28.52. Model: Electric current is conserved.

Visualize: Please refer to Figure P28.52. For the top, middle, and bottom segments the subscripts "top," "mid," and "bot" are used.

Solve: **(a)** Since current is conserved, $I_{top} = I_{mid} = I_{bot} = 10$ A.

(b) The current density is $J = I/A = I/\pi R^2$. Thus,

$$J_{top} = J_{bot} = \frac{10\text{ A}}{\pi(0.001\text{ m})^2} = 3.18\times10^6\text{ A/m}^2 \qquad J_{mid} = \frac{10\text{ A}}{\pi(0.0005\text{ m})^2} = 1.27\times10^7\text{ A/m}^2$$

(c) The electric field is $E = J/\sigma$. Thus,

$$E_{top} = E_{bot} = \frac{J_{top}}{\sigma} = \frac{3.18\times10^6\text{ A/m}^2}{3.5\times10^7\ \Omega^{-1}\text{m}^{-1}} = 0.0909\text{ N/C} \qquad E_{mid} = \frac{J_{mid}}{\sigma} = \frac{1.27\times10^7\text{ A/m}^2}{3.5\times10^7\ \Omega^{-1}\text{m}^{-1}} = 0.364\text{ N/C}$$

(d) The drift speed is $v_d = J/ne$. Thus

$$(v_d)_{top} = (v_d)_{bot} = \frac{J_{top}}{ne} = \frac{3.18\times10^6\text{ A/m}^2}{(6.0\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})} = 3.31\times10^{-4}\text{ m/s}$$

$$(v_d)_{mid} = \frac{J_{mid}}{ne} = \frac{1.27\times10^7\text{ A/m}^2}{(6.0\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})} = 1.33\times10^{-3}\text{ m/s}$$

(e) The mean time between collisions is $\tau = mv_d/eE$. Thus,

$$\tau_{top} = \tau_{bot} = \frac{m(v_d)_{top}}{eE_{top}} = \frac{(9.11\times10^{-31}\text{ kg})(3.31\times10^{-4}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.0909\text{ N/C})} = 2.07\times10^{-14}\text{ s}$$

$$\tau_{mid} = \frac{m(v_d)_{mid}}{eE_{mid}} = \frac{(9.11\times10^{-31}\text{kg})(1.33\times10^{-3}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.364\text{ N/C})} = 2.08\times10^{-14}\text{ s}$$

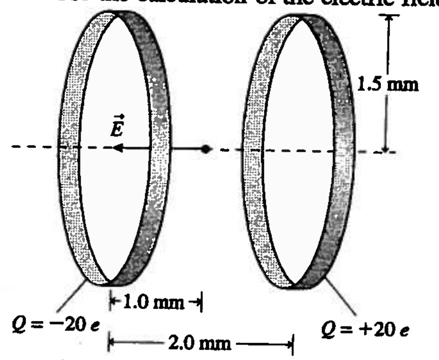
The collision times are the same in all three segments because collisions are part of the microphysics of electrons inside the metal, independent of the macrophysics of fields and currents.

(f) The electron current is $i = N_e/\Delta t = I/e$. Because $I_1 = I_2 = I_3 = 10$ A,

$$\left(\frac{N_e}{\Delta t}\right)_{top} = \left(\frac{N_e}{\Delta t}\right)_{mid} = \left(\frac{N_e}{\Delta t}\right)_{bot} = \frac{10\text{ A}}{1.60\times10^{-19}\text{ C}} = 6.25\times10^{19}\text{ s}^{-1}$$

Quantity	Top	Middle	Bottom
I	10 A	10 A	10 A
J	3.18×10^6 A/m	1.27×10^7 A/m^2	3.18×10^6 A/m^2
E	0.0909 N/C	0.364 N/C	0.0909 N/C
v_d	3.31×10^{-4} m/s	1.33×10^{-3} m/s	3.31×10^{-4} m/s
τ	2.07×10^{-14} s	2.07×10^{-14} s	2.07×10^{-14} s
i	6.25×10^{19} s^{-1}	6.25×10^{19} s^{-1}	6.25×10^{19} s^{-1}

28.47. Model: Use the calculation of the electric field of a ring of charge from Chapter 26.
Visualize:



Both the rings contribute equally to the field strength. The radius of each ring is R = 1.5 mm. The left ring is negatively charged and the right ring is positively charged because 20 electrons have been transferred from the right ring to the left ring.

