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phy260S08

Homework 9

Due at 11:00pm on Saturday, April 12, 2008

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Coulomb's Law Tutorial

Description: Use Coulomb's law to compute the (vector) force between two, and subsequently multiple, electric charges (both positive and negative). One charge acts at an angle of pi/4 radians relative to the given coordinate axes, and thus trigonometry is required to solve the problem.

Learning Goal: To understand how to calculate forces between charged particles, particularly the dependence on the sign of the charges and the distance between them.

Coulomb's law describes the force that two charged particles exert on each other (by Newton's third law, those two forces must be equal and opposite). The force F_{21} exerted *by* particle 2 (with charge q_2) *on* particle 1 (with charge q_1) is proportional to the charge of each particle and inversely proportional to the square of the distance r between them:

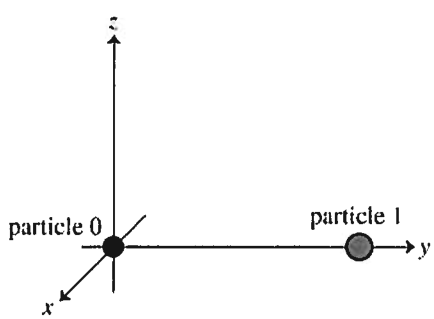
$$\vec{F}_{21} = \frac{k\, q_2\, q_1}{r^2}\hat{r}_{21},$$

where $k = \frac{1}{4\pi\epsilon_0}$ and $\hat{r}_{21}$ is the unit vector pointing *from* particle 2 *to* particle 1. The force vector will be parallel or antiparallel to the direction of $\hat{r}_{21}$, parallel if the product $q_1\, q_2 > 0$ and antiparallel if $q_1\, q_2 < 0$; the force is *attractive* if the charges are of opposite sign and *repulsive* if the charges are of the same sign.

Part A

Consider two positively charged particles, one of charge q_0 (particle 0) fixed at the origin, and another of charge q_1 (particle 1) fixed on the y-axis at $(0,\, d_1,\, 0)$. What is the net force $\vec{F}$ *on* particle 0 *due to* particle 1?

Express your answer (a vector) using any or all of k, q_0, q_1, d_1, $\hat{x}$, $\hat{y}$, and $\hat{z}$.



ANSWER:

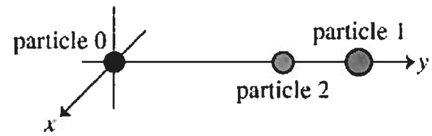
$\vec{F}$ = $\frac{-kq_0q_1\hat{y}}{d_1^2}$

$\frac{\frac{-1}{4\pi\epsilon_0}q_0q_1\hat{y}}{d_1^2}$

Part B

Now add a third, negatively charged, particle, whose charge is $-q_2$ (particle 2). Particle 2 fixed on the y-axis at position $(0,\, d_2,\, 0)$. What is the new net force on particle 0, *from* particle 1 and particle 2?

Express your answer (a vector) using any or all of k, q_0, q_1, q_2, d_1, d_2, $\hat{x}$, $\hat{y}$, and $\hat{z}$.



ANSWER:

$$\vec{F} = \begin{array}{c} kq_0\left(\frac{-q_1}{d_1^2}+\frac{q_2}{d_2^2}\right)\hat{y} \\ \frac{1}{4\pi\epsilon_0}q_0\left(\frac{-q_1}{d_1^2}+\frac{q_2}{d_2^2}\right)\hat{y} \end{array}$$

Part C

Particle 0 experiences a repulsion *from* particle 1 and an attraction *toward* particle 2. For certain values of d_1 and d_2, the repulsion and attraction should balance each other, resulting in no net force. For what ratio d_1/d_2 is there no net force on particle 0?

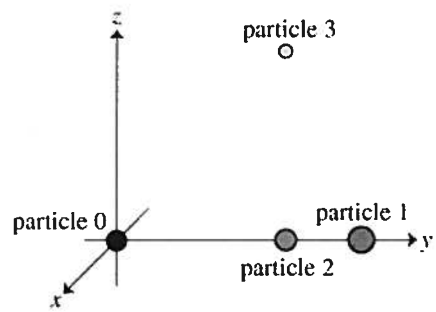
Express your answer in terms of any or all of the following variables: k, q_0, q_1, q_2.

ANSWER:

$$d_1/d_2 = \sqrt{\frac{q_1}{q_2}}$$

Part D

Now add a fourth charged particle, particle 3, with positive charge q_3, fixed in the yz-plane at $(0, d_2, d_2)$. What is the net force $\vec{F}$ on particle 0 due *solely* to this charge?



Part D.1 Find the magnitude of force from particle 3

What is the magnitude of the force on particle 0 from particle 3, fixed at $(0, d_2, d_2)$?

Hint D.1.a Distance to particle 3

Use the Pythagorean theorem to find the straight line distance between the origin and $(0, d_2, d_2)$.

Express your answer using k, q_0, q_3, d_2.

ANSWER:

$$F_3 = \begin{array}{c} \frac{kq_0q_3}{2d_2^2} \\ \frac{\frac{1}{4\pi\epsilon_0}q_0q_3}{2d_2^2} \end{array}$$

Hint D.2 Vector components

The force vector points from q_3 to q_0. Because q_3 is symmetrically located between the y-axis and the z-axis, the angle between $\hat{r}_{30}$, the unit vector pointing *from* particle 3 *to* particle 0, and the y-axis is $\pi/4$ radians. You have already calculated the magnitude of the vector above. Now break up the force vector into its y and z components.

Express your answer (a vector) using k, q_0, q_3, d_2, $\hat{x}$, $\hat{y}$, and $\hat{z}$. Include only the force caused by particle 3.

