From Hot to Cool: The 2nd Law

a) Heat flows from the bucket to its surroundings. As a result $Q < 0 \Rightarrow \Delta S = \frac{Q}{T} < 0$.

b) Again, heat flows from the envioronment to the ice cube, Thus, $Q > 0 \Rightarrow \Delta S \angle Q > 0$.

c) Burning the wood, turns the wood into a gas, The result is an increase in "disorder"
$\Rightarrow \Delta S > 0$

d) $\Delta S = \frac{Q}{T} = \frac{+25.0\,J}{293K} = 8.53 \times 10^{-2}\ J/K$

Note that T is measured in Kelvins.

e) $\Delta S = \frac{Q}{T} = \frac{-25.0 \times 10^3 J}{500K} = -50.0\ J/K$

F) $\Delta S = \frac{Q}{T} = \frac{+25.0 \times 10^3 J}{400K} = 62.5\ J/K$

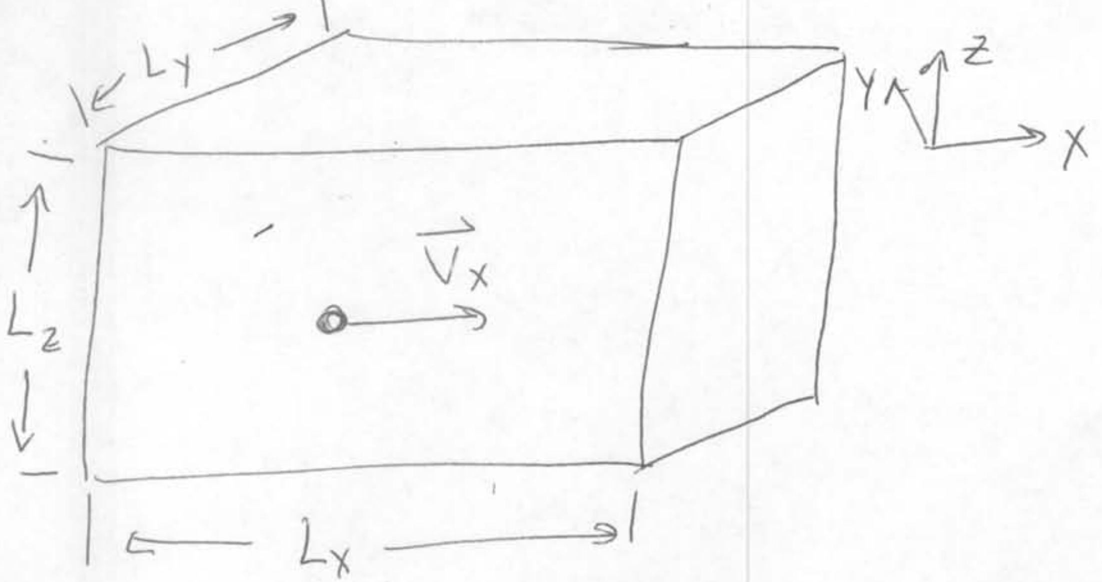
G) $\Delta S_{sys} = \Delta S_1 + \Delta S_2 = \frac{Q_1}{T_1} + \frac{Q_2}{T_2}$

$$= \frac{+25.0 \times 10^3\ J}{400\ K} + \frac{(-25.0 \times 10^3\ J)}{500\ K} =$$

$$= 62.5\ J/K - 50\ J/K = 12.5\ J/K$$

Note that $\Delta S_{sys} > 0$ in accord w/ the 2nd law

The Ideal Gas Law



a) The force F_x is given by $\vec{F}_x = \frac{d\vec{p}_x}{dt}$ from Newton's 1st Law. Thus, the average $\langle F_x \rangle$ is give by $\frac{\Delta P_x}{\Delta t}$, where ΔP_x is the change in the particles momentum + Δt is the time between succesive collisions w/ the right wal

$$\Delta P_x = P_x^F - P_x^I = -mV_x - mV_x = -2mV_x$$

$$\Delta t = 2L_x / V_x$$

Thus,

This gives the force exerted on the wall by the particle. (Newton's 3rd Law)

$$\langle F_x \rangle = -\left(\frac{-2mV_x}{2L_x/V_x}\right) = \boxed{\frac{mV_x^2}{L_x}}$$

b) The pressure due to 1 particle is given by for the right wall

$$P_1 = \frac{\langle F_x \rangle}{L_y L_z} = \frac{m V_x^2}{L_x L_y L_z} = \frac{m V_x^2}{V}$$

where $V = L_x L_y L_z$ is the volume of the container. From the Equipartition Thm, the average kinetic energy is given by:

$$\frac{1}{2} m \langle V_x^2 \rangle = \frac{1}{2} kT \Rightarrow \langle V_x^2 \rangle = \frac{k}{m} T .$$

