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# phy260S08

## homework 12

Due at 11:00pm on Friday, May 9, 2008

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## Problem 29.41

**Description:** Two small metal spheres with masses 2.0 g and 4.0 g are tied together by a r-long massless string and are at rest on a frictionless surface. Each is charged to + Q. (a) What is the energy of this system? (b) What is the tension in the string?

Two small metal spheres with masses 2.0 g and 4.0 g are tied together by a 4.60 $\mathrm{cm}$-long massless string and are at rest on a frictionless surface. Each is charged to $+\ 2.10\ \mu\mathrm{C}$.

### Part A

What is the energy of this system?

ANSWER: $\frac{9.0\cdot10^9 Q^2}{r}$ J

### Part B

What is the tension in the string?

ANSWER: $\frac{9.0\cdot10^9 Q^2}{r^2}$ N

## Problem 29.45

**Description:** Two d-diameter disks spaced r apart form a parallel-plate capacitor. The electric field between the disks is E. (a) What is the voltage across the capacitor? (b) How much charge is on each disk? (c) An electron is launched from the negative plate...

Two 2.10 $\mathrm{cm}$-diameter disks spaced 1.50 $\mathrm{mm}$ apart form a parallel-plate capacitor. The electric field between the disks is $4.50\times10^5\ \mathrm{V/m}$.

### Part A

What is the voltage across the capacitor?

ANSWER: $Er$ V

### Part B

How much charge is on each disk?

ANSWER: $3.14\left(\frac{d}{2}\right)^2 E\cdot 8.85\cdot10^{-12}$ C

**Part C**

An electron is launched from the negative plate. It strikes the positive plate at a speed of $2.10\times10^{7}$ $\mathrm{m/s}$. What was the electron's speed as it left the negative plate?

ANSWER:

$$\sqrt{v^2-\frac{\frac{2\cdot1.6\cdot10^{-19}Er}{9.11}}{10^{-31}}}$$ m/s

## Problem 29.50

**Description:** (a) What is the escape speed of an electron launched from the surface of a d-diameter plastic sphere that has been charged to Q?

**Part A**

What is the escape speed of an electron launched from the surface of a 0.800 $\mathrm{cm}$-diameter plastic sphere that has been charged to 8.00 $\mathrm{nC}$?

ANSWER:

$$\sqrt{\frac{\frac{\frac{2\cdot1.60\cdot10^{-19}}{9.11}}{10^{-31}}\cdot9.0\cdot10^{9}Q\cdot2}{d}}$$ m/s

## Electric Potential Energy versus Electric Potential

**Description:** Introduces concept of electric potential energy and its relationship to the electrostatic force by analogy with the gravitational potential/force. Compares electric potential energy with electric potential.

**Learning Goal:** To understand the relationship and differences between electric potential and electric potential energy.

In this problem we will learn about the relationships between electric force $\vec{F}$, electric field $\vec{E}$, potential energy $U$, and electric potential $V$. To understand these concepts, we will first study a system with which you are already familiar: the uniform gravitational field.

**Gravitational Force and Potential Energy**

First we review the force and potential energy of an object of mass $m$ in a uniform gravitational field that points downward (in the $-\hat{z}$ direction), like the gravitational field near the earth's surface.

**Part A**

Find the force $\vec{F}(z)$ on an object of mass $m$ in the uniform gravitational field when it is at height $z=0$.

**Express $\vec{F}(z)$ in terms of $m$, $z$, $\hat{z}$, and $g$.**

ANSWER:

$\vec{F}(z) =$ $-mg\hat{z}$

$-mg\hat{z}$

Because we are in a uniform field, the force does not depend on the object's location. Therefore, the variable $z$ does not appear in the correct answer.

**Part B**

Now find the gravitational potential energy $U(z)$ of the object when it is at an arbitrary height $z$. Take zero potential to be at position $z = 0$.

**Express $U(z)$ in terms of $m$, $z$, and $g$. Note that because potential energy is a scalar, and not a vector, there will be no unit vector in the answer.**

ANSWER: $U(z) =$ $mgz$
$mgz$

### Part C

In what direction does the object accelerate when released with initial velocity upward?

ANSWER:
- upward
- downward
- upward or downward depending on its mass
- downward only if the ratio of $g$ to initial velocity is large enough

### Electric Force and Potential Energy

Now consider the analogous case of a particle with charge $q$ placed in a uniform electric field of strength $E$, pointing downward (in the $-\hat{z}$ direction)

### Part D

Find $\vec{F}(z)$, the electric force on the charged particle at height $z$.

**Hint D.1 Relationship between force and electric field**

The force on a particle of charge $q$ in an electric field $\vec{E} = -E\hat{z}$ is given by $\vec{F} = q\vec{E}$.

