**19.52. Model:** The heat engine follows a closed cycle with process 1 $\to$ 2 and process 3 $\to$ 4 being isochoric and process 2 $\to$ 3 and process 4 $\to$ 1 being isobaric. For a monatomic gas, $C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$.

**Visualize:** Please refer to Figure P19.52.

**Solve:** **(a)** The first law of thermodynamics is $Q = \Delta E_{th} + W_S$. For the isochoric process 1 $\to$ 2, $W_{S\,1\to 2} = 0$ J. Thus,

$$Q_{1\to 2} = 3750\text{ J} = \Delta E_{th} = nC_V \Delta T$$
$$\Rightarrow \Delta T = \frac{3750\text{ J}}{nC_V} = \frac{3750\text{ J}}{(1.0\text{ mol})(\frac{3}{2}R)} = \frac{3750\text{ J}}{(1.0\text{ mol})(\frac{3}{2})(8.31\text{ J/mol K})} = 301\text{ K}$$
$$\Rightarrow T_2 - T_1 = 300.8\text{ K} \Rightarrow T_2 = 300.8\text{ K} + 300\text{ K} = 601\text{ K}$$

To find volume $V_2$,

$$V_2 = V_1 = \frac{nRT_1}{p_1} = \frac{(1.0\text{ mol})(8.31\text{ J/mol K})(300\text{ K})}{3.0\times 10^5\text{ Pa}} = 8.31\times 10^{-3}\text{ m}^3$$

The pressure $p_2$ can be obtained from the isochoric condition as follows:

$$\frac{p_2}{T_2} = \frac{p_1}{T_1} \Rightarrow p_2 = \frac{T_2}{T_1}p_1 = \left(\frac{601\text{ K}}{300\text{ K}}\right)(3.00\times 10^5\text{ Pa}) = 6.01\times 10^5\text{ Pa}$$

With the above values of $p_2$, $V_2$ and $T_2$, we can now obtain $p_3$, $V_3$ and $T_3$. We have

$$V_3 = 2V_2 = 1.662\times 10^{-2}\text{ m}^3 \qquad p_3 = p_2 = 6.01\times 10^5\text{ Pa}$$
$$\frac{T_3}{V_3} = \frac{T_2}{V_2} \Rightarrow T_3 = \frac{V_3}{V_2}T_2 = 2T_2 = 1202\text{ K}$$

For the isobaric process 2 $\to$ 3,

$$Q_{2\to 3} = nC_P\Delta T = (1.0\text{ mol})(\tfrac{5}{2}R)(T_3 - T_2) = (1.0\text{ mol})(\tfrac{5}{2})(8.31\text{ J/mol K})(601\text{ K}) = 12{,}480\text{ J}$$
$$W_{S\ 2\to 3} = p_3(V_3 - V_2) = (6.01\times 10^5\text{ Pa})(8.31\times 10^{-3}\text{ m}^3) = 4990\text{ J}$$
$$\Delta E_{th} = Q_{2\to 3} - W_{S\ 2\to 3} = 12{,}480\text{ J} - 4990\text{ J} = 7490\text{ J}$$

We are now able to obtain $p_4$, $V_4$ and $T_4$. We have

$$V_4 = V_3 = 1.662\times 10^{-2}\text{ m}^3 \qquad p_4 = p_1 = 3.00\times 10^5\text{ Pa}$$
$$\frac{T_4}{p_4} = \frac{T_3}{p_3} \Rightarrow T_4 = \frac{p_4}{p_3}T_3 = \left(\frac{3.00\times 10^5\text{ Pa}}{6.01\times 10^5\text{ Pa}}\right)(1202\text{ K}) = 600\text{ K}$$

For isochoric process 3 $\to$ 4,

$$Q_{3\to 4} = nC_V\Delta T = (1.0\text{ mol})(\tfrac{3}{2}R)(T_4 - T_3) = (1.0\text{ mol})(\tfrac{3}{2})(8.31\text{ J/mol K})(-602) = -7500\text{ J}$$
$$W_{S\ 3\to 4} = 0\text{ J} \Rightarrow \Delta E_{th} = Q_{3\to 4} - W_{S\ 3\to 4} = -7500\text{ J}$$

For isobaric process 4 $\to$ 1,

$$Q_{4\to 1} = nC_P\Delta T = (1.0\text{ mol})\tfrac{5}{2}(8.31\text{ J/mol K})(300\text{ K} - 600\text{ K}) = -6230\text{ J}$$
$$W_{S\ 4\to 1} = p_4(V_1 - V_4) = (3.00\times 10^5\text{ Pa})\times(8.31\times 10^{-3}\text{ m}^3 - 1.662\times 10^{-2}\text{ m}^3) = -2490\text{ J}$$
$$\Delta E_{th} = Q_{4\to 1} - W_{S\ 4\to 1} = -6230\text{ J} - (-2490\text{ J}) = -3740\text{ J}$$

| | $W_S$ (J) | $Q$ (J) | $\Delta E_{th}$ (J) |
|---|---|---|---|
| $1 \to 2$ | 0 | 3750 | 3750 |
| $2 \to 3$ | 4990 | 12,480 | 7490 |
| $3 \to 4$ | 0 | –7500 | –7500 |
| $4 \to 1$ | –2490 | –6230 | –3740 |
| Net | 2500 | 2500 | 0 |

**(b)** The thermal efficiency of this heat engine is

$$\eta = \frac{W_{out}}{Q_H} = \frac{W_{out}}{Q_{1\to2} + Q_{2\to3}} = \frac{2500\ \text{J}}{3750\ \text{J} + 12{,}480\ \text{J}} = 0.154 = 15.4\%$$

**Assess:** For a closed cycle, as expected, $(W_s)_{net} = Q_{net}$ and $(\Delta E_{th})_{net} = 0$ J