We can replace V_x^2 in P_1 by the average value $\langle v_x^2 \rangle$. Thus, $P_1 = \frac{kT}{V}$. For N particles we multiply by N to find

$$\boxed{P = \frac{NkT}{V}}$$

c) Equipartion implies that

$$\frac{1}{2} m \langle V_x^2 \rangle = \frac{1}{2} m \langle V_y^2 \rangle = \frac{1}{2} kT \Rightarrow \langle V_x^2 \rangle = \langle V_y^2 \rangle.$$

The pressure on the wall $P \propto \frac{1}{L_y L_z}$ and

thus depends on the size of the Box.

$\langle F_x \rangle \propto \frac{1}{L_x}$ and is independent of L_y and L_z.

d) If you heat a fixed quantity of gas, you increase T while holding N fixed. Then,

$PV = NkT \Rightarrow PV$ must increase. In addition if you hold P fixed then V itself, must increase.

Dust Equipartition

a). Equipartion states that you get $\frac{1}{2}kT$ in thermal energy for every degree of freedom. Thus, for the kinetic energy K, we find:

$$K = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m \left[\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right].$$

There are 3 degrees of freedom (v_x, v_y, v_z). Thus

$$K = 3\left(\frac{1}{2}kT\right) = \frac{3}{2}\left(1.38 \times 10^{-23}\ J/K\right)(290\ K)$$

$$= 600 \times 10^{-23}\ J = \boxed{6.0 \times 10^{-21}\ J}$$

b) The root-mean-square velocity is $v_{rms} = \sqrt{\langle v^2 \rangle}$.

From part (a) we know that

$$K = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT \Rightarrow \langle v^2 \rangle = 3kT/m.$$

The mass is given by

$$m = \rho V = \rho \frac{4\pi}{3}\left(\frac{d}{2}\right)^3 = \rho \pi d^3 / 6$$

Thus

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{\rho \pi d^3/6}} = \sqrt{18 \frac{kT}{\rho \pi d^3}}$$

c) $$v_{rms} = \left(\frac{18 \cdot (1.38 \times 10^{-23}\ J/K)(290K)}{2000\ kg/m^3 \cdot (3.14) \cdot (5 \times 10^{-6} m)^3} \right)^{1/2}$$

$$= 3.03 \times 10^{-4}\ m/s = 0.303\ mm/s$$

Velocity + Energy Scaling

a) Equipartition states that the average translational kinetic energy is

$$E_{trans} = \frac{3}{2} k_B T$$

(3 degrees of freedom, V_x^2, V_y^2 and V_z^2). It is independent of mass. Therefore the answer is

$T_E = 300$ K

b) $\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$ ~~$\Rightarrow V_{rms} = \sqrt{\langle v^2 \rangle}$~~

$\Rightarrow T = \frac{1}{3} \frac{m}{k} \langle v^2 \rangle = \frac{1}{3} \frac{m}{k} V_{rms}^2$ w/ $V_{rms} = \sqrt{\langle v^2 \rangle}$

Thus,

$$\frac{T_{rms}^{H_2}}{T_{rms}^{O_2}} = \frac{\frac{1}{3} \frac{m_{H_2}}{k} V_{rms}^2}{\frac{1}{3} \frac{m_{O_2}}{k} V_{rms}^2} = \frac{m_{H_2}}{m_{O_2}}$$

$$\Rightarrow T_{rms}^{H_2} = \frac{m_{H_2}}{m_{O_2}} T_{rms}^{O_2} = \left(\frac{2u}{32u}\right) 300K = \boxed{19K}$$