ANSWER:

$$\vec{F} = \frac{-kq_0q_3}{2d_2^2}\frac{\sqrt{2}}{2}(\hat{y}+\hat{z})$$

$$\frac{\frac{-1}{4\pi\epsilon_0}q_0q_3}{2d_2^2}\frac{\sqrt{2}}{2}(\hat{y}+\hat{z})$$

PSS 25.1: What's the Point?

Description: Given information about the placement of two charges and the forces they exert on a third charge, find the position of the third charge. (PSS 25.1: Electrostatic forces and Coulomb's Law)

Learning Goal: To practice Problem-Solving Strategy 25.1 for problems involving electrostatic forces.

Two charged particles, with charges $q_1 = q$ and $q_2 = 4q$, are located a distance d apart on the x axis. A third charged particle, with charge $q_3 = q$, is placed on the x axis such that the magnitude of the force that charge 1 exerts on charge 3 is equal to the force that charge 2 exerts on charge 3.

Find the position of charge 3. Assume that all three charges are positive.

MODEL: Identify point charges or objects that can be modeled as point charges.

VISUALIZE: Use a *pictorial representation* to establish a coordinate system, show the positions of the charges, show the force vectors on the charges, define distances and angles, and identify what you are trying to find. This is the process of translating words into symbols.

SOLVE: The mathematical representation is based on Coulomb's law:

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{K|q_1||q_2|}{r^2}.$$

- Show the directions of the forces--repulsive for like charges and attractive for opposite charges--on the pictorial representation.
- When possible, do graphical vector addition on the pictorial representation. Although this is not exact, it tells you the type of answer you should expect.
- Write each force vector in terms of its x and y components, then add the components to find the net force. Use the pictorial representation to determine which components are positive and which are negative.

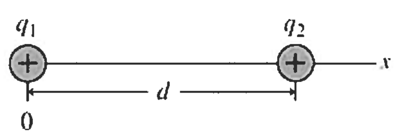
ASSESS: Check that your result has the correct units, is reasonable, and answers the question.

Model

Model the charged particles as point charges.

Visualize

Starting with the figure given here,
complete the pictorial representation for this problem, including all of the elements listed in the problem-solving strategy. Use your drawing to answer the following questions.



Part A

Which of the following are correct statements about the location of charge 3?

Hint A.1 **How to approach the problem**

Consider placing charge 3 at a few different points on your diagram (on the x axis), and sketch qualitatively the force that it would experience due to charges 1 and 2 at each point. Recall that the magnitude of the force between two charges depends on both the distance between them and the magnitude of each charge. Try to identify the places where the magnitudes, but not necessarily directions, of the forces on charge 3 due to charges 1 and 2 are the same.

Hint A.2 **Which charge is closer to charge 3?**

Charge 2 has four times the magnitude of charge 1, yet we are trying to place charge 3 so that the force on it due to charge 1 is equal to that due to charge 2. By inspecting the formula for the Coloumb force between two charges, what can you say about the relative distance from charge 3 to charge 1 versus that from charge 3 to charge 2?

Check all that apply.

ANSWER:

- [ ] Charge 3 must be located between charges 1 and 2.
- [ ] Charge 3 must be located to the left of charge 1.
- [ ] Charge 3 must be located to the right of charge 2.
- [x] There is more than one possible location for charge 3.
- [x] Charge 3 must be closer to charge 1 than to charge 2.

Charge 3 may be either between the two other charges or to the left of charge 1. To see why the latter is a possibility, imagine placing charge 3 a very short distance to the left of charge 1. Since it is much closer to charge 1 than to charge 2, the force due to charge 1 will be greater. Now imagine placing charge 3 a very long distance to the left of charge 1. The difference in distance from charge 3 to each of the other two charges is negligible, so charge 3 will feel a stronger force due to charge 2, since the magnitude of charge 2 is larger. Somewhere in between these two extremes, there must be a point where the forces due to charge 1 and charge 2 are equal.

In the remainder of this problem, we'll see if we can come up with two possible values for x_3, the position of charge 3.

Part B

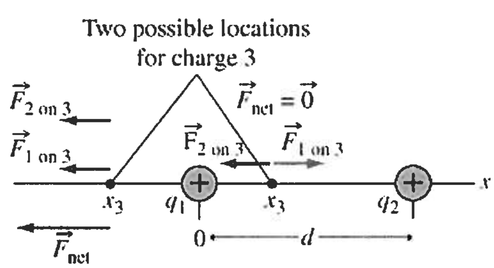
Assume that charge 3 is placed in between the other two charges at a point that satisfies the conditions given in the problem statement.

What can you determine about the net force acting on charge 3?

ANSWER:

- ( ) It is directed to the left.
- ( ) It is directed to the right.
- ( ) It is perpendicular to the x axis.
- (•) It is zero.

Here is a more complete pictorial representation based on the answers to the questions so far.



Solve

Now use the information and the insights that you have accumulated to construct the necessary mathematical expressions and to derive the solution.

Part C

Find the two possible values of x_3.

Hint C.1 **How to approach the problem**

Derive two expressions for the force on charge 3 due to the other two charges. Set $F_{1\text{ on }3} = F_{2\text{ on }3}$ and solve for x_3. You should obtain a quadratic equation that has two real-valued solutions.

Part C.2 **Find $F_{1\,\mathrm{on}\,3}$**

Find the magnitude of the force $\vec{F}_{1\,\mathrm{on}\,3}$.

Express your answer in terms of q, d, x_3, and the Coulomb constant K. You may not need all of these quantities.

ANSWER: $F_{1\,\mathrm{on}\,3} = \frac{Kq^2}{x_3^2}$

Part C.3 **Find $F_{2\,\mathrm{on}\,3}$**

Find the magnitude of the force $\vec{F}_{2\,\mathrm{on}\,3}$.

