Assignment Display Mode: View Printable Answers

# phy260S08

## homework 13

**Due at 11:00pm on Wednesday, May 14, 2008**

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## Problem 30.50

**Description:** A 15-cm-long nichrome wire is connected across the terminals of a 1.5 V battery. (a) What is the electric field inside the wire? (b) What is the current density inside the wire? (c) If the current in the wire is 2.0 A, what is the wire's diameter?

A 15-cm-long nichrome wire is connected across the terminals of a 1.5 V battery.

### Part A

What is the electric field inside the wire?

ANSWER: $1.00 \cdot 10^{1}$ V/m

### Part B

What is the current density inside the wire?

ANSWER: $6.67 \cdot 10^{6}$ $A/m^2$

### Part C

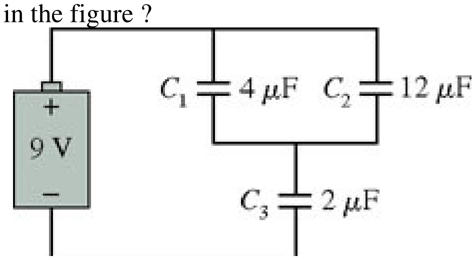
If the current in the wire is 2.0 A, what is the wire's diameter?

ANSWER: 0.618 mm

## Problem 30.64

**Description:** What are the charge on and the potential difference across each capacitor in the figure ? (a) Q_1... (b) V_1... (c) Q_2... (d) V_2... (e) Q_3... (f) V_3...

What are the charge on and the potential difference across each capacitor in the figure ?



**Part A**

$Q_1$

ANSWER: 4.00 $\mu C$

**Part B**

$V_1$

ANSWER: 1.00 V

**Part C**

$Q_2$

ANSWER: 12.0 $\mu C$

**Part D**

$V_2$

ANSWER: 1.00 V

**Part E**

$Q_3$

ANSWER: 16.0 $\mu C$

**Part F**

$V_3$

ANSWER: 8.00 V

## Problem 30.74

**Description:** A parallel-plate capacitor is constructed from two a * a electrodes spaced d_1 apart. The capacitor plates are charged to Q, then disconnected from the battery. (a) How much energy is stored in the capacitor? (b) Insulating handles are used to ...

A parallel-plate capacitor is constructed from two 7.00 $\mathrm{cm} \times 7.00\ \mathrm{cm}$ electrodes spaced 0.500 $\mathrm{mm}$ apart. The capacitor plates are charged to 10.0 $\mathrm{nC}$, then disconnected from the battery.

**Part A**

How much energy is stored in the capacitor?

ANSWER: $\frac{d_1 (Q^2)(10^{18})}{2 \cdot 8.85 a^2}$ $\mu J$

Part B

Insulating handles are used to pull the capacitor plates apart until the spacing is 2.00 $\mathrm{mm}$. Now how much energy is stored in the capacitor?

ANSWER: $\frac{d_2 (Q^2)(10^{18})}{2 \cdot 8.85 a^2}$ $\mu J$

## Kirchhoff's Rules and Applying Them

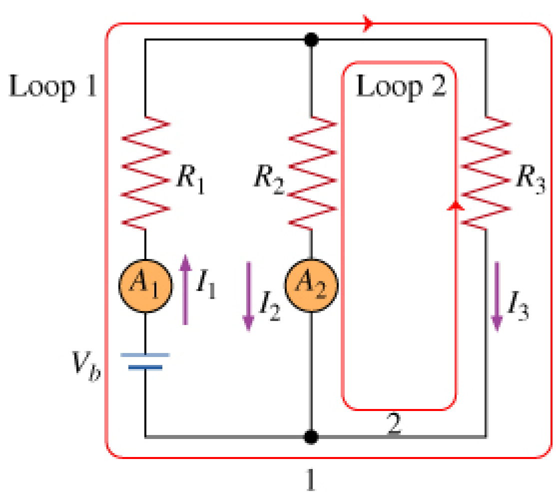
**Description:** Questions elicit the physics behind Kirchoff's rules. Then gives example and suggestions on how to use them.

**Learning Goal:** To understand the origins of both of Kirchhoff's rules and how to use them to solve a circuit problem.