**Express $\vec{F}(z)$ in terms of $q$, $E$, $z$, and $\hat{z}$.**

ANSWER: $\vec{F}(z) =$ $-qE\hat{z}$
$-qE\hat{z}$

### Part E

Now find the potential energy $U(z)$ of this charged particle when it is at height $z$. Take zero potential to be at position $z = 0$.

**Express $U(z)$ (a scalar quantity) in terms of $q$, $E$, and $z$.**

ANSWER: $U(z) =$ $qEz$
$qEz$

### Part F

In what direction does the charged particle accelerate when released with upward initial velocity?

ANSWER:
- upward
- downward
- upward or downward depending on its charge
- downward only if the ratio of $qE$ to initial velocity is large enough

**Electric Field and Electric Potential**

The electric potential $V$ is defined by the relationship $U = qV$, where $U$ is the electric potential energy of a particle with charge $q$.

Part G

Find the electric potential $V$ of the uniform electric field $\vec{E} = E\hat{z}$. Note that this field is not pointing in the same direction as the field in the previous section of this problem. Take zero potential to be at position $z = 0$.

**Express $V$ in terms of $q$, $E$, and $z$.**

ANSWER: $V =$ $-Ez$
$-Ez$

Part H

The SI unit for electric potential is the *volt*. The volt is a derived unit, which means that it can be written in terms of other SI units. What are the dimensions of the volt in terms of the fundamental SI units?

**Express your answer in terms of the standard abbreviations for the fundamental SI units: m (meters), kg (kilograms), s (seconds), and C (coulombs)**

ANSWER: volts = $\frac{kgm^2}{s^2C}$
$\frac{kgm^2}{s^3A}$

Part I

The electric field can be derived from the electric potential, just as the electrostatic force can be determined from the electric potential energy. The relationship between electric field and electric potential is $\vec{E} = -\vec{\nabla}V$, where $\vec{\nabla}$ is the gradient operator:

$$\vec{\nabla}V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}.$$

The partial derivative $\frac{\partial V}{\partial x}$ means the derivative of $V$ with respect to $x$, holding all other variables constant.

Consider again the electric potential $V = -Ez$ corresponding to the field $\vec{E} = E\hat{z}$. This potential depends on the $z$ coordinate only, so $\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0$ and $\frac{\partial V}{\partial z} = \frac{dV}{dz}$.

Find an expression for the electric field $\vec{E}$ in terms of the derivative of $V$.

**Express your answer as a vector in terms of the unit vectors $\hat{x}$, $\hat{y}$, and/or $\hat{z}$. Use $dV/dz$ for the derivative of $V$ with respect to $z$.**

ANSWER: $\vec{E} = \frac{-dV}{dz}\hat{z}$

Part J

A positive test charge will accelerate toward regions of _______ electric potential and _______ electric potential energy.

Part J.1 **Direction of the electric field**

In what direction do electric fields point?

ANSWER:
- (selected) from regions of higher electric potential to lower electric potential
- from regions of lower electric potential to higher electric potential

Hint J.2 **Formula for the force on a charge in an electric field**

The force $\vec{F}$ on a charge $q$ in an electric field $\vec{E}$ is given by

$$\vec{F} = q\vec{E}.$$

This is similar to the equation $F = mg$ for the force on a mass in a uniform gravitational field.

Hint J.3 **Formula for electric potential energy**

The electric potential energy $U$ of a charge $q$ at electric potential $V$ is given by

$$U = qV.$$

This is similar to the equation $U_g = mV_g = mgh$, for the gravitational potential energy of a particle with mass $m$.

**Choose the appropriate answer combination to fill in the blanks correctly.**

ANSWER:
- higher; higher
- higher; lower
- lower; higher
- (selected) lower; lower

**Part K**

A negative test charge will accelerate toward regions of ________ electric potential and ________ electric potential energy.

Part K.1 **Direction of the electric field**

In what direction do electric fields point?

ANSWER:
- (selected) from regions of higher electric potential to lower electric potential
- from regions of lower electric potential to higher electric potential

Hint K.2 **Formula for the force on a charge in an electric field**

The force $\vec{F}$ on a charge $q$ in an electric field $\vec{E}$ is given by

$$\vec{F} = q\vec{E}.$$

This is similar to the equation $F = mg$ for the force on a mass in a uniform gravitational field.

Hint K.3 **Formula for electric potential energy**

The electric potential energy $U$ of a charge $q$ at electric potential $V$ is given by

$$U = qV.$$

This is similar to the equation $U_g = mV_g = mgh$ for the gravitational potential energy of a particle with mass $m$. It works even for negative charges. Negative masses are not known to exist.

