔE_{th}
1 → 2	3.04	16.97	13.93
2 → 3	0	–10.13	–10.13
3 → 1	–1.52	–5.32	–3.80
Net	1.52	1.52	0

(b) The efficiency of the engine is

$$\eta = \frac{W_{net}}{Q_H} = \frac{1.52 \text{ J}}{16.97 \text{ J}} = 0.0896 = 8.96\%$$

(c) The power output of the engine is

$$500\ \frac{\text{revolutions}}{\text{min}}\times\frac{1 \text{ min}}{60 \text{ s}}\times\frac{W_{net}}{\text{revolution}} = \frac{500}{60}\times1.52 \text{ J/s} = 12.7 \text{ W}$$

Assess: For a closed cycle, as expected, $(W_s)_{net} = Q_{net}$ and $(\Delta E_{th})_{net} = 0$ J.