This problem introduces Kirchhoff's two rules for circuits:

- *Kirchhoff's loop rule*: The sum of the voltage changes across the circuit elements forming any closed loop is zero.
- *Kirchhoff's junction rule*: The algebraic sum of the currents into (or out of) any junction in the circuit is zero.

The figure shows a circuit that illustrates the concept of *loops*, which are colored red and labeled loop 1 and loop 2. Loop 1 is the loop around the entire circuit, whereas loop 2 is the smaller loop on the right. To apply the loop rule you would add the voltage changes of all circuit elements around the chosen loop. The figure contains two junctions (where three or more wires meet)--they are at the ends of the resistor labeled $R_3$. The battery supplies a constant voltage $V_b$, and the resistors are labeled with their resistances. The ammeters are ideal meters that read $I_1$ and $I_2$ respectively.




The direction of each loop and the direction of each current arrow that you draw on your own circuits are arbitrary. Just assign voltage drops consistently and sum both voltage drops and currents algebraically and you will get correct equations. If the actual current is in the opposite direction from your current arrow, your answer for that current will be negative. The direction of any loop is even less imporant: The equation obtained from a counterclockwise loop is the same as that from a clockwise loop except for a negative sign in front of every term (i.e., an inconsequential change in overall sign of the equation because it equals zero).

Part A

The junction rule describes the conservation of which quantity? Note that this rule applies only to circuits that are in a steady state.

**Hint A.1 At the junction**

Think of the analogy with water flow. If a certain current of water comes to a split in the pipe, what can you say (mathematically) about the sum of the three water currents at this junction? If this were not true, water would accumulate at the junction.

ANSWER:

- ◉ current
- ○ voltage
- ○ resistance

**Part B**

Apply the junction rule to the junction labeled with the number 1 (at the bottom of the resistor of resistance $R_2$).

**Hint B.1 Elements in series**

The current through resistance $R_1$ is not labeled. You should recognize that the current $I_1$ passing through the ammeter also passes through resistance $R_1$ because there is no junction in between the resistor and the ammeter that could allow it to go elsewhere. Similarly, the current passing through the battery must be $I_1$ also. Circuit elements connected in a string like this are said to be *in series* and the same current must pass through each element. This fact greatly reduces the number of independent current values in any practical circuit.

**Answer in terms of given quantities, together with the meter readings $I_1$ and $I_2$ and the current $I_3$.**

ANSWER:

$$\Sigma I = 0 = \begin{array}{l} I_3 + I_2 - I_1 \\ -I_3 - I_2 + I_1 \end{array}$$

If you apply the juncion rule to the junction above $R_2$, you should find that the ezpression you get is equivalent to what you just obtained for the junction labeled 1. Obviously the conservation of charge or current flow enforces the same relationship among the currents when they separate as when they recombine.

**Part C**

Apply the loop rule to loop 2 (the smaller loop on the right). Sum the voltage changes across each circuit element around this loop going in the direction of the arrow. Remember that the current meter is ideal.

**Hint C.1 Elements in series have same current**

The current through the ammeter is $I_1$, and this current has to go through the resistor of resistance $R_1$ because there is no junction in between that could add or subtract current. Similarly, the current passing through the battery must be $I_1$ also. Circuit elements connected in a string like this are said to be *in series* and the same current must pass through each element. This fact greatly reduces the number of independent current values in any practical circuit.

**Hint C.2 Sign of voltage across resistors**

In determining the signs, note that if your chosen loop traverses a particular resistor in the *same* direction as the current through that resistor, then the end it enters through will have a more positive potential than the end from which it exits by the amount $IR$. Thus the voltage change across that resistor will be negative. Conversely, if your chosen loop traverses the resistor in the opposite direction from its current arrow, the voltage changes across the resistor will be positive. Let these conventions govern your equations (i.e., don't try to figure out the direction of current flow when using the Kirchhoff loop--decide when you put the current arrows on the resistors and stick with that choice).

**Hint C.3** **Voltage drop across ammeter**

An ideal ammeter has zero resistance. Hence there is no voltage drop across it.