Part C.3.a **Find the distance between charges 2 and 3**

What is the distance r_{23} between charge 2 and charge 3?

Express your answer in terms of d and x_3.

ANSWER: $r_{23} = d - x_3$

Express your answer in terms of q, d, x_3, and the Coulomb constant K. You may not need all of these quantities.

ANSWER: $F_{2\,\mathrm{on}\,3} = \frac{4Kq^2}{(d-x_3)^2}$

Express your answer in terms of q, d, and the Coulomb constant K. You may not need all of these quantities.

ANSWER: $x_{3,1}, x_{3,2} = \frac{d}{3}\ \ -d$
$-d\ \ \frac{d}{3}$

Assess

When you work on a problem on your own, without the computer-provided feedback, only you can assess whether your answer seems right. The following question will help you practice the skills necessary for such an assessment.

Part D

Based on the problem-solving strategy and all the information you obtained up through the Visualize step, which of the following general conditions must the answer to this problem satisfy to be correct?

Check all that apply.

ANSWER:

- [ ] $x_{3,1}$ and $x_{3,2}$ must represent positions that are equidistant from charge 1.
- [x] Both $x_{3,1}$ and $x_{3,2}$ must represent positions that are closer to charge 1 than to charge 2.
- [ ] Both $x_{3,1}$ and $x_{3,2}$ must represent positions that are closer to charge 2 than to charge 1.
- [x] Either $x_{3,1}$ or $x_{3,2}$ must represent a position between charge 1 and charge 2.
- [x] Both $x_{3,1}$ and $x_{3,2}$ must have the correct units.

It is a good idea to create your own list of conditions that an answer must meet when you are solving any problem, then test whatever answer you obtain against these conditions. This is an excellent way to spot mistakes.

Magnitude and Direction of Electric Fields

Description: Short quantitative problem for which students must find the charge producing an electric field and sum the electric fields produced by two charges. This problem is based on Young/Geller Conceptual Analysis 17.6.

A small object A, electrically charged, creates an electric field. At a point P located 0.250 m directly north of A, the field has a value of 40.0 $\mathrm{N/C}$ directed to the south.

Part A

What is the charge of object A?

Hint A.1 **How to approach the problem**

Recall that the electric field at a point P due to a point charge is proportional to the magnitude of the charge and inversely proportional to the square of the distance of P from the charge. Furthermore, the direction of the field is determined by the sign of the charge.

Part A.2 **Find an expression for the charge**

Which of the following expressions gives the correct magnitude of charge q that produces an electric field of magnitude E at a distance r from the charge? In the following expressions k is a constant that has units of $\mathrm{N \cdot m^2/C^2}$.

Hint A.2.a **Magnitude of the electric field of a point charge**

Given a point charge q, the magnitude of the electric field E at a distance r from the charge is given by

$$E = k\frac{|q|}{r^2},$$

where the constant of proportionality is $k = 8.99\times10^9\ \mathrm{N \cdot m^2/C^2}$.

ANSWER:

- ○ $q = k\frac{E}{d^2}$
- ○ $q = kEd^2$
- ○ $q = k\frac{d^2}{E}$
- ◉ $q = \frac{Ed^2}{k}$

Part A.3 **Find the sign of the charge**

What is the sign of the charge that produces an electric field that points toward the charge?

ANSWER:

- ○ positive
- ◉ negative

Since the electric field produced by A at P points south toward A, the charge of A must be negative.

ANSWER:

- ○ 1.11×10^{-9} C
- ○ -1.11×10^{-9} C
- ○ 2.78×10^{-10} C
- ◉ -2.78×10^{-10} C
- ○ 5.75×10^{12} C
- ○ -5.75×10^{12} C

Part B

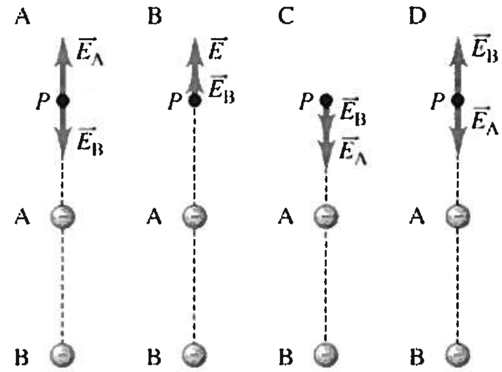
If a second object B with the same charge as A is placed at 0.250 m south of A (so that objects A and B and point P follow a straight line), what is the magnitude of the total electric field produced by the two objects at *P*?

Hint B.1 **How to approach the problem**

Since the electric field is a vector quantity, you need to apply the principle of superposition to find the total field at P. The principle of superposition in terms of electric fields says that the total electric field at any point due to two or more charges is the vector sum of the fields that would be produced at that point by the individual charges.

Part B.2 **Find the vector sum of the electric fields**

Which of the following diagrams, where $\vec{E}_{PA}$ and $\vec{E}_{PB}$ are the electric fields produced by A and B, respectively, correctly represents the situation described in this problem?



ANSWER: C

Now find the magnitude of the vector sum.

Part B.3 **Find the electric field produced by B at P**

What is the magnitude of the electric field E_2 produced by the second object B at point P?

Hint B.3.a **Magnitude of the electric field of a point charge**

Given a point charge q, the magnitude of the electric field E at a distance r from the charge is given by

$$E = k\frac{|q|}{r^2},$$

where the constant of proportionality is $k = 8.99\times10^9\ \mathrm{N\cdot m^2/C^2}$.

Part B.3.b **Find the distance from P to the second object**

How far (d_{PB}) is P from B? Recall that P is located 0.250 m north of A and B is located 0.250 m south of A.

