20.72. Solve: **(a)** The peak power of the light pulse is

$$P_{\text{peak}} = \frac{\Delta E}{\Delta t} = \frac{500 \text{ mJ}}{10 \text{ ns}} = \frac{0.500 \text{ J}}{1.0 \times 10^{-8} \text{ s}} = 5.0 \times 10^{7} \text{ W}$$

(b) The average power is

$$P_{\text{avg}} = \frac{E_{\text{total}}}{1 \text{ s}} = \frac{10 \times 500 \text{ mJ}}{1 \text{ s}} = \frac{5 \text{ J}}{1 \text{ s}} = 5 \text{ W}$$

The laser delivers pulses of very high power. But the laser spends most of its time "off," so the *average* power is very much less than the peak power.

(c) The intensity is

$$I_{\text{laser}} = \frac{P}{a} = \frac{5.0 \times 10^{7} \text{ W}}{\pi (5.0\ \mu\text{m})^2} = \frac{5.0 \times 10^{7} \text{ W}}{7.85 \times 10^{-11} \text{ m}^2} = 6.4 \times 10^{17} \text{ W/m}^2$$

(d) The ratio is

$$\frac{I_{\text{laser}}}{I_{\text{sun}}} = \frac{6.4 \times 10^{17} \text{ W/m}^2}{1.1 \times 10^{3} \text{ W/m}^2} = 5.8 \times 10^{14}$$

Course PHYSICS260

Assignment 3

Due at 11:00pm on Wednesday, February 20, 2008

Standard Expression for a Traveling Wave

Description: Identify independant variables and parameters in the standard travelling wave; find phase, wavelength, period, and velocity of the wave using omega and k.
Learning Goal: To understand the standard formula for a sinusoidal traveling wave.
One formula for a wave with a *y* displacement (e.g., of a string) traveling in the *x* direction is

$$y(x,t) = A\sin(kx - \omega t).$$

All the questions in this problem refer to this formula and to the wave it describes.

Part A

Which of the following are independent variables?

Hint A.1 What are independent variables?

Independent variables are those that are freely varied to control the value of the function. The independent variables typically appear on the horizontal axis of a plot of the function.

ANSWER:

- [ ] x only
- [ ] t only
- [ ] A only
- [ ] k only
- [ ] ω only
- [x] x and t
- [ ] ω and t
- [ ] A and k and ω

Part B

Which of the following are parameters that determine the characteristics of the wave?

Hint B.1 What are parameters?

Parameters are constants that determine the characteristics of a particular function. For a wave these include the amplitude, frequency, wavelength, and period of the wave.

ANSWER:

- [ ] x only
- [ ] t only
- [ ] A only
- [ ] k only
- [ ] ω only
- [ ] x and t
- [ ] ω and t
- [x] A and k and ω

Part C

What is the phase $\phi(x,t)$ of the wave?

Hint C.1 Definition of phase

The phase is the argument of the trig function, which is expressed in radians.

Express the phase in terms of one or more given variables (A, k, x, t, and ω) and any needed constants like π.

ANSWER: $\phi(x,t) = kx - \omega t$

Part D

What is the wavelength λ of the wave?

Hint D.1 Finding the wavelength

Consider the form of the wave at time $t = 0$. The wave crosses the *y* axis, sloping upward at $x = 0$. The wavelength is the *x* position at which the wave next crosses the *y* axis, sloping upward (i.e., the length of one complete cycle of oscillation).

Express the wavelength in terms of one or more given variables (A, k, x, t, and ω) and any needed constants like π.

ANSWER: $\lambda = \frac{2\pi}{k}$

Part E

What is the period T of this wave?

Express the period in terms of one or more given variables (A, k, x, t, and ω) and any needed constants like π.

ANSWER: $T = \frac{2\pi}{\omega}$

Part F

What is the speed of propagation v of this wave?

Hint F.1 How to find v

If you've done the previous parts of this problem, you have found the wavelength and the period of this wave. The speed of propagation is a function of these two quantities: $v = \lambda/T$.

Express the speed of propagation in terms of one or more given variables (A, k, x, t, and ω) and any needed constants like π.

ANSWER: $v = \frac{\omega}{k}$

The Doppler Effect on a Train

Description: Find the frequency heard by a listener in motion with respect to the source of sound that is itself moving.

A train is traveling at 30.0 $\mathrm{m/s}$ relative to the ground in still air. The frequency of the note emitted by the train whistle is 262 Hz.

Part A

What frequency $f_{approach}$ is heard by a passenger on a train moving at a speed of 18.0 $\mathrm{m/s}$ relative to the ground in a direction opposite to the first train and approaching it?

Hint A.1 How to approach the problem

The listener is in motion with respect to the source of sound; therefore, you need to consider the Doppler effect. Moreover, you also need to take into account that the source of sound is moving. If you have a formula that gives the frequency shift when both source and listener are moving, you can apply it directly.