**Choose the appropriate answer combination to fill in the blanks correctly.**

ANSWER:
- ○ higher; higher
- ● higher; lower
- ○ lower; higher
- ○ lower; lower

A charge in an electric field will experience a force in the direction of *decreasing* potential energy. Since the electric potential energy of a negative charge is equal to the charge times the electric potential ($U = qV$), the direction of *decreasing* electric potential energy is the direction of *increasing* electric potential.

## Introduction to Capacitance

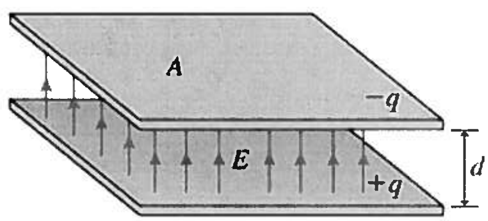
**Description:** Introduces the concept of capacitance, and the basic formula for air-filled parallel-plate capacitance

**Learning Goal:** To understand the meaning of capacitance and ways of calculating capacitance

When a positive charge $q$ is placed on a conductor that is insulated from ground, an electric field emanates from the conductor to ground, and the conductor will have a nonzero potential $V$ relative to ground. If more charge is placed on the conductor, this voltage will increase proportionately. The ratio of charge to voltage is called the *capacitance* $C$ of this conductor: $C = q/V$.

Capacitance is one of the central concepts in electrostatics, and specially constructed devices called *capacitors* are essential elements of electronic circuits. In a capacitor, a second conducting surface is placed near the first (they are often called *electrodes*), and the relevant voltage is the voltage between these two electrodes.

This tutorial is designed to help you understand capacitance by assisting you in calculating the capacitance of a parallel-plate capacitor, which consists of two plates each of area $A$ separated by a small distance $d$ with air or vacuum in between. In figuring out the capacitance of this configuration of conductors, it is important to keep in mind that the voltage difference is the line integral of the electric field between the plates.




**Part A**

What property of objects is best measured by their capacitance?

ANSWER:
- ○ ability to conduct electric current
- ○ ability to distort an external electrostatic field
- ● ability to store charge
- ○ ability to store electrostatic energy

Capacitance is a measure of the ability of a system of two conductors to store electric charge and energy. Capacitance is defined as $C = \frac{Q}{V}$. This ratio remains constant as long as the system retains its geometry and the amount of dielectric does not change. Capacitors are special devices designed to combine a large capacitance with a small size. However, any pair of conductors separated by a dielectric (or vacuum) has some capacitance. Even an isolated electrode has a small capacitance. That is, if a charge $Q$ is placed on it, its potential $V$ with respect to ground will change, and the ratio $\frac{Q}{V}$ is its capacitance $C$.

**Part B**

Assume that charge $-q$ is placed on the top plate, and $+q$ is placed on the bottom plate. What is the magnitude of the electric field $E$ between the plates?

**Part B.1** **How do you find the magnitude of the electric field?**

What is the easiest way to obtain $E$?

ANSWER:

- (•) Use Gauss's law and the fact that $E = 0$ outside the capacitor.
- ( ) Use Gauss's law and the symmetry of the lower plate.
- ( ) Use Coulomb's law integrating over all charge on the bottom plate.
- ( ) Use Coulomb's law integrating over all charge on both plates.

**Part B.2** **What is the electric flux integral due to the electric field?**

Apply Gauss's law to a small box whose top side is just above the lower plate and whose bottom is just below it, where $E = 0$. Start by finding the electric flux integral $\Phi_E$.

**Express this integral in terms of the area $a$ of the top side of the box and the magnitude $E$ of the electric field between the plates.**

ANSWER: $\Phi_E = aE$

**Part B.3** **Find the enclosed charge**

Find the amount of charge $q_{\rm encl}$ enclosed in a small box whose top side is just above the lower plate and whose bottom is just below it, where $E = 0$.

**Part B.3.a** **Find the surface charge**

What is $\sigma$, the charge per unit area on the lower plate?

**Express $\sigma$ in terms of any necessary constants and quantities given in the introduction.**

ANSWER: $\sigma = \frac{q}{A}$

**Express the enclosed charge in terms of the cross-sectional area of the box $a$ and other quantities given in the introduction.**

ANSWER: $q_{\rm encl} = \frac{aq}{A}$

**Hint B.4** **Recall Gauss's law**

Gauss's law states that $\Phi_E = \frac{q_{\rm encl}}{\epsilon_0}$.

**Express $E$ in terms of $q$ and other quantities given in the introduction, in addition to $\epsilon_0$ and any other constants needed.**

ANSWER: $E = \frac{q}{A\epsilon_0}$

**Part C**

What is the voltage $V$ between the plates of the capacitor?