**Express the voltage drops in terms of $V_b$, $I_2$, $I_3$, the given resistances, and any other given quantities.**

ANSWER: $\Sigma(\Delta V) = 0 =$ $I_3R_3 - I_2R_2$
$- I_3R_3 + I_2R_2$

**Part D**

Now apply the loop rule to loop 1 (the larger loop spanning the entire circuit). Sum the voltage changes across each circuit element around this loop going in the direction of the arrow.

**Express the voltage drops in terms of $V_b$, $I_1$, $I_3$, the given resistances, and any other given quantities.**

ANSWER: $\Sigma(\Delta V) = 0 =$ $V_b - I_1R_1 - I_3R_3$
$-(V_b - I_1R_1 - I_3R_3)$

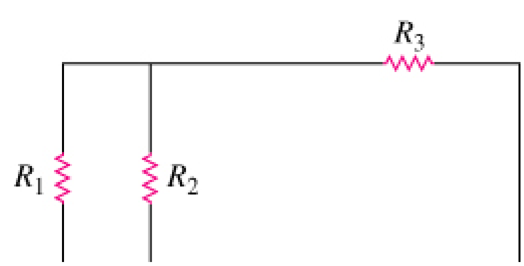
There is one more loop in this circuit, the inner loop through the battery, both ammeters, and resistors $R_1$ and $R_2$. If you apply Kirchhoff's loop rule to this additional loop, you will generate an extra equation that is redundant with the other two. In general, you can get enough equations to solve a circuit by either

1. selecting all of the internal loops (loops with no circuit elements inside the loop) or
2. using a number of loops (not necessarily internal) equal to the number of internal loops, with the extra proviso that at least one loop pass through each circuit element.

## Equivalent Resistance

**Description:** Find the equivalent resistance of a network of resistors with series and parallel connections. The network geometry gets progressively more complicated by adding more resistors.

Consider the network of four resistors shown in the diagram, where $R_1$ = 2.00 $\Omega$, $R_2$ = 5.00 $\Omega$, $R_3$ = 1.00 $\Omega$, and $R_4$ = 7.00 $\Omega$. The resistors are connected to a constant voltage of magnitude $V$.



**Part A**

Find the equivalent resistance $R_A$ of the resistor network.

**Hint A.1 How to reduce the network of resistors**

The network of resistors shown in the diagram is a combination of series and parallel connections. To determine its equivalent resistance, it is most convenient to reduce the network in successive stages. First compute the equivalent resistance of the parallel connection between the resistors $R_1$ and $R_2$, and imagine replacing the connection with a resistor with such resistance. The resulting network will consist of three resistors in series. Then find their equivalent resistance, which will also be the equivalent resistance of the original network.

**Part A.2 Find the resistance equivalent to $R_1$ and $R_2$**

Find the equivalent resistance $R_{12}$ of the parallel connection between the resistors $R_1$ and $R_2$.

**Hint A.2.a Two resistors in parallel**

Consider two resistors of resistance $R_a$ and $R_b$ that are connected in parallel. They are equivalent to a resistor with resistance $R_{eq}$, which satisfies the following relation:

$$\frac{1}{R_{eq}} = \frac{1}{R_a} + \frac{1}{R_b}.$$

**Express your answer in ohms.**

ANSWER: $R_{12} = \frac{R_1 R_2}{R_1 + R_2}$ $\Omega$

If you replace the resistors $R_1$ and $R_2$ with an equivalent resistor with resistance $R_{12}$, the resulting network will consist of three resistors $R_{12}$, $R_3$, and $R_4$ connected in series. Their equivalent resistance is also the equivalent resistance of the original network.

**Hint A.3 Three resistors in series**

Consider three resistors of resistance $R_a$, $R_b$, and $R_c$ that are connected in series. They are equivalent to a resistor with resistance $R_{eq}$, which is given by

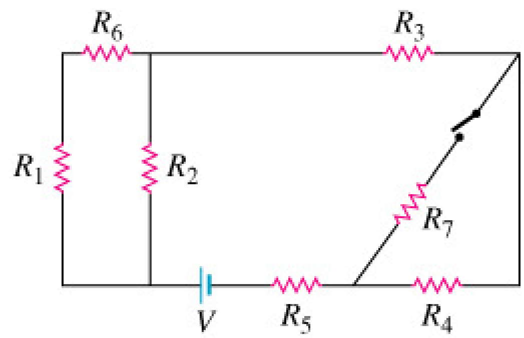
$$R_{eq} = R_a + R_b + R_c.$$

**Express your answer in ohms.**

ANSWER: $R_A = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_4$ $\Omega$

**Part B**

Two resistors of resistance $R_5$ = 3.00 $\Omega$ and $R_6$ = 3.00 $\Omega$ are added to the network, and an additional resistor of resistance $R_7$ = 3.00 $\Omega$ is connected by a switch, as shown in the diagram.. Find the equivalent resistance $R_{\rm B}$ of the new resistor network when the switch is open.