Express your answer in meters.

ANSWER: d_{PB} = $2d$ m

Express your answer in newtons per coulomb.

ANSWER: E_2 = E_4 N/C

ANSWER:
- 40.0 N/C
- 50.0 N/C
- 30.0 N/C
- 10.0 N/C

Problem 25.52

Description: A positive point charge Q is located at x =a and a negative point charge -Q is at x = - a. A positive charge q can be placed anywhere on the x-axis. (a) Find an expression for (F_net)_x, the x-component of the net force on q, when |x| < ...

[Print]

A positive point charge Q is located at $x = a$ and a negative point charge $-Q$ is at $x = -a$. A positive charge q can be placed anywhere on the x-axis.

Part A

Find an expression for $(F_{net})_x$, the x-component of the net force on q, when $|x| < a$.

ANSWER:

- ○ $-\frac{2 \cdot K \cdot Q \cdot q(a^2+x^2)}{(a^2-x^2)}$
- ⊙ $-\frac{2 \cdot K \cdot Q \cdot q(a^2+x^2)}{(a^2-x^2)^2}$
- ○ $\frac{2 \cdot K \cdot Q \cdot q(a^2+x^2)}{(a^2-x^2)^2}$
- ○ $-\frac{K \cdot Q \cdot q(a^2+x^2)}{(a^2-x^2)^2}$

Part B

Find an expression for $(F_{net})_x$, the x-component of the net force on q, when $|x| > a$.

ANSWER:

- ○ $\frac{4 \cdot K \cdot Q \cdot q \cdot a \cdot x}{(x^2-a^2)^2}$
- ○ $-\frac{4 \cdot K \cdot Q \cdot q \cdot a \cdot x}{(x^2-a^2)^2}$
- ⊙ $\frac{4 \cdot K \cdot Q \cdot q \cdot a \cdot |x|}{(x^2-a^2)^2}$
- ○ $\frac{4 \cdot K \cdot Q \cdot q \cdot a \cdot |x|}{(x^2-a^2)}$

Problem 25.65

Description: A q charge is located at position (x,y) = (1.0 (cm), 2.0 (cm)). At what (x, y) position(s) is the electric field (a) - 225,000 i_unit N/C ? (b) 161,000 i_unit+80,500 j_unit N/C ? (c) ... (d) 21,600 i_unit-28,800 j_unit N/C ? ...

A 92.3 nC charge is located at position $(x, y) = (1.0 \text{ cm}, 2.0 \text{ cm})$. At what (x, y) position(s) is the electric field

Part A

$-225,000\hat{i}$ N/C ?

ANSWER: x = $1 - \frac{2\sqrt{q}}{\sqrt{(10)}}$ cm

Part B

$(161,000\hat{i} + 80,500\hat{j})$ N/C ?

ANSWER: x = $1 + \frac{2\sqrt{q}}{\sqrt{(10)}}$ cm

Part C

ANSWER: y = $2 + \frac{1\sqrt{q}}{\sqrt{(10)}}$ cm

Part D

$(21,600\hat{i} - 28,800\hat{j})$ N/C ?

ANSWER: $x = 1 + \frac{3\sqrt{q}}{\sqrt{(10)}}$ cm

Part E

ANSWER: $y = 2 - \frac{4\sqrt{q}}{\sqrt{(10)}}$ cm

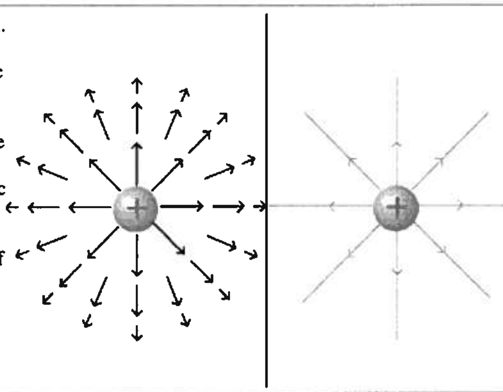
Visualizing Electric Fields

Description: Select the correct drawing of electric field lines for several situations and answer questions about why other choices are incorrect. Then, these ideas are demonstrated with an applet.

Learning Goal: To understand the nature of electric fields and how to draw field lines.

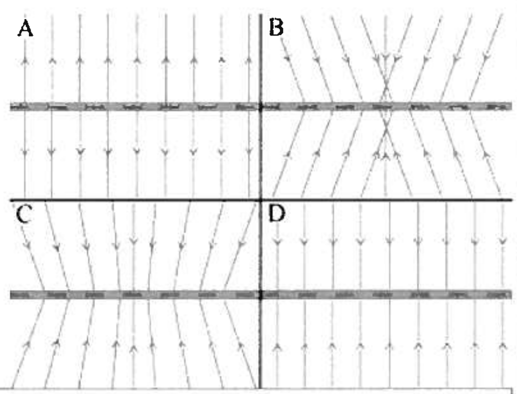
Electric field lines are a tool used to visualize electric fields. A field line is drawn beginning at a positive charge and ending at a negative charge. Field lines may also appear from the edge of a picture or disappear at the edge of the picture. Such lines are said to begin or end *at infinity*. The field lines are directed so that the electric field at any point is tangent to the field line at that point.

The figure shows two different ways to visualize an electric field. On the left, vectors are drawn at various points to show the direction and magnitude of the electric field. On the right, electric field lines depict the same situation. Notice that, as stated above, the electric field lines are drawn such that their tangents point in the same direction as the electric field vectors on the left. Because of the nature of electric fields, field lines never cross. Also, the vectors shrink as you move away from the charge, and the electric field lines spread out as you move away from the charge. The spacing between electric field lines indicates the strength of the electric field, just as the length of vectors indicates the strength of the electric field. The greater the spacing between field lines, the weaker the electric field. Although the advantage of field lines over field vectors may not be apparent in the case of a single charge, electric field lines present a much less cluttered and more intuitive picture of more complicated charge arrangements.