Hint C.1 **The electric field is the derivative of the potential**

The voltage difference is the integral of the electric field from one plate to the other; in symbols, $V = E \cdot d$.

**Express $V$ in terms of the quantities given in the introduction and any required physical constants.**

ANSWER: $V = \frac{dq}{A\epsilon_0}$

Part D

Now find the capacitance $C$ of the parallel-plate capacitor.

**Express $C$ in terms of quantities given in the introduction and constants like $\epsilon_0$.**

ANSWER: $C = \frac{\epsilon_0 A}{d}$

You have derived the general formula for the capacitance of a parallel-plate capacitor with plate area $A$ and plate separation $d$. It is worth remembering.

Part E

Consider an air-filled charged capacitor. How can its capacitance be increased?

Hint E.1 **What does capacitance depend on?**

Capacitance depends on the inherent properties of the system of conductors, such as its geometry and the presence of dielectric, not on the charge placed on the conductors.

ANSWER:

- ○ Increase the charge on the capacitor.
- ○ Decrease the charge on the capacitor.
- ○ Increase the spacing between the plates of the capacitor.
- ◉ Decrease the spacing between the plates of the capacitor.
- ○ Increase the length of the wires leading to the capacitor plates.

Part F

Consider a charged parallel-plate capacitor. Which combination of changes would quadruple its capacitance?

ANSWER:

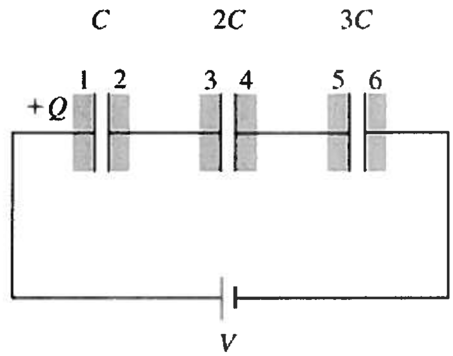
- ○ Double the charge and double the plate area.
- ○ Double the charge and double the plate separation.
- ○ Halve the charge and double the plate separation.
- ○ Halve the charge and double the plate area.
- ◉ Halve the plate separation; double the plate area.
- ○ Double the plate separation; halve the plate area.

## Capacitors in Series

**Description:** Contains several questions that help practice basic calculations for capacitors connected in series; there is a similar skill-builder ("Capacitors in Parallel") about capacitors connected in parallel.

**Learning Goal:** To understand how to calculate capacitance, voltage, and charge for a combination of capacitors connected in series.

Consider the combination of capacitors shown in the figure. Three capacitors are connected to each other in series, and then to the battery. The values of the capacitances are $C$, $2C$, and $3C$, and the applied voltage is $V$. Initially, all of the capacitors are completely discharged; after the battery is connected, the charge on plate 1 is $Q$.




Part A

What are the charges on plates 3 and 6?

Hint A.1 **Charges on capacitors connected in series**

When the plates of two adjacent capacitors are connected, the sum of the charges on the two plates must remain zero, since the pair is isolated from the rest of the circuit; that is, $Q_2 + Q_3 = 0$ and $Q_4 + Q_5 = 0$, where $Q_i$ is the charge on plate $i$.

Hint A.2 **The charges on a capacitor's plates**

When electrostatic equilibrium is reached, the charges on the two plates of a capacitor must have equal magnitude and opposite sign.

ANSWER:

- ○ $+Q$ and $+Q$
- ○ $-Q$ and $-Q$
- ⊙ $+Q$ and $-Q$
- ○ $-Q$ and $+Q$
- ○ 0 and $+Q$
- ○ 0 and $-Q$

Part B

If the voltage across the first capacitor (the one with capacitance $C$) is $V'$, then what are the voltages across the second and third capacitors?

Hint B.1 **Definition of capacitance**

The capacitance $C$ is given by $Q/V$, where $Q$ is the charge of the capacitor and $V$ is the voltage across it.

Hint B.2 **Charges on the capacitors**

As established earlier, the absolute value of the charge on each plate is $Q$: It is the same for all three capacitors and thus for all six plates.

ANSWER:

- ○ $2V'$ and $3V'$
- ⊙ $\frac{V'}{2}$ and $\frac{V'}{3}$
- ○ $V'$ and $V'$
- ○ 0 and $V'$

Part C

Find the voltage $V_1$ across the first capacitor.

**Hint C.1 How to analyze voltages**

According to the law of conservation of energy, the sum of the voltages across the capacitors must equal the voltage of the battery.