If you only have formulas for the Doppler shift when either the source or listener is moving but not both, you will need to calculate the frequency shift in two steps. First, consider a point somewhere between the two trains and compute the frequency of the train whistle that would be heard by a stationary listener at that point. Then consider that point to be the (stationary) *source* of the sound that is observed by the passenger moving in the second train.

Hint A.2 Doppler shift equations for moving source or observer

When a source of sound with frequency f_0 moves at speed v_S with respect to a stationary observer, the observer will hear a sound of frequency f given by

$$f = \frac{f_0}{1 \pm v_S/v}.$$

When a source of sound with frequency f_0 is stationary with respect to an observer moving with speed v_L, the observer will hear a sound of frequency f given by

$$f = (1 \pm v_L/v) f_0.$$

In both cases, v is the speed of sound and the sign to use depends on whether the source is moving toward or away from the observer.

Hint A.3 Doppler equations when both the source and the listener are in motion

If f_S is the frequency of a sound wave emitted by a source in motion at speed v_S, the frequency f_L of the sound wave heard by a listener in motion at speed v_L is given by

$$f_L = \frac{v \pm v_L}{v \pm v_S} f_S,$$

where v is the speed of sound.

Part A.4 Determine the appropriate signs

In the Doppler equation for a moving source and a moving observer,

$$f_L = \frac{v + v_L}{v \pm v_S} f_S,$$

where v is the speed of sound, which signs belong in the numerator and denominator if the source and the observer are both moving toward each other? Recall that speed and frequency are both scalar quantities.

Hint A.4.a Qualitative understanding of the Doppler shift

If the source and listener are approaching each other, the frequency observed will be higher than the frequency emitted. Similarly, if the the source and listener are receding from each other, the frequency observed will be lower than the frequency emitted.

ANSWER:

- ○ A minus sign in the numerator and a minus sign in the denominator.
- ○ A plus sign in the numerator and a plus sign in the denominator.
- ◉ A plus sign in the numerator and a minus sign in the denominator.
- ○ A minus sign in the numerator and a plus sign in the denominator.

Note that these are the signs that should be used when solving for $f_{\rm approach}$.

Express your answer in hertz.

ANSWER: $f_{approach} = \frac{344 + 18.0}{344 - 30.0} \cdot 262$ Hz

Part B

What frequency f_{recede} is heard by a passenger on a train moving at a speed of 18.0 m/s relative to the ground in a direction opposite to the first train and receding from it?

Hint B.1 How to approach the problem

The listener is in motion with respect to the source of sound; therefore, you need to consider the Doppler effect. Moreover, you also need to take into account that the source of sound is moving. If you have a formula that gives the frequency shift when both source and listener are moving, you can apply it directly.

If you only have formulas for the Doppler shift when either the source or listener is moving but not both, you will need to calculate the frequency shift in two steps. First, consider a point somewhere between the two trains and compute the frequency of the train whistle that would be heard by a stationary listener at that point. Then consider that point to be the (stationary) *source* of the sound that is observed by the passenger moving in the second train.

Hint B.2 Doppler shift equations for moving source or observer

When a source of sound with frequency f_0 moves at speed v_S with respect to a stationary observer, the observer will hear a sound of frequency f, given by

$$f = \frac{f_0}{1 \pm v_S/v}.$$

When a source of sound with frequency f_0 is stationary with respect to an observer moving with speed v_L, the observer will hear a sound of frequency f given by

$$f = (1 \pm v_L/v)f_0.$$

In both cases, v is the speed of sound and the sign to use depends on whether the source is moving toward or away from the observer.

Hint B.3 Doppler equations when both the source and the listener are in motion

If f_S is the frequency of a sound wave emitted by a source in motion at speed v_S, the frequency f_L of the sound wave heard by a listener in motion at speed v_L is given by

$$f_L = \frac{v \pm v_L}{v \pm v_S} f_S,$$

where v is the speed of sound.

Part B.4 Determine the appropriate signs

In the Doppler equation for a moving source and a moving observer,

$$f_L = \frac{v \pm v_L}{v \pm v_S} f_S,$$

where v is the speed of sound, which signs belong in the numerator and denominator if the source and the observer are both moving away from each other? Recall that speed and frequency are both scalar quantities.

Hint B.4.a Qualitative understanding of the Doppler shift

If the source and listener are approaching each other, the frequency observed will be higher than the frequency emitted. Similarly, if the the source and listener are receding from each other, the frequency observed will be lower than the frequency emitted.

ANSWER:

- ○ A minus sign in the numerator and a minus sign in the denominator.
- ○ A plus sign in the numerator and a plus sign in the denominator.
- ○ A plus sign in the numerator and a minus sign in the denominator.
- ◉ A minus sign in the numerator and a plus sign in the denominator.

Note that these are the signs that should be used when solving for f_{recede}.

Express your answer in hertz.