18.22. Solve: (a) The mean free path is

$$\lambda = \frac{1}{4\sqrt{2}\pi(N/V)r^2}$$

where $r \approx 0.5 \times 10^{-10}$ m is the atomic radius for helium and N/V is the gas number density. From the ideal gas law,

$$\frac{N}{V} = \frac{p}{kT} = \frac{0.10 \text{ atm} \times 101{,}300 \text{ Pa / atm}}{(1.38 \times 10^{-23} \text{ J / K})(10 \text{ K})} = 7.34 \times 10^{25} \text{ m}^{-3}$$

$$\Rightarrow \lambda = \frac{1}{4\sqrt{2}\pi(7.34 \times 10^{25} \text{ m}^{-3})(0.5 \times 10^{-10} \text{ m})^2} = 3.07 \times 10^{-7} \text{ m} = 307 \text{ nm}$$

(b) The root-mean-square speed is

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J / K})(10 \text{ K})}{4 \times (1.661 \times 10^{-27} \text{ kg})}} = 250 \text{ m / s}$$

where we used $A = 4$ u as the atomic mass of helium.

(c) The average energy per atom is $\varepsilon_{\text{avg}} = \frac{3}{2}k_B T = \frac{3}{2}(1.38 \times 10^{-23} \text{ J / K})(10 \text{ K}) = 2.07 \times 10^{-22}$ J.

18.57. Solve: (a) The thermal energy is

$$E_{th} = (E_{th})_{N_2} + (E_{th})_{O_2} = \tfrac{5}{2}N_{N_2}k_B T + \tfrac{5}{2}N_{O_2}k_B T = \tfrac{5}{2}N_{total}k_B T$$

where N_{total} is the total number of molecules. The identity of the molecules makes no difference since both are diatomic. The number of molecules in the room is

$$N_{total} = \frac{pV}{k_B T} = \frac{(101{,}300\text{ Pa})(2\text{ m}\times 2\text{ m}\times 2\text{ m})}{(1.38\times 10^{-23}\text{ J/K})(273\text{ K})} = 2.15\times 10^{26}$$

The thermal energy is

$$E_{th} = \tfrac{5}{2}(2.15\times 10^{26})(1.38\times 10^{-23}\text{ J/K})(273\text{ K}) = 2.03\times 10^{6}\text{ J}$$

(b) A 1 kg ball at height $y = 1$ m has a potential energy $U = mgy = 9.8$ J. The ball would need 9.8 J of initial kinetic energy to reach this height. The fraction of thermal energy that would have to be conveyed to the ball is

$$\frac{9.8\text{ J}}{2.03\times 10^{6}\text{ J}} = 4.83\times 10^{-6}$$

(c) A temperature change ΔT corresponds to a thermal energy change $\Delta E_{th} = \tfrac{5}{2}N_{total}k_B\Delta T$. But $\tfrac{5}{2}N_{total}k_B = E_{th}/T$. Using this, we can write

$$\Delta E_{th} = \frac{E_{th}}{T}\Delta T \Rightarrow \Delta T = \frac{\Delta E_{th}}{E_{th}}T = \frac{-9.8\text{ J}}{2.03\times 10^{6}\text{ J}}273\text{ K} = -0.00132\text{ K}$$

The room temperature would decrease by 0.00132 K or 0.00132°C.

(d) The situation with the ball at rest on the floor and in thermal equilibrium with the air is a very probable distribution of energy and thus a state with high entropy. Although energy would be conserved by removing energy from the air and transferring it to the ball, this would be a very *improbable* distribution of energy and thus a state of low entropy. The ball will not be spontaneously launched from the ground because this would require a decrease in entropy, in violation of the second law of thermodynamics.

As another way of thinking about the situation, the ball and the air are initially at the same temperature. Once even the slightest amount of energy is transferred from the air to the ball, the air's temperature will be less than that of the ball. Any further flow of energy from the air to the ball would be a situation in which heat energy is flowing from a colder object to a hotter object. This cannot happen because it would violate the second law of thermodynamics.

18.50. Solve: (a) From Equation 18.26 $v_{\rm rms} = \sqrt{3k_{\rm B}T/m}$. For an adiabatic process

$$T_{\rm f}V_{\rm f}^{\gamma-1} = T_{\rm i}V_{\rm i}^{\gamma-1} \Rightarrow T_{\rm f} = T_{\rm i}\left(\frac{V_{\rm i}}{V_{\rm f}}\right)^{\gamma-1} \Rightarrow T_{\rm f} = T_{\rm i}(8)^{\frac{5}{3}-1} = 4T_{\rm i}$$

The root-mean-square speed increases by a factor of 2 with an increase in temperature.