**Express your answer in terms of $V$.**

ANSWER: $V_1 = \frac{6V}{11}$

**Part D**

Find the charge $Q$ on the first capacitor.

**Express your answer in terms of $C$ and $V_1$.**

ANSWER: $Q = CV_1$

**Part E**

Using the value of $Q$ just calculated, find the equivalent capacitance $C_{eq}$ for this combination of capacitors in series.

**Hint E.1 Using the definition of capacitance**

The "equivalent" capacitor has the same charge as each of the individual capacitors: $Q$. Use the general formula $C = Q/V$ to find $C_{eq}$.

**Express your answer in terms of $C$.**

ANSWER: $C_{eq} = \frac{6C}{11}$

The formula for combining three capacitors in series is

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

How do you think this formula may be generalized to $n$ capacitors?

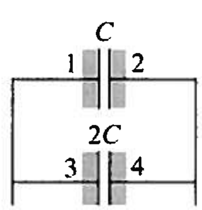
## Capacitors in Parallel

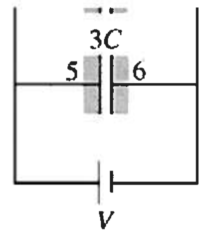
**Description:** A series of questions on basic calculations for capacitors connected in parallel; there is a similar skill-builder ("Capacitors in Series") about capacitors connected in series.

**Learning Goal:** To understand how to calculate capacitance, voltage, and charge for a parallel combination of capacitors.

Frequently, several capacitors are connected together to form a collection of capacitors. We may be interested in determining the overall capacitance of such a collection. The simplest configuration to analyze involves capacitors connected in series or in parallel. More complicated setups can often (though not always!) be treated by combining the rules for these two cases. Consider the example of a parallel combination of capacitors: Three capacitors are connected to each other and to a battery as shown in the figure.

The individual capacitances are $C$, $2C$, and $3C$, and the battery's voltage is $V$.






**Part A**

If the potential of plate 1 is $V$, then, in equilibrium, what are the potentials of plates 3 and 6? Assume that the negative terminal of the battery is at zero potential.

**Hint A.1 Electrostatic equilibrium**

When electrostatic equilibrium is reached, all objects connected by a conductor (by wires, for example) must have the same potential. Which plates on this diagram are at the same potential?

ANSWER:

- $V$ and $V$
- $2V$ and $3V$
- $V$ and $0$
- $\frac{V}{2}$ and $\frac{V}{3}$

**Part B**

If the charge of the first capacitor (the one with capacitance $C$) is $Q$, then what are the charges of the second and third capacitors?

**Hint B.1 Definition of capacitance**

Capacitance $C$ is given by $Q/V$, where $Q$ is the charge of the capacitor and $V$ is the voltage across it.

**Hint B.2 Voltages across the capacitors**

As established earlier, the voltage across each capacitor is $V$. The voltage is always the same for capacitors connected in parallel.

ANSWER:

- $2Q$ and $3Q$
- $\frac{Q}{2}$ and $\frac{Q}{3}$
- $Q$ and $Q$
- $0$ and $0$

**Part C**

Suppose we consider the system of the three capacitors as a single "equivalent" capacitor. Given the charges of the three individual capacitors calculated in the previous part, find the total charge $Q_{tot}$ for this equivalent capacitor.

**Express your answer in terms of $V$ and $C$.**

ANSWER: $Q_{tot}$ = $6CV$

**Part D**

Using the value of $Q_{tot}$, find the equivalent capacitance $C_{eq}$ for this combination of capacitors.

**Hint D.1 Using the definition of capacitance**

Use the general formula $C = Q/V$ to find $C_{eq}$. The charge on the "equivalent" capacitor is $Q_{tot}$, and the voltage across this capacitor is the voltage across the battery, $V$.

**Express your answer in terms of $C$.**

ANSWER: $C_{eq} = 6C$

The formula for combining three capacitors in parallel is

$$C_{parallel} = C_1 + C_2 + C_3.$$

How do you think this formula may be generalized to $n$ capacitors?

## Energy in Capacitors and Electric Fields

**Description:** Several questions about the energy of charged capacitors, energy density, and energy of an electrostatic field. Students are asked to calculate the energy of a charged capacitor by two different methods.

**Learning Goal:** To be able to calculate the energy of a charged capacitor and to understand the concept of energy associated with an electric field.

The energy of a charged capacitor is given by $U = QV/2$, where $Q$ is the charge of the capacitor and $V$ is the potential difference across the capacitor. The energy of a charged capacitor can be described as the energy associated with the electric field created inside the capacitor.