Part A

Which of the following figures correctly depicts the field lines from an infinite uniformly negatively charged sheet? Note that the sheet is being viewed edge-on in all pictures.



Hint A.1 Description of the field

Recall that the field around an infinite charged sheet is always perpendicular to the sheet and that the field strength does not change, regardless of distance from the sheet.

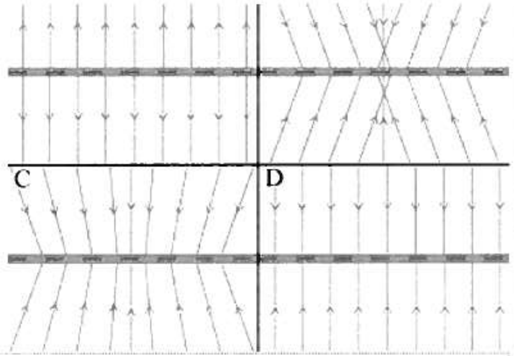
ANSWER:
- A
- B
- C
- D (selected)

Part B

In the diagram from part A , what is wrong with figure B? (Pick

only those statements that apply to figure B.)

Check all that apply.

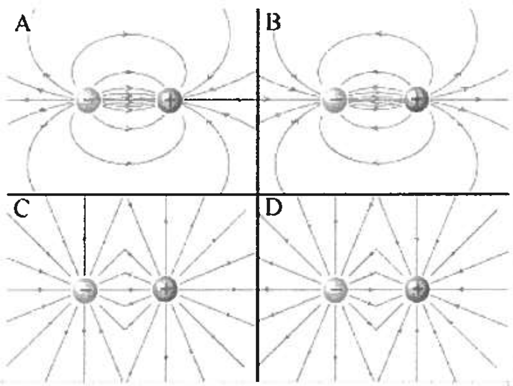


ANSWER:

- [x] Field lines cannot cross each other.
- [x] The field lines should be parallel because of the sheet's symmetry.
- [ ] The field lines should spread apart as they leave the sheet to indicate the weakening of the field with distance.
- [ ] The field lines should always end on negative charges or at infinity.

Part C

Which of the following figures shows the correct electric field lines for an electric dipole?



ANSWER:

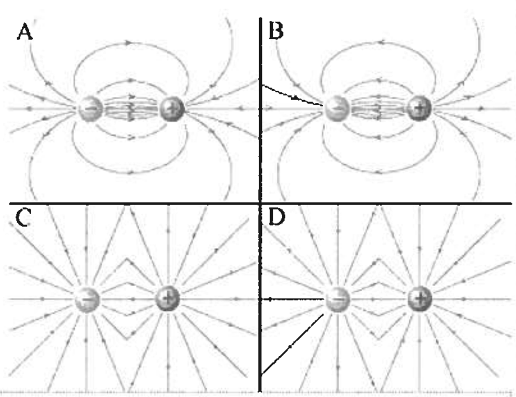
- ( ) A
- (•) B
- ( ) C
- ( ) D

This applet shows two charges. You can alter the charge on each independently or alter the distance between them. You should try to get a feeling for how altering the charges or the distance affects the field lines.

Part D

In the diagram from part C , what is wrong with figure D? (Pick only those statements that apply to figure D.)

Check all that apply.

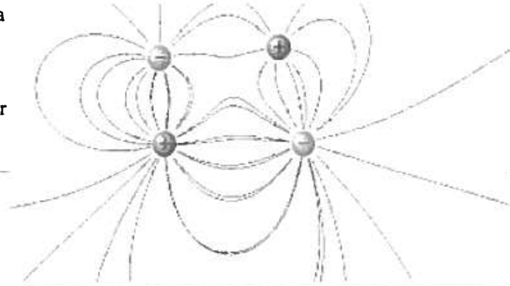


ANSWER:

- [ ] Field lines cannot cross each other.
- [ ] The field lines should turn sharply as you move from one charge to the other.
- [x] The field lines should be smooth curves.
- [x] The field lines should always end on negative charges or at infinity.

In even relatively simple setups as in the figure, electric field lines are quite helpful for understanding the field

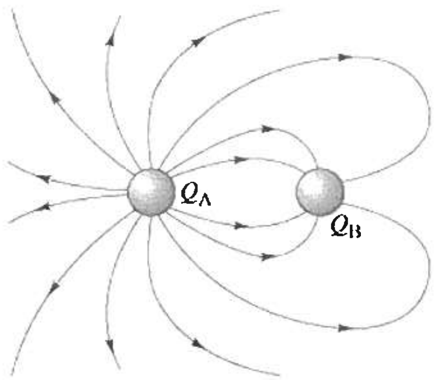
qualitatively (understanding the general direction in which a certain charge will move from a specific position, identifying locations where the field is roughly zero or where the field points a specific direction, etc.). A good figure with electric field lines can help you to organize your thoughts as well as check your calculations to see whether they make sense.



Part E

In the figure , the electric field lines are shown for a system of two point charges, Q_A and Q_B. Which of the following could represent the magnitudes and signs of Q_A and Q_B?

In the following, take q to be a positive quantity.