Solve: From Chapter 26, the electric field of a ring of charge $+Q$ at $z = -1.0$ mm on the axis where the axis of each ring is $\hat{k}$ is

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{\left(z^2+R^2\right)^{3/2}}\hat{k} = \left(9\times10^9\ \text{N m}^2/\text{C}^2\right)\frac{\left(-1.0\times10^{-3}\ \text{m}\right)\left(20\times1.6\times10^{-19}\ \text{C}\right)}{\left[\left(-1.0\times10^{-3}\ \text{m}\right)^2+\left(1.5\times10^{-3}\ \text{m}\right)^2\right]^{3/2}}\hat{k}$$

$$= -4.92\times10^{-3}\hat{k}\ \text{N/C}$$

The left ring with charge $-Q$ makes an equal contribution

$$\vec{E}_- = -4.92\times10^{-3}\hat{k}\ \text{N/C}$$

$$\Rightarrow \vec{E}_{\text{net}} = \vec{E}_+ + \vec{E}_- = -9.84\times10^{-3}\hat{k}\ \text{N/C}$$

The negative sign with E_+, E_-, and E_{net} means these electric fields are in the $-z$ direction. Using $J = I/A = \sigma E$, the current is

$$I = \pi(1.5\times10^{-3}\ \text{m})^2(3.5\times10^7\ \Omega^{-1}\text{m}^{-1})(9.84\times10^{-3}\ \text{N/C}) = 2.43\ \text{A}$$

Assess: This result is consistent with the value given in Table 26.1 for the electric field strength in a current-carrying wire.

Solve: **(a)** The beam current is 1.5 mA. This means the beam transports a charge of 1.5×10^{-3} C in 1 s. The number of protons delivered in one second is

$$\frac{1.5\times10^{-3}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 9.375\times10^{15}$$

(b) From $J = I/A$, the current in the ring of width dr at a distance r from the center is

$$dI = JdA = J_{\text{edge}}\left(\frac{r}{R}\right)(2\pi r dr) = J_{\text{edge}}\frac{2\pi r^2 dr}{R}$$

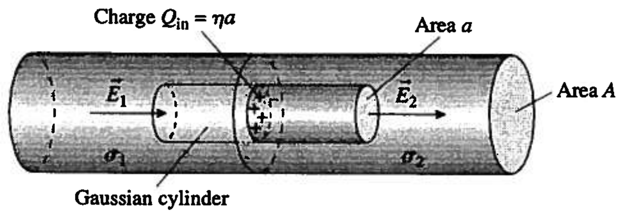
The total current $I = 1.5$ mA is found by integrating this expression:

$$I_{\text{total}} = \int dI = 1.5\times10^{-3}\ \text{A} = \frac{J_{\text{edge}}}{R}\int_0^R 2\pi r^2 dr = \frac{2\pi J_{\text{edge}} R^2}{3}$$

$$\Rightarrow J_{\text{edge}} = \left(1.5\times10^{-3}\ \text{A}\right)\frac{3}{2\pi\left(2.5\times10^{-3}\ \text{m}\right)^2} = 115\ \text{A}/\text{m}^2$$

28.57. Model: The currents in the two segments of the wire are the same.

Visualize: The electric fields $\vec{E}_1$ and $\vec{E}_2$ point in the direction of the current. Establish a cylindrical Gaussian surface with end area a that extends into both segments of the wire.



Solve: **(a)** Because current is conserved, $I_1 = I_2 = I$. The cross-section areas of the two wires are the same, so the current densities are the same: $J_1 = J_2 = I/A$. Thus the electric fields in the two segments have strengths

$$E_1 = \frac{J_1}{\sigma_1} = \frac{I}{A\sigma_1} \qquad E_2 = \frac{J_2}{\sigma_2} = \frac{I}{A\sigma_2}$$

The electric field enters the Gaussian surface on the left (negative flux) and exits on the right. No flux passes through the wall of cylinder, so the net flux is $\Phi_e = E_2 a - E_1 a$. The Gaussian cylinder encloses charge $Q_{in} = \eta a$ on the boundary between the segments. Gauss's law is

$$\Phi_e = \frac{Q_{in}}{\varepsilon_0} \Rightarrow E_2 a - E_1 a = \frac{Ia}{A}\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right) = \frac{\eta a}{\varepsilon_0}$$

Thus the surface charge density on the boundary is

$$\eta = \frac{\varepsilon_0 I}{A}\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)$$

(b) From the expression obtained in part (a)

$$\eta = \frac{Q}{\pi R^2} = \frac{I\varepsilon_0}{(\pi R^2)}\left(\frac{1}{\sigma_{\text{iron}}} - \frac{1}{\sigma_{\text{copper}}}\right)$$

$$\Rightarrow Q = (5\ \text{A})\left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right)\left(\frac{1}{1.0\times10^7\ \Omega^{-1}\text{m}^{-1}} - \frac{1}{6.0\times10^7\ \Omega^{-1}\text{m}^{-1}}\right) = 3.68\times10^{-18}\ \text{C}$$

Assess: This charge corresponds to a deficit of a mere 23 electrons on the boundary between the metals.

Assignment Display Mode: View Printable Answers

[Print]

Course PHYSICS260

Assignment 11

Due at 11:00pm on Wednesday, April 30, 2008

View Grading Details

Introduction to Electric Current

Description: Mostly conceptual questions about electric current in metals.