In this problem, you will derive two more formulas for the energy of a charged capacitor; you will then use a parallel-plate capacitor as a vehicle for obtaining the formula for the energy density associated with an electric field. It will be useful to recall the definition of capacitance, $C = Q/V$, and the formula for the capacitance of a parallel-plate capacitor,

$C = \epsilon_0 A/d$, where $A$ is the area of each of the plates and $d$ is the plate separation. As usual, $\epsilon_0$ is the permittivity of free space.

First, consider a capacitor of capacitance $C$ that has a charge $Q$ and potential difference $V$.

**Part A**

Find the energy $U$ of the capacitor in terms of $C$ and $Q$ by using the definition of capacitance and the formula for the energy in a capacitor.

**Express your answer in terms of $C$ and $Q$.**

ANSWER: $U = \frac{Q^2}{2C}$

**Part B**

Find the energy $U$ of the capacitor in terms of $C$ and $V$ by using the definition of capacitance and the formula for the energy in a capacitor.

**Express your answer in terms of $C$ and $V$.**

ANSWER: $U = \frac{CV^2}{2}$

All three of these formulas are equivalent:

$$U = \frac{QV}{2} = \frac{Q^2}{2C} = \frac{CV^2}{2}.$$

Depending on the problem, one or another may be more convenient to use. However, any one of them would give you the correct answer. Note that these formulas work for *any* type of capacitor.

Part C

A parallel-plate capacitor is connected to a battery. The energy of the capacitor is $U_0$. The capacitor remains connected to the battery while the plates are slowly pulled apart until the plate separation doubles. The new energy of the capacitor is $U$. Find the ratio $U/U_0$.

Part C.1 **Determine what remains constant**

As the plates are being pulled apart slowly, what quantity or quantities remain constant?

ANSWER:
- capacitance only
- (•) voltage only
- charge only
- both voltage and capacitance
- both voltage and charge

Since the geometry of the capacitor is changing, its capacitance changes, too. However, the voltage remains constant, since it must equal the voltage provided by the battery.

Part C.2 **Identify which formula to use**

Which formula for energy is most convenient to use in this case?

ANSWER:
- $\frac{QV}{2}$
- $\frac{Q^2}{2C}$
- (•) $\frac{CV^2}{2}$

ANSWER: $\frac{U}{U_0}$ = 0.5

Part D

A parallel-plate capacitor is connected to a battery. The energy of the capacitor is $U_0$. The capacitor is then disconnected from the battery and the plates are slowly pulled apart until the plate separation doubles. The new energy of the capacitor is $U$. Find the ratio $U/U_0$.

Part D.1 **Determine what remains constant**

As the plates are being pulled apart, what quantity or quantities remain constant?

ANSWER:
- capacitance only
- voltage only
- (•) charge only
- both voltage and charge
- both voltage and capacitance

The charge remains constant, since the capacitor is disconnected and the charge therefore literally has nowhere to go.

**Part D.2 Identify which formula to use**

Which formula for energy is most convenient to use in this case?

ANSWER:

- $\frac{QV}{2}$
- (selected) $\frac{Q^2}{2C}$
- $\frac{CV^2}{2}$

ANSWER: $\frac{U}{U_0}$ = 2

In this part of the problem, you will express the energy of various types of capacitors in terms of their geometry and voltage.

**Part E**

A parallel-plate capacitor has area $A$ and plate separation $d$, and it is charged to voltage $V$. Use the formulas from the problem introduction to obtain the formula for the energy $U$ of the capacitor.

**Express your answer in terms of $A$, $d$, $V$, and appropriate constants.**

ANSWER: $U = \frac{\epsilon_0 A V^2}{2d}$

Let us now recall that the energy of a capacitor can be thought of as the energy of the electric field inside the capacitor. The energy of the electric field is usually described in terms of *energy density* $u$, the energy per unit volume.

A parallel-plate capacitor is a convenient device for obtaining the formula for the energy density of an electric field, since the electric field inside it is nearly *uniform*. The formula for energy density can then be written as

$$u = \frac{U}{V},$$

where $U$ is the energy of the capacitor and $V$ is the *volume* of the capacitor (*not* its voltage).

**Part F**

A parallel-plate capacitor has area $A$ and plate separation $d$, and it is charged so that the electric field inside is $E$. Use the formulas from the problem introduction to find the energy $U$ of the capacitor.

Hint F.1 **How to approach the problem**

Recall that for the uniform electric field $E$ between the plates of a parallel-plate capacitor, $V = Ed$, where $V$ is the potential difference between the plates and $d$ is the distance between the two plates. You can use this relation to rewrite the equation for energy $U = \frac{1}{2}CV^2$ in terms of the electric field and the geometry of the capacitor (i.e., the area of the plates and the distance between them).