ANSWER:

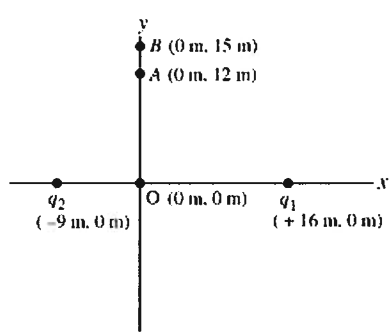
- ○ $Q_A = +q$, $Q_B = -q$
- ◉ $Q_A = +7q$, $Q_B = -3q$
- ○ $Q_A = +3q$, $Q_B = -7q$
- ○ $Q_A = -3q$, $Q_B = +7q$
- ○ $Q_A = -7q$, $Q_B = +3q$

Very far from the two charges, the system looks like a single charge with value $Q_A + Q_B = +4q$. At large enough distances, the field lines will be indistinguishable from the field lines due to a single point charge $+4q$.

Electric Field due to Multiple Point Charges

Description: Calculate the electric field due to two charges of the same sign. Then add a third charge at a new point and find its charge (including sign) to cancel the field of the first two charges at a particular point.

Two point charges are placed on the x axis.
The first charge, $q_1 = 8.00$ nC, is placed a distance 16.0 m from the origin along the positive x axis; the second charge, $q_2 = 6.00$ nC, is placed a distance 9.00 m from the origin along the negative x axis.



Part A

Calculate the electric field at point A, located at coordinates (0 m, 12.0 m).

Hint A.1 How to approach the problem

Find the contributions to the electric field at point A separately for q_1 and q_2, then add them together (using vector addition) to find the total electric field at that point. You will need to use the Pythagorean theorem to find the distance of each charge from point A.

Part A.2 Calculate the distance from each charge to point A

Calculate the distance from each charge to point A.

Enter the two distances, separated by a comma, in meters to three significant figures.

ANSWER: $r_{A1}, r_{A2} = \sqrt{x_1^2 + y_a^2}$ $\sqrt{x_2^2 + y_a^2}$ m

Part A.3 Determine the directions of the electric fields

Which of the following describes the directions of the electric fields $\vec{E}_{A1}$ and $\vec{E}_{A2}$ created by charges q_1 and q_2 at point A?

ANSWER:
- $\vec{E}_{A1}$ points up and left and $\vec{E}_{A2}$ points up and right.
- $\vec{E}_{A1}$ points up and left and $\vec{E}_{A2}$ points down and left.
- $\vec{E}_{A1}$ points down and right and $\vec{E}_{A2}$ points up and right.
- $\vec{E}_{A1}$ points down and right and $\vec{E}_{A2}$ points down and left.

In this case, the electric fields due to the two charges have both x and y components that are nonzero. To find the total field, add these two components separately.

Part A.4 Calculate the components of $\vec{E}_{A1}$

Calculate the x and y components of the electric field $\vec{E}_{A1}$ at point A due to charge q_1.

Part A.4.a Calculate the magnitude of the total field

Calculate the magnitude of the field E_{A1} at point A due to charge q_1 only.

Express your answer in newtons per coulomb to three significant figures.

ANSWER: $E_{A1} = \frac{kq_1}{x_1^2 + y_a^2}$ N/C

Hint A.4.b How to find the components of the total field

Once you have found the magnitude of the field, use trigonometry to determine the x and y components of the field. The electric field of a positive point charge points directly away from the charge, so the direction of the electric field at point A due to charge q_1 will be along the line joining the two. Use the position coordinates of q_1 and point A to find the angle that the line joining the two makes with the x or y axis. Then use this angle to resolve the electric field vector into components.

Express your answers in newtons per coulomb, separated by a comma, to three significant figures.

ANSWER: $E_{A1x}, E_{A1y} = \frac{-kq_1x_1}{(x_1^2 + y_a^2)^{\frac{3}{2}}}$ $\frac{kq_1y_a}{(x_1^2 + y_a^2)^{\frac{3}{2}}}$ N/C

Part A.5 Calculate the components of $\vec{E}_{A2}$

Calculate the x and y components of the electric field at point A due to charge q_2.

Part A.5.a Calculate the magnitude of the total field

Calculate the magnitude of the field E_{A2} at point A due to charge q_2 only.

Express your answer in newtons per coulomb to three significant figures.

ANSWER: $E_{A2} = \frac{kq_2}{x_2^2 + y_a^2}$ N/C

Hint A.5.b How to find the components of the total field

Once you have found the magnitude of the field, use trigonometry to determine the *x* and *y* components of the field. The electric field of a positive point charge points directly away from the charge, so the direction of the electric field at point A due to charge q_2 will be along the line joining the two. Use the position coordinates of q_2 and point A to find the angle that the line joining the two makes with the *x* or *y* axis. Then use this angle to resolve the electric field vector into components.

Express your answers in newtons per coulomb, separated by a comma, to three significant figures.

ANSWER: $E_{A2x}, E_{A2y} = \frac{kq_2x_2}{(x_2^2+y_a^2)^{\frac{3}{2}}}\ \ \frac{kq_2y_a}{(x_2^2+y_a^2)^{\frac{3}{2}}}$ N/C

Give the *x* and *y* components of the electric field as an ordered pair. Express your answer in newtons per coulomb to three significant figures.

ANSWER: $E_{Ax}, E_{Ay} = 0\ \ \frac{kq_2y_a}{(x_2^2+y_a^2)^{\frac{3}{2}}} + \frac{kq_1y_a}{(x_1^2+y_a^2)^{\frac{3}{2}}}$ N/C

Part B

An unknown additional charge q_3 is now placed at point B, located at coordinates (0 m, 15.0 m).

Find the magnitude and sign of q_3 needed to make the total electric field at point A equal to zero.

Hint B.1 How to approach the problem

You have already calculated the electric field at point A due to q_1 and q_2. Now find the charge q_3 needed to make an opposite field at point A, so when the two are added together the total field is zero.