ANSWER: $f_{recede} = \frac{344 - 18.0}{344 + 30.0} \cdot 262$ Hz

Problem 20.12

Description: The displacement of a wave traveling in the positive x-direction is D (x, t) = (3.5 (cm)) sin (2.7x-124t), where x is in m and t is in s. (a) What is the frequency of this wave? (b) What is the wavelength of this wave? (c) What is the speed...

Part A

What is the frequency of this wave?

$$f=\frac{\omega}{2\pi}$$

ANSWER: 19.7 Hz

Part B

What is the wavelength of this wave?

$$\lambda=\frac{2\pi}{k}$$

ANSWER: 2.33 m

Part C

What is the speed of this wave?

$$v=\frac{\omega}{k}$$

ANSWER: 45.9 m/s

Problem 20.18

Description: (a) A spherical wave with a wavelength of 2.0 m is emitted from the origin. At one instant of time, the phase at r=4.0 m is pi (rad). At that instant, what is the phase at r=3.5 m? (b) What is the phase at r=4.5 m at the same instant?

Part A

$$\frac{2\pi}{k}=2.0m$$

$$k=\pi m^{-1}$$

$$\theta=kr-\omega t\Big|_{kr=4\pi}=\pi$$

$$\omega t=3\pi$$

$$\theta=kr-\omega t\Big|_{kr=3.5\pi}=3.5\pi-3\pi=0.5\pi$$

ANSWER: $\frac{\pi}{2}$ rad

Part B

What is the phase at $r=4.5\ \mathrm{m}$ at the same instant?

$$\theta=kr-\omega t\Big|_{kr=4.5\pi}=4.5\pi-3\pi=1.5\pi$$

ANSWER: $\frac{3}{2}\pi$ rad

Problem 20.66

Description: A string that is under T of tension has linear density mu. A sinusoidal wave with amplitude A and wavelength lambda travels along the string. (a) What is the maximum velocity of a particle on the string?

A string that is under 49.0 N of tension has linear density 4.20 $\mathrm{g/m}$. A sinusoidal wave with amplitude 3.00 cm and wavelength 1.80 m travels along the string.

Part A

What is the maximum velocity of a particle on the string?

$$v = \sqrt{\frac{T}{\mu}}$$

$$v_{\max} = A\omega = A\left(v_{travel}k\right) = A\sqrt{\frac{T}{\mu}}\frac{2\pi}{\lambda} = \frac{2\pi A}{\lambda}\sqrt{\frac{T}{\mu}}$$

ANSWER: $\frac{2\pi A}{\lambda}\sqrt{\frac{T}{\mu}}$ m/s

Problem 20.71

Description: The sound intensity r_1 from a wailing tornado siren is I_1. (a) What is the intensity at r_2? (b) The weakest intensity likely to be heard over background noise is approx 1 mu W/m^2. Estimate the maximum distance at which the siren can be heard.

The sound intensity 40.0 m from a wailing tornado siren is 0.190 $\mathrm{W/m^2}$.

Part A

What is the intensity at 1000 m?

$$I_1 r_1^2 = I_2 r_2^2$$

ANSWER: $\frac{I_1 r_1^2}{r_2^2} \cdot 1000000$ $\mu\mathrm{W/m^2}$

Part B

The weakest intensity likely to be heard over background noise is $\approx 1\mu\mathrm{W/m^2}$. Estimate the maximum distance at which the siren can be heard.

$$r_2 = \sqrt{\frac{I_1 r_1^2}{I_2}}\Big|_{I_2=1} = r_1\sqrt{I_1}$$

ANSWER: $r_1\sqrt{I_1}$ km

Problem 20.74

A physics professor demonstrates the Doppler effect by tying a 600 Hz sound generator to a 1.0-m-long rope and whirling it around her head in a horizontal circle at 100 rpm.

Part A

What is the highest frequency heard by a student in the classroom?

$$\omega = 100rpm = \frac{2\pi * 100}{60} = 6.28s^{-1}$$

$$v_{\max} = \omega A = 6.28m/s$$

$$f_{\substack{\max \\ \min}} = \frac{f_0}{1 \mp v_{\max}/v} = \begin{matrix} 619Hz \\ 582Hz \end{matrix}$$

ANSWER: 619 Hz

Part B

What is the lowest frequency heard by a student in the classroom?

ANSWER: 582 Hz

Problem 20.76

A starship approaches its home planet at a speed of 0.1 c. When it is 54×10^6 km away, it uses its green laser beam $(\lambda = 540 \text{ nm})$ to signal its approach.

Part A

How long does the signal take to travel to the home planet?

$$t = \frac{l}{c} = \frac{5.4*10^{10}m}{3.0*10^8 m/s} = 180s$$

ANSWER: 180 s

Part B

At what wavelength is the signal detected on the home planet?

$$\lambda = \lambda_0 - 0.1cT = \lambda_0 - 0.1c\left(\frac{\lambda_0}{c}\right) = 486\text{nm}$$

ANSWER: 488 nm