**Express your answer in terms of $A$, $d$, $E$, and appropriate constants.**

ANSWER: $U = \frac{\epsilon_0 A d E^2}{2}$

As mentioned before, we can think of the energy of the *capacitor* as the energy of the *electric field inside the capacitor*.

Part G

Find the energy density $u$ of the electric field in a parallel-plate capacitor. The magnitude of the electric field inside the capacitor is $E$.

Hint G.1 **How to approach the problem**

Since the electric field outside a parallel-plate capacitor is essentially zero, the volume that you are looking for is the volume of the space between the two plates.

Hint G.2 **Volume between the plates**

Recall that the volume $V$ of a solid with two parallel bases of the same shape and sides perpendicular to the bases is given by $V = Ah$, where $A$ is the area of each of the bases and $h$ is the distance between the bases. Note that the space between the plates of a parallel-plate capacitor is such a solid.

**Express your answer in terms of $E$ and appropriate constants.**

ANSWER: $u = \frac{\epsilon_0 E^2}{2}$

Note that the answer for $u$ does not contain any reference to the geometry of the capacitor: $A$ and $d$ do not appear in the formula. In fact, the formula

$$u = \frac{\epsilon_0 E^2}{2}$$

describes the energy density in *any* electrostatic field, whether created by a capacitor or any other source.

| Summary | 0 of 8 items complete (0% avg. score)<br>0 of 80 points |
|---|---|

**29.58. Model:** The electric field inside a capacitor is uniform.
**Solve:** (a) While the capacitor is attached to the battery, the plates are at the same potentials as the terminals of the battery. Thus, the potential difference across the capacitor is $\Delta V_C = 15$ V. The electric field strength inside the capacitor is

$$E = \frac{\Delta V_C}{d} = \frac{15\ \text{V}}{0.5\times10^{-2}\ \text{m}} = 3000\ \text{V/m}$$

Because $E = \eta/\varepsilon_0 = Q/A\varepsilon_0$, the charge on each plate is

$$Q = EA\varepsilon_0 = (3000\ \text{V/m})\ \pi\ (0.05\ \text{m})^2\ (8.85\times10^{-12}\ \text{C}^2/\text{N m}^2) = 2.09\times10^{-10}\ \text{C}$$

(b) After the electrodes are pulled away to a new separation of $d' = 1.0$ cm, the potential difference across the capacitor stays the same as before. That is, $\Delta V_C' = \Delta V_C = 15$ V. The electric field strength inside the capacitor is

$$E' = \frac{\Delta V'}{d'} = \frac{15\ \text{V}}{0.01\ \text{m}} = 1500\ \text{V}$$

The charge on each plate is

$$Q' = E'A\varepsilon_0 = (1500\ \text{V})\pi(0.05\ \text{m})^2(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2) = 1.04\times10^{-10}\ \text{C}$$

(c) The potential difference $\Delta V_C' = \Delta V_C = 15$ V is unchanged when the electrodes are expanded to area $A' = 4A$. The electric field between the plates is

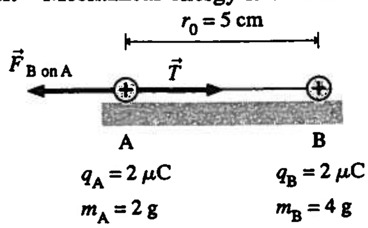
$$E' = E = \frac{\Delta V_C'}{d'} = \frac{15\ \text{V}}{0.5\times10^{-2}\ \text{m}} = 3000\ \text{V/m}$$

The new charge is

$$Q' = E'A'\varepsilon_0 = 4EA\varepsilon_0 = 4Q = 8.34\times10^{-12}\ \text{C}$$

**29.41. Model:** Mechanical energy is conserved. Metal spheres are point particles and they have point charges.

**Visualize:**




**Solve:** **(a)** The system could have both kinetic and potential energy, although here $K = 0$ J. The energy of the system is

$$E_0 = K_0 + U_0 = 0\text{ J} + \frac{q_A q_B}{4\pi\varepsilon_0 r_0} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{0.05\text{ m}} = 0.720\text{ J}$$

**(b)** In static equilibrium, the net force on sphere A is zero. Thus

$$T = F_{\text{B on A}} = \frac{|q_A||q_B|}{4\pi\varepsilon_0 r_0^2} = \frac{(9.0\times10^9\text{ N m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{(0.05\text{ m})^2} = 14.4\text{ N}$$

**(c)** The spheres move apart due to the repulsive electric force between them. The surface is frictionless, so they continue to slide without stopping. When they are very far apart ($r_1 \to \infty$), their potential energy $U_1 \to 0$ J. Energy is conserved, so we have

$$E_1 = K_1 + U_1 = \tfrac{1}{2}m_A v_{A1}^2 + \tfrac{1}{2}m_B v_{B1}^2 + 0\text{ J} = E_0$$