**Hint B.1 How to reduce the extended network of resistors**

Since the switch is open, no current passes through the resistor $R_7$, which can be ignored then. As you did in Part A, reduce the network in successive stages. Note that the new resistor $R_5$ is in series with the resistors $R_3$ and $R_4$, while the new resistor $R_6$ is in series with $R_1$.

**Part B.2 Find the resistance equivalent to $R_1$, $R_2$, and $R_6$**

Find the resistance $R_{126}$ equivalent to the resistor connection with $R_1$, $R_2$, and $R_6$.

**Part B.2.a Find the resistance equivalent to $R_1$ and $R_6$**

Find the resistance $R_{16}$ equivalent to the connection between $R_1$ and $R_6$.

**Hint B.2.a.i Two resistors in series**

Consider two resistors of resistance $R_{\rm a}$, and $R_{\rm b}$ that are connected in series. They are equivalent to a resistor with resistance $R_{\rm eq}$, which is given by

$$R_{\rm eq} = R_{\rm a} + R_{\rm b}.$$

**Express your answer in ohms.**

ANSWER: $R_{16}$ = $R_1 + R_6$ $\Omega$

If you replace the resistors $R_1$ and $R_6$ with their equivalent resistor (of resistance $R_{16}$), the resistor $R_2$ will result in parallel with $R_{16}$.

**Hint B.2.b Two resistors in parallel**

Consider two resistors of resistance $R_\mathrm{a}$ and $R_\mathrm{b}$ that are connected in parallel. They are equivalent to a resistor with resistance $R_\mathrm{eq}$, which satisfies the following relation:

$$\frac{1}{R_\mathrm{eq}} = \frac{1}{R_\mathrm{a}} + \frac{1}{R_\mathrm{b}}.$$

**Express your answer in ohms.**

ANSWER: $R_{126} = \frac{(R_1 + R_6)\,R_2}{R_1 + R_6 + R_2}$ Ω

If you replace the resistors $R_1$, $R_2$, and $R_6$ with an equivalent resistor with resistance $R_{126}$, the resulting network will consist of four resistors—$R_{126}$, $R_3$, $R_4$, and $R_5$—all connected in series. Their equivalent resistance is also the equivalent resistance of the original network.

**Hint B.3 Four resistors in series**

Consider four resistors of resistance $R_\mathrm{a}$, $R_\mathrm{b}$, $R_\mathrm{c}$, and $R_\mathrm{d}$ that are connected in series. They are equivalent to a resistor with resistance $R_\mathrm{eq}$, which is given by

$$R_\mathrm{eq} = R_\mathrm{a} + R_\mathrm{b} + R_\mathrm{c} + R_\mathrm{d}.$$

**Express your answer in ohms.**

ANSWER: $R_\mathrm{B} = \frac{(R_1 + R_6)\,R_2}{R_1 + R_6 + R_2} + R_3 + R_4 + R_5$ Ω

## Part C

Find the equivalent resistance $R_\mathrm{C}$ of the resistor network described in Part B when the switch is closed.

**Hint C.1 How to reduce the network of resistors when the switch is closed**

When the switch is closed, current passes through the resistor $R_7$; therefore the resistor must be included in the calculation of the equivalent resistance. Also when the switch is closed, the resistor $R_4$ is no longer connected in series with the resistors $R_3$ and $R_5$, as was the case when the switch was open. Instead, now $R_4$ is in parallel with $R_7$ and their equivalent resistor will be in series with $R_3$ and $R_5$.

**Part C.2 Find the resistance equivalent to $R_4$ and $R_7$**

Find the equivalent resistance $R_{47}$ of the parallel connection between the resistors $R_4$ and $R_7$.