(b) From Equation 18.3 $\lambda = \left[4\sqrt{2}\pi(N/V)r^2\right]^{-1}$. A decrease in volume decreases the mean free path by a factor of 1/8.

(c) For an adiabatic process,

$$T_{\rm f}V_{\rm f}^{\gamma-1} = T_{\rm i}V_{\rm i}^{\gamma-1} \Rightarrow T_{\rm f} = T_{\rm i}\left(\frac{V_{\rm i}}{V_{\rm f}}\right)^{\gamma-1} = T_{\rm i}(8)^{\frac{5}{3}-1} = 4T_{\rm i}$$

Because the decrease in volume increases $T_{\rm f}$, the thermal energy increases by a factor of 4.

(d) The molar specific heat at constant volume is $C_{\rm V} = \frac{3}{2}R$, a constant. It does not change.

18.43. Solve: (a) The number of molecules of helium is

$$N_{\text{helium}} = \frac{pV}{k_B T} = \frac{(2.0\times 1.013\times 10^5\ \text{Pa})(100\times 10^{-6}\ \text{m}^3)}{(1.38\times 10^{-23}\ \text{J/K})(373\ \text{K})} = 3.936\times 10^{21}$$

$$\Rightarrow n_{\text{helium}} = \frac{3.936\times 10^{21}}{6.022\times 10^{23}\ \text{mol}^{-1}} = 6.536\times 10^{-3}\ \text{mol}$$

The initial internal energy of helium is

$$E_{\text{helium i}} = \tfrac{3}{2} N_{\text{helium}} k_B T = 30.4\ \text{J}$$

The number of molecules of argon is

$$N_{\text{argon}} = \frac{pV}{k_B T} = \frac{(4.0\times 1.013\times 10^5\ \text{Pa})(200\times 10^{-6}\ \text{m}^3)}{(1.38\times 10^{-23}\ \text{J/K})(673\ \text{K})} = 8.726\times 10^{21}$$

$$\Rightarrow n_{\text{argon}} = \frac{8.726\times 10^{21}}{6.022\times 10^{23}\ \text{mol}^{-1}} = 1.449\times 10^{-2}\ \text{mol}$$

The initial thermal energy of argon is

$$E_{\text{argon i}} = \tfrac{3}{2} N_{\text{argon}} k_B T = 121.6\ \text{J}$$

(b) The equilibrium condition for monatomic gases is

$$(\varepsilon_{\text{helium f}})_{\text{avg}} = (\varepsilon_{\text{argon f}})_{\text{avg}} = (\varepsilon_{\text{total}})_{\text{avg}}$$

$$\Rightarrow \frac{E_{\text{helium f}}}{n_{\text{helium}}} = \frac{E_{\text{argon f}}}{n_{\text{argon}}} = \frac{E_{\text{tot}}}{n_{\text{tot}}} = \frac{(30.4+121.6)\ \text{J}}{(6.54\times 10^{-3}+1.449\times 10^{-2})\ \text{mol}} = 7228\ \text{J/mol}$$

$$\Rightarrow E_{\text{helium f}} = (7228\ \text{J/mol})n_{\text{helium}} = (7228\ \text{J/mol})(6.54\times 10^{-3}\ \text{mol}) = 47.3\ \text{J}$$

$$E_{\text{argon f}} = (7228\ \text{J/mol})n_{\text{argon}} = (7228\ \text{J/mol})(1.449\times 10^{-2}\ \text{mol}) = 104.7\ \text{J}$$

(c) The amount of heat transferred is

$$E_{\text{helium f}} - E_{\text{helium i}} = 47.3\ \text{J} - 30.4\ \text{J} = 16.9\ \text{J} \qquad E_{\text{argon f}} - E_{\text{argon i}} = 104.7\ \text{J} - 121.6\ \text{J} = -16.9\ \text{J}$$

The helium gains 16.9 J of heat energy and the argon loses 16.9 J. Thus 16.9 J are transferred from the argon to the helium.