Momentum is also conserved: $P_{\text{after}} = m_A v_{A1} + m_B v_{B1} = P_{\text{before}} = 0$ kg m/s. Note that these are *velocities* and that $v_{A1}$ is a negative number. From the momentum equation,

$$v_{A1} = -\frac{m_B v_{B1}}{m_A}$$



Substituting this into the energy equation,

$$E_0 = \tfrac{1}{2}m_A\left(-\frac{m_B v_{B1}}{m_A}\right)^2 + \tfrac{1}{2}m_B v_{B1}^2 = \tfrac{1}{2}m_B\left(\frac{m_B}{m_A}+1\right)v_{B1}^2$$

$$\Rightarrow v_{B1} = \sqrt{\frac{2E_0}{m_B(m_B/m_A+1)}} = \sqrt{\frac{2(0.720\text{ J})}{(0.004\text{ kg})(4\text{ g}/2\text{ g}+1)}} = 10.95\text{ m/s}$$

Using this result, we can then find $v_{A1} = -m_B v_{B1}/m_A = -21.9$ m/s. These are the velocities, so the final *speeds* are 21.9 m/s for the 2 g sphere and 10.95 m/s for the 4 g sphere.

**29.45. Model:** Energy is conserved. The electron's potential energy inside the capacitor can be found from the capacitor's electric potential.

**Solve:** (a) The voltage across the capacitor is

$$\Delta V_C = Ed = (5.0\times10^5 \text{ V/m})(2.0\times10^{-3}\text{ m}) = 1000 \text{ V}$$

(b) Because $E = \eta/\varepsilon_0$ for a parallel-plate capacitor, with $\eta = Q/A$, the charge on each plate is

$$Q = \pi R^2 E\varepsilon_0 = \pi(1.0\times10^{-2}\text{ m})^2(5.0\times10^5\text{ V/m})(8.85\times10^{-12}\text{ C}^2/\text{N m}^2) = 1.39\times10^{-9}\text{ C}$$

(c) The electron has charge $q = -e$, and its potential energy at a point where the capacitor's potential is $V$ is $U = -eV$. Since the electron is launched from the negative (lower potential) plate toward the positive (higher potential) plate, its potential energy becomes more negative (because of the negative sign of the electron charge). That is, the potential energy decreases, which must lead to an increase in the kinetic energy. Conversely, the electron's speed as it is launched is smaller than $2.0\times10^7$ m/s. The conservation of energy equation is

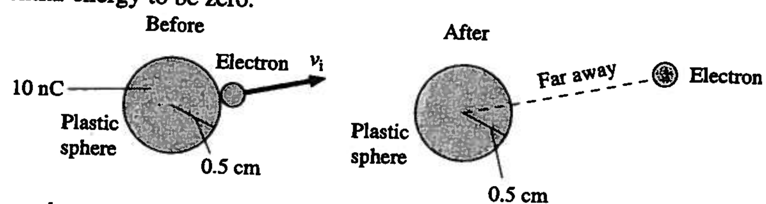
$$K_f + qV_f = K_i + qV_i \Rightarrow \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2 + q(V_f - V_i)$$

$$\Rightarrow v_i^2 = v_f^2 + \frac{2}{m}(-e)(1000\text{ V})$$

$$\Rightarrow v_i = \sqrt{(2.0\times10^7\text{ m/s})^2 - \frac{2(1.60\times10^{-19}\text{ C})(1000\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 7.0\times10^6\text{ m/s}$$

**29.50. Model:** Energy is conserved. The electron ends up so far away from the plastic sphere that we can consider its potential energy to be zero.

**Visualize:**




The minimum speed to escape is the speed that allows the electron to reach $r_f = \infty$ when $v_f = 0$ m/s.

**Solve:** The conservation of energy equation $K_f + U_f = K_i + U_i$ is

$$0\text{ J} + 0\text{ J} = \tfrac{1}{2}mv_i^2 + qV_i \Rightarrow 0\text{ J} = \tfrac{1}{2}mv_i^2 + (-e)\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\right)$$

$$\Rightarrow v_i = \sqrt{\frac{2e}{m}\frac{1}{4\pi\varepsilon_0}\frac{q}{R}} = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})}{9.11\times10^{-31}\text{ kg}}(9.0\times10^9\text{ N m}^2/\text{C}^2)\left(\frac{10\times10^{-9}\text{ C}}{0.5\times10^{-2}\text{ m}}\right)} = 7.95\times10^7\text{ m/s}$$