**Hint C.2.a Two resistors in parallel**

Consider two resistors of resistance $R_a$ and $R_b$ that are connected in parallel. They are equivalent to a resistor with resistance $R_{eq}$, which satisfies the following relation:

$$\frac{1}{R_{eq}} = \frac{1}{R_a} + \frac{1}{R_b}.$$

**Express your answer in ohms.**

ANSWER: $R_{47} = \frac{R_4 R_7}{R_4 + R_7}$ Ω

If you replace the resistors $R_4$ and $R_7$ with their equivalent resistor (of resistance $R_{47}$), and the resistors $R_1$, $R_2$ and $R_6$ with their equivalent resistor (of resistance $R_{126}$), calculated in Part B, the resulting network will consist of four resistors—$R_{126}$, $R_3$, $R_5$, and $R_{47}$—all connected in series. Their equivalent resistance is also the equivalent resistance of the original network.

Hint C.3 **Four resistors in series**

Consider four resistors of resistance $R_a$, $R_b$, $R_c$, and $R_d$ that are connected in series. They are equivalent to a resistor with resistance $R_{eq}$, which is given by

$$R_{eq} = R_a + R_b + R_c + R_d.$$

**Express your answer in ohms.**

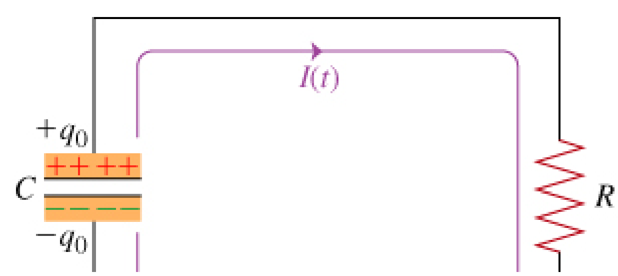
ANSWER: $R_C = \frac{(R_1 + R_6) R_2}{R_1 + R_6 + R_2} + R_3 + R_5 + \frac{R_7 R_4}{R_7 + R_4}$ Ω

## An R-C Circuit

**Description:** Derive exponential decay in an R-C circuit with the capacitor initially charged.

**Learning Goal:** To understand the behavior of the current and voltage in a simple R-C circuit

A capacitor with capacitance $C$ is initially charged with charge $q_0$. At time $t = 0$ a resistor with resistance $R$ is connected across the capacitor.



**Part A**

Use the Kirchhoff loop rule and Ohm's law to express the voltage across the capacitor $V(t)$ in terms of the current $I(t)$ flowing through the circuit.

**Express your answer in terms of $I(t)$ and $R$.**

ANSWER: $V(t) = I(t)R$

**Part B**

We would like to use the relation $V(t) = I(t)R$ to find the voltage and current in the circuit as functions of time.

To do so, we use the fact that current can be expressed in terms of the voltage. This will produce a differential equation relating the voltage $V(t)$ to its derivative. Rewrite the right-hand side of this relation, replacing $I(t)$ with an expression involving the time derivative of the voltage.

**Part B.1 How to approach the problem**

Which of the following are true statements about the various time-dependent quantities in this problem?

a. $V(t)$ and $I(t)$ are simply related by the definition of capacitance.
b. $V(t)$ is related to $q(t)$ by the definition of capacitance.
c. $I(t)$ is simply related to the derivative of $q(t)$.
d. The integral of $I(t)$ is simply related to $q(t)$.

ANSWER:

- ○ a only
- ○ b only
- ● all but a
- ○ all but b
- ○ a and c
- ○ none of them

**Part B.2 Find the relation between $I(t)$ and $V(t)$**

What is the relationship between the current $I(t)$ in the circuit and the voltage $V(t)$ across the capacitor? Use $dV(t)/dt$ for the derivative of the voltage.

**Part B.2.a Find the relation between $I(t)$ and $q(t)$**

What is the relationship between the current in the circuit $I(t)$ and the charge $q(t)$ on the capacitor? Use $dq(t)/dt$ for the derivative of the charge.

**Express your answer in terms of $dq(t)/dt$.**

ANSWER:

$I(t) = \frac{-dq(t)}{dt}$

**Part B.2.b Find the relationship between $q(t)$ and $V(t)$**

What relationship between $q(t)$ and $V(t)$ does the presence of the capacitor enforce?