(d) The equilibrium condition for monatomic gases is

$$(\varepsilon_{\text{helium}})_{\text{avg}} = (\varepsilon_{\text{argon}})_{\text{avg}} \Rightarrow \frac{E_{\text{helium f}}}{N_{\text{helium}}} = \frac{E_{\text{argon f}}}{N_{\text{argon}}} = \tfrac{3}{2} k_B T_f$$

Substituting the above values,

$$\frac{47.3\ \text{J}}{3.936\times 10^{21}} = \frac{104.7\ \text{J}}{8.726\times 10^{21}} = \tfrac{3}{2}(1.38\times 10^{-23}\ \text{J/K})T_f \Rightarrow T_F = 580\ \text{K}$$

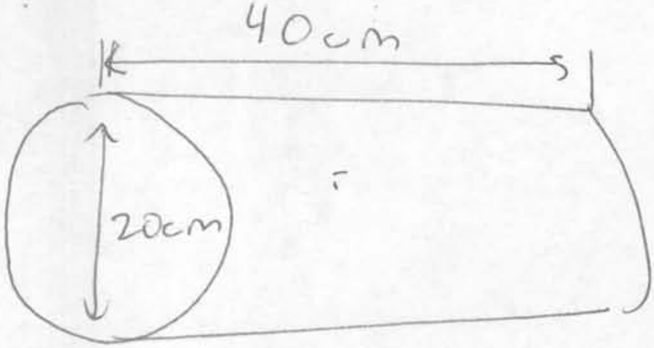
(e) The final pressure of the helium and argon are

$$p_{\text{helium f}} = \frac{N_{\text{helium}} k_{\text{B}} T}{V_{\text{helium}}} = \frac{(3.936 \times 10^{21})(1.38 \times 10^{-23} \text{ J/K})(580 \text{ K})}{100 \times 10^{-6} \text{ m}^3} = 3.15 \times 10^5 \text{ Pa} = 3.11 \text{ atm}$$

$$p_{\text{argon f}} = \frac{N_{\text{argon}} k_{\text{B}} T}{V_{\text{argon}}} = \frac{(8.726 \times 10^{21})(1.38 \times 10^{-23} \text{ J/K})(580 \text{ K})}{200 \times 10^{-6} \text{ m}^3} = 3.49 \times 10^5 \text{ Pa} = 3.45 \text{ atm}$$

Written Problem

4.0×10^{22} atoms Ne at $100°C$



a) The number density is

$$\frac{N}{V} = \frac{4.0 \times 10^{22}}{40\,cm\,\pi \left(\frac{20}{2}cm\right)^2} = 3.18 \times 10^{24}\ m^{-3}$$

b) The root-mean-square velocity is given by

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23}\ J/k)(373k)}{20u\ (1.661 \times 10^{-27}\ kg/u)}}$$

$$= 682\ m/s$$

where one Ar has mass 20u.

c) $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$

On average, any direction is equally likely. Thus,

$\langle v_x^2 \rangle = \frac{1}{3} \langle v^2 \rangle$ and

$$(v_x)_{rms} = \sqrt{\langle v_x^2 \rangle} = \frac{\sqrt{\langle v^2 \rangle}}{\sqrt{3}} = \frac{v_{rms}}{\sqrt{3}} = \frac{682 \, m/s}{\sqrt{3}} = 394 \, m/s$$

d) The rate at which a particle hits the right wall on average is

$$\frac{1}{2} \frac{N}{V} A (v_x)_{rms} = \frac{1}{2} (3.18 \times 10^{24} \, m^{-3}) \pi \left(\frac{20}{2} cm\right)^2 \left(\frac{m}{100 \, cm}\right)^2 394 \, m/s$$

$$= 1.97 \times 10^{25} \, s^{-1}$$

e) The pressure is:

$$P = \frac{1}{3} \frac{N}{V} m \, v_{rms}^2 = \frac{1}{3} (3.18 \times 10^{24} \, m^{-3}) \, 20u \times (1.661 \times 10^{-27} \, kg/u)(682 \, m/s)^2$$

$$= 16.4 \, kPa$$

f) $P = \frac{NkT}{V} = (3.18 \times 10^{24} \, m^{-3})(1.38 \times 10^{-23} \, J/K)(373 \, K) = 16.4 \, kPa$