Part B.2 Determine the sign of the charge

Which sign of charge q_3 is needed to create an electric field $\vec{E}_{A3}$ that points in the opposite direction of the total field due to the other two charges, q_1 and q_2?

ANSWER:
- (selected) positive
- negative

Hint B.3 Calculating the magnitude of the new charge

Keep in mind that the magnitude of the field due to q_3 is $E_{A3} = kq_3/r_{A3}^2$, and the field must be equal in magnitude to the field due to charges q_1 and q_2.

Express your answer in nanocoulombs to three significant figures.

ANSWER: $q_3 = \left(\left(\frac{q_2y_a}{(x_2^2+y_a^2)^{\frac{3}{2}}} + \frac{q_1y_a}{(x_1^2+y_a^2)^{\frac{3}{2}}}\right)(y_b-y_a)^2\right)\cdot 10^9$ nC

Electric Dipole in an Electric Field

Description: Find the dipole moment of a pair of charges, then find the electric field exerting a torque on that dipole.

Point charges $q_1 = -4.40$ nC and $q_2 = +4.40$ nC are separated by distance 3.70 mm, forming an electric dipole.

Part A

Find the magnitude of the electric dipole moment.

Hint A.1 How to approach the problem

Use the equation for the dipole moment (not the electric field or repulsive force on the charges). Also, check that your answer has dimensions appropriate for a dipole moment.

Hint A.2 Formula for dipole moment

The formula for the magnitude of the dipole moment p of a pair of opposite charges, each with magnitude q = 4.40 nC, separated by a distance d = 3.70 mm is $p = qd$.

Express your answer in coulomb meters to three significant figures.

ANSWER: qd $\mathrm{C \cdot m}$

Part B

What is the direction of the electric dipole moment?

ANSWER:
- (selected) from q_1 to q_2
- from q_2 to q_1

Part C

The charges are in a uniform electric field whose direction makes an angle 36.1° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude 8.00×10^{-9} $\mathrm{N \cdot m}$?

Part C.1 **Find an equation for the torque**

Which of the following is an expression for the magnitude of the torque?

ANSWER:
- qdE
- (selected) $qdE\sin(\theta)$
- $\frac{qd}{E}$
- $\frac{qd}{E\sin(\theta)}$

Hint C.2 **How to obtain E**

Once the torque is expressed in terms of the dipole moment, the electric field, and the angle between the two, you can solve for E.

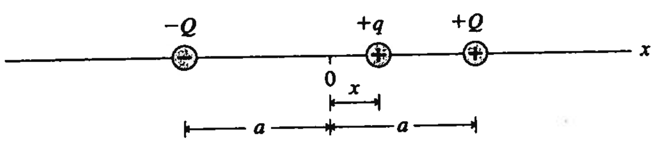
Express your answer in newtons per coulomb to three significant figures.

ANSWER: $\frac{\tau}{qd\sin(\theta)}$ $\mathrm{N/C}$

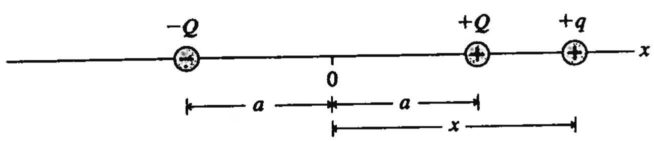
Summary	1 of 8 items complete (11.88% avg. score) 9.5 of 80 points

25.52. Model: The charged particles are point charges.

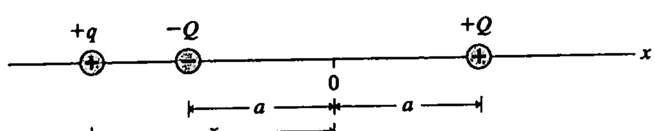
Visualize: $|x| < a$:



$x > a$:



$x < -a$:



Solve: **(a)** The force on q is the vector sum of the force from $-Q$ and $+Q$. We have

$$\vec{F}_{+Q \text{ on } +q} = \left(\frac{K|+Q||+q|}{(a-x)^2}, \text{ away from } +Q\right) = \frac{KQq}{(a-x)^2}(-\hat{i})$$

$$\vec{F}_{-Q \text{ on } +q} = \left(\frac{K|-Q||+q|}{(a+x)^2}, \text{ toward } -Q\right) = \frac{KQq}{(a+x)^2}(-\hat{i})$$

$$\Rightarrow (F_{\text{net}})_x = -KQq\left[\frac{1}{(a-x)^2} + \frac{1}{(a+x)^2}\right] = -\frac{2KQq(a^2+x^2)}{(a^2-x^2)^2}$$

To arrive at the final expression we used $(a-x)^2(a+x)^2 = [(a-x)(a+x)]^2 = (a^2-x^2)^2$.

(b) There are two cases when $|x| > a$. For $x > a$,

$$\vec{F}_{+Q \text{ on } +q} = \left(\frac{K|+Q||+q|}{(x-a)^2}, \text{ away from } +Q\right) = \frac{KQq}{(x-a)^2}(+\hat{i}) \qquad \vec{F}_{-Q \text{ on } +q} = \left(\frac{K|-Q||+q|}{(x+a)^2}, \text{ toward } -Q\right) = \frac{KQq}{(x+a)^2}(-\hat{i})$$

$$\Rightarrow (F_{\text{net}})_x = KQq\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right] = \frac{-4KQqax}{(x^2-a^2)^2}$$

For $x < -a$ (that is, for negative values of x),

$$\vec{F}_{+Q\text{ on }+q} = \frac{KQq}{(x-a)^2}(-\hat{i}) \qquad \vec{F}_{-Q\text{ on }+q} = \frac{KQq}{(a+x)^2}(+\hat{i})$$