**Express your answer in terms of $V(t)$ and any other quantities given in the problem introduction.**

ANSWER: $q(t) = CV(t)$

**Express your answer in terms of $dV(t)/dt$ and quantities given in the problem introduction.**

ANSWER: $I(t) = \frac{-CdV(t)}{dt}$

**Express your answer in terms of $dV(t)/dt$ and quantities given in the problem introduction.**

ANSWER: $V(t) = \frac{-CdV(t)}{dt}R$

**Part C**

Now solve the differential equation $V(t) = -CR\frac{dV(t)}{dt}$ for the initial conditions given in the problem introduction to find the voltage as a function of time for any time $t$.

**Part C.1 Find the voltage at time $t = 0$**

What is the voltage $V_0$ across the capacitor described in the problem introduction at time $t = 0$?

**Express your answer in terms of quantities given in the problem introduction.**

ANSWER: $V_0 = \frac{q_0}{C}$

**Hint C.2 Method 1: Guessing the form of the solution**

A function whose derivative is proportional to itself is $V(t) = A\exp\lambda t$. Find $A$ and $\lambda$ in the above equation that satisfy the boundary condition $V(t=0) = V_0$ as well as the given differential equation.

**Hint C.3 Method 2: Separation of variables**

Separating variables, the differential equation can be rewritten as

$$\frac{dV}{V} = -\frac{dt}{CR}.$$

Integrating both sides, you would get

$$\int_{V_0}^{V} \frac{dV}{V} = \int_0^t -\frac{dt}{CR}.$$

Evaluate the integrals on both sides to obtain an expression for $V(t)$.

**Express your answer in terms of $q_0$, $C$, $R$, and $t$.**

ANSWER: $V(t) = \frac{q_0}{C}e^{\frac{-t}{RC}}$

If there were a battery in the circuit with EMF $\mathcal{E}$, the equation for $V(t)$ would be $V(t) = \mathcal{E} - RC\frac{dV(t)}{dt}$.

This differential equation is no longer homogeneous in $V(t)$ (homogeneous means that if you multiply any solution by a constant it is still a solution). However, it can be solved simply by the substitution $V_b(t) = V(t) - \mathcal{E}$. The effect of this substitution is to eliminate the $\mathcal{E}$ term and yield an equation for $V_b(t)$ that is identical to the equation you solved for $V(t)$. If a battery is added, the initial condition is usually that the capacitor has zero charge at time $t = 0$. The solution under these conditions will look like $V(t) = \mathcal{E}(1 - e^{-t/(RC)})$. This solution implies that the voltage across the capacitor is zero at time $t = 0$ (since the capacitor was uncharged then) and rises asymptotically to $\mathcal{E}$ (with the result that current essentially stops flowing through the circuit).

### Part D

Given that the voltage across the capacitor as a function of time is $V(t) = \frac{q_0}{C}e^{-t/(RC)}$, what is the current $I(t)$ flowing through the resistor as a function of time (for $t > 0$)? It might be helpful to look again at Part A of this problem.

**Part D.1 Apply Ohm's law**

Because the capacitor and resistor are connected in series, the voltage across each is the same. Given the voltage $V(t)$ across the capacitor for $t > 0$, what is the current that flows in the resistor?

**Express your answer in terms of $V(t)$ and any quantities given in the problem introduction.**

ANSWER: $I(t) = \frac{V(t)}{R}$

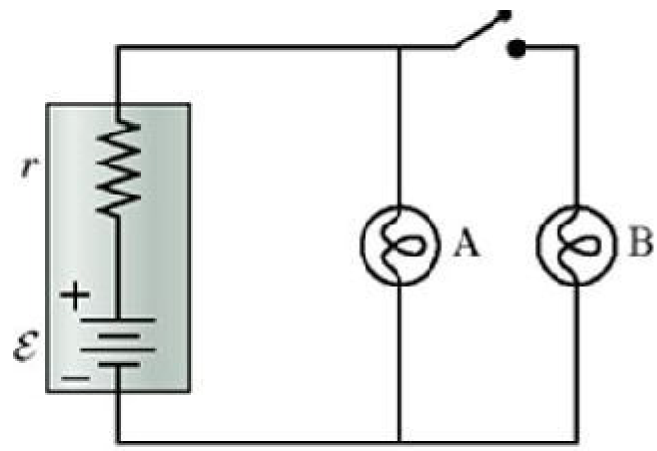
**Express your answer in terms of $t$ and any quantities given in the problem introduction.**

ANSWER: $I(t) = \frac{q_0}{RC} e^{\frac{-t}{RC}}$

## Problem 31.62

**Description:** For an ideal battery r=0 Omega, closing the switch in the figure does not affect the brightness of bulb A. In practice, bulb A dims just a little when the switch closes. To see why, assume that the 1.5 V battery has an internal resistance r =0.50 ...