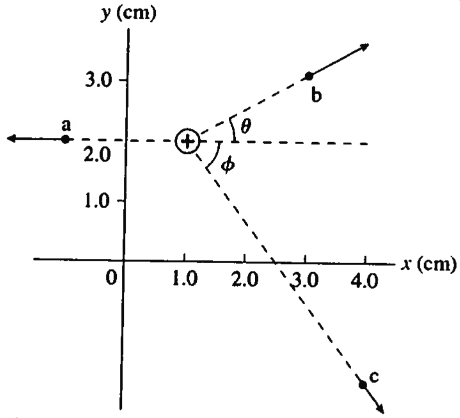
$$\Rightarrow (F_{\text{net}})_x = -\frac{4KQqax}{(x^2-a^2)^2}$$

That is, the net force is to the right when $x > a$ and to the right when $x < -a$. We can combine these two cases into a single equation for $|x| > a$:

$$(F_{\text{net}})_x = \frac{4KQqa|x|}{(x^2-a^2)^2}$$

Here, the force is always to the right when $|x| > a$.

25.65. Model: The electric field is that of a positive charge at $(x, y) = (1.0\text{ cm}, 2.0\text{ cm})$.
Visualize:



Solve: (a) The electric field of a positive charge points straight away from the charge, so we can roughly locate the points of interest based simply on whether the signs of E_x and E_y are positive or negative. For point a, the electric field has no y-component and the x-component points to the left, so point a must be to the left of the charge along a horizontal line. Using the field of a point charge,

$$E_x = E = \frac{K|q|}{r_a^2} \Rightarrow r_a = \sqrt{\frac{K|q|}{|E_x|}} = \sqrt{\frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(10.0\times10^{-9}\text{ C})}{225{,}000\text{ N/C}}} = 0.0200\text{ m} = 2.0\text{ cm}$$

Thus, $(x_a, y_a) = (-1\text{ cm}, 2\text{ cm})$.
(b) Point b is above and to the right of the charge. The magnitude of the field at this point is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(161{,}000\text{ N/C})^2 + (80{,}500\text{ N/C})^2} = 180{,}000\text{ N/C}$$

Using the field of a point charge,

$$E = \frac{K|q|}{r_b^2} \Rightarrow r_b = \sqrt{\frac{K|q|}{|\vec{E}|}} = \sqrt{\frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(10.0\times10^{-9}\text{ C})}{180{,}000\text{ N/C}}} = 2.236\text{ cm}$$

This gives the total distance but not the horizontal and vertical components. However, we can determine the angle θ because $\vec{E}_b$ points straight away from the positive charge. Thus,

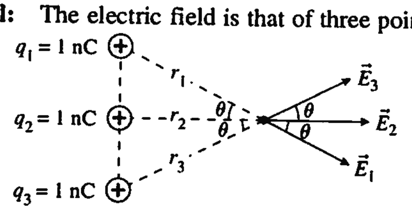
$$\theta = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}\left(\frac{80{,}500\text{ N/C}}{161{,}000\text{ N/C}}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°$$

The horizontal and vertical distances are then $d_x = r_b\cos\theta = (2.236\text{ cm})\cos 26.57° = 2.00\text{ cm}$ and $d_y = r_b\sin\theta = 1.00\text{ cm}$. Thus, point b is at position $(x_b, y_b) = (3\text{ cm}, 3\text{ cm})$.
(c) To calculate point c, which is below and to the right of the charge, a similar procedure is followed. We first find $E = 36{,}000\text{ N/C}$ from which we find the total distance $r_c = 5.00$ cm. The angle ϕ is

$$\phi = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}\left(\frac{28{,}800}{21{,}600}\right) = 53.13°$$

which gives distances $d_x = r_c\cos\phi = 3.00\text{ cm}$ and $d_y = r_c\sin\phi = 4.00\text{ cm}$. Thus, point c is at position $(x_c, y_c) = (4\text{ cm}, -2\text{ cm})$.

25.66. Model: The electric field is that of three point charges.
Visualize:



Solve: **(a)** In the figure, the distances are $r_1 = r_3 = \sqrt{(1\text{ cm})^2 + (3\text{ cm})^2} = 3.162\text{ cm}$ and the angle is $\theta = \tan^{-1}(1/3) = 18.43°$. Using the equation for the field of a point charge,

$$E_1 = E_3 = \frac{K|q_1|}{r_1^2} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(1.0\times10^{-9}\text{ C})}{(0.03162\text{ m})^2} = 9000\text{ N/C}$$

We now use the angle θ to find the components of the field vectors:

$$\vec{E}_1 = E_1\cos\theta\hat{i} - E_1\sin\theta\hat{j} = \left(8540\hat{i} - 2840\hat{j}\right)\text{ N/C} \qquad \vec{E}_3 = E_3\cos\theta\hat{i} + E_3\sin\theta\hat{j} = \left(8540\hat{i} + 2840\hat{j}\right)\text{ N/C}$$

$\vec{E}_2$ is easier since it has only an x-component. Its magnitude is

$$E_2 = \frac{K|q_2|}{r_2^2} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(1.0\times10^{-9}\text{ C})}{(0.0300\text{ m})^2} = 10{,}000\text{ N/C} \Rightarrow \vec{E}_2 = E_2\hat{i} = 10{,}000\hat{i}\text{ N/C}$$

(b) The electric field is defined in terms of an electric *force* acting on charge q: $\vec{E} = \vec{F}/q$. Since forces obey a principle of superposition ($\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \cdots$) it follows that the electric field due to several charges also obeys a principle of superposition.

(c) The net electric field at a point 3 cm to the right of q_2 is $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 27{,}100\hat{i}\text{ N/C}$. The y-components of $\vec{E}_1$ and $\vec{E}_2$ cancel, giving a net field pointing along the x-axis.