For an ideal battery $(r = 0\ \Omega)$, closing the switch in the figure does not affect the brightness of bulb A. In practice, bulb A dims *just a little* when the switch closes. To see why, assume that the 1.5 V battery has an internal resistance $r = 0.50\ \Omega$ and that the resistance of a glowing bulb is $R = 6\ \Omega$.




### Part A

What is the current through bulb A when the switch is open?

ANSWER: 0.231 A

### Part B

What is the current through bulb A after the switch has closed?

ANSWER: 0.214 A

### Part C

By what percent does the current through A change when the switch is closed?

ANSWER: $-7.36$ %

### Part D

Would the current through A change when the switch is closed if $r = 0\ \Omega$?

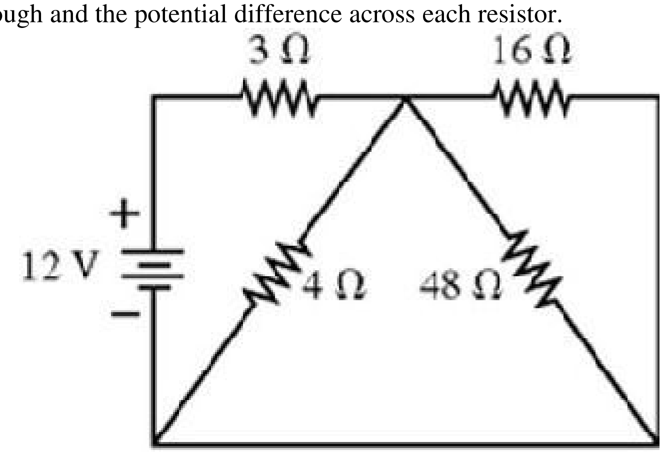
ANSWER:

Yes
No

## Problem 31.68

**Description:** For the circuit shown in the figure , find the current through and the potential difference across each resistor. (a) Find the current through 3 Omega resistor. (b) Find the potential difference across 3 Omega resistor. (c) Find the current through ...

For the circuit shown in the figure , find the current through and the potential difference across each resistor.




**Part A**

Find the current through 3 $\Omega$ resistor.

**ANSWER:** 2.00 A

**Part B**

Find the potential difference across 3 $\Omega$ resistor.

**ANSWER:** 6.00 V

**Part C**

Find the current through 4 $\Omega$ resistor.

**ANSWER:** 1.50 A

**Part D**

Find the potential difference across 4 $\Omega$ resistor.

**ANSWER:** 6.00 V

**Part E**

Find the current through 48 $\Omega$ resistor.

ANSWER: 0.125 A

**Part F**

Find the potential difference across 48 $\Omega$ resistor.

ANSWER: 6.00 V

**Part G**

Find the current through 16 $\Omega$ resistor.

ANSWER: 0.375 A

**Part H**

Find the potential difference across 16 $\Omega$ resistor.

ANSWER: 6.00 V

## Problem 31.74

**Description:** A 0.25 mu(F) capacitor is charged to 50 V. It is then connected in series with a 25 Omega resistor and a 100 Omega resistor and allowed to discharge completely. (a) How much energy is dissipated by the 25 Omega resistor?

A $0.25\ \mu\mathrm{F}$ capacitor is charged to 50 V. It is then connected in series with a $25\ \Omega$ resistor and a $100\ \Omega$ resistor and allowed to discharge completely.

**Part A**

How much energy is dissipated by the $25\ \Omega$ resistor?

ANSWER: $62.5 \cdot 10^{-6}$ J

| Summary | 0 of 9 items complete (0% avg. score)<br>0 of 90 points |
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