Learning Goal: To understand the nature of electric current and the conditions under which it exists.

Electric current is defined as the motion of electric charge through a *conductor*. Conductors are materials that contain movable charged particles. In metals, the most commonly used conductors, such charged particles are electrons. The more electrons that pass through a cross section of a conductor per second, the greater the current. The conventional definition of current is

$$I = \frac{Q_{\rm total}}{\Delta t},$$

where I is the current in a conductor and $Q_{\rm total}$ is the total charge passing through a cross section of the conductor during the time interval Δt.

The motion of free electrons in metals not subjected to an electric field is *random*: Even though the electrons move fairly rapidly, the net result of such motion is that $Q_{\rm total} = 0$ (i.e., equal numbers of electrons pass through the cross section in opposite directions). However, when an electric field is imposed, the electrons continue in their random motion, but in addition, they tend to move in the direction of the force applied by the electric field.

In summary, the two conditions for electric current in a material are the presence of *movable charged particles* in the material and the presence of an *electric field*.

Quantitatively, the motion of electrons under the influence of an electric field is described by the *drift speed*, which tends to be much smaller than the speed of the random motion of the electrons. The number of electrons passing through a cross section of a conductor depends on the drift speed (which, in turn, is determined by both the microscopic structure of the material and the electric field) and the cross-sectional area of the conductor.

In this problem, you will be offered several conceptual questions that will help you gain an understanding of electric current in metals.

Part A

You are presented with several long cylinders made of different materials. Which of them are likely to be good conductors of electric current?

Check all that apply.

ANSWER:

- [x] copper
- [x] aluminum
- [ ] glass
- [ ] quartz
- [ ] cork
- [ ] plywood
- [ ] table salt
- [x] gold

Part B

Metals are good conductors of electric current for which of the following reasons?

ANSWER:
- ◯ They possess high concentrations of protons.
- ◯ They possess low concentrations of protons.
- ◉ They possess high concentrations of free electrons.
- ◯ They possess low concentrations of free electrons.

Part C

Which of the following is the most likely drift speed of the electrons in the filament of a light bulb?

ANSWER:
- ◯ 10^{-8} m/s
- ◉ 10^{-4} m/s
- ◯ 10 m/s
- ◯ 10^{4} m/s
- ◯ 10^{8} m/s

Part D

You are presented with several wires made of the same conducting material. The radius and drift speed are given for each wire in terms of some unknown units r and v. Rank the wires in order of decreasing electron current.

Hint D.1 What the wires have in common

Since the wires are made of the same material, the charge carriers and their densities are the same for all the wires.

Hint D.2 A formula for electric current

Other conditions being equal, the current is proportional to the product of the cross-sectional area of the wire and the drift velocity, that is,

$$I = n|q|v_{\rm d}A,$$

where I is the current, $v_{\rm d}$ is the drift velocity, A is the cross-sectional area, n is the density of charge carriers, and q is the charge on the carriers.

Rank from most to least electron current. To rank items as equivalent, overlap them.

ANSWER:

View

Part E

The drift speed of the electrons in a wire depends strongly on which of the following factors?

ANSWER:
- ◯ the cross-sectional area of the wire
- ◯ the mass of the wire
- ◯ the temperature of the wire
- ◉ the internal electric field in the wire

Part F

What quality must the charge density on the surface of a conducting wire possess if an electric field is to act on the negatively charged electrons inside the wire?

ANSWER:

The charge density must be
- positive.
- negative.
- (selected) nonuniform.
- uniform.

Down To The Wire

Description: Given the current in a wire and the characteristics of the material of the wire, find the current density, drift velocity of the electrons in the wire and the average time between the collisions.

A current of $I = 8.0\ \mathrm{A}$ is flowing in a typical extension cord of length $L = 3.00\ \mathrm{m}$. The cord is made of copper wire with diameter $d = 1.5\ \mathrm{mm}$.

The charge of the electron is $e = 1.6 \times 10^{-19}\ \mathrm{C}$. The resisitivity of copper is $\rho = 1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m}$. The concentration of free electrons in copper is $n = 8.5 \times 10^{28}\ \mathrm{m}^{-3}$.

Part A

Find the drift velocity v_d of the electrons in the wire.

Part A.1 Find the current density first

Find J, the magnitude of the current density in this wire.

Hint A.1.a Diameter and area

Using the diameter of the wire, find its cross-sectional area.

Express your answer in amperes per meter squared. Use two significant figures.

ANSWER: $J = 4.5 \cdot 10^{6}\ \mathrm{A/m^2}$

Hint A.2 Current density and the drift speed

The current density is proportional to the drift speed: $J = nev_d$.

Express your answer in meters per second, to two significant figures.

ANSWER: $v_d = 3.3 \cdot 10^{-4}\ \mathrm{m/s}$

Note that this wire is carrying more current density than is carried by most household wiring in everyday use. With the given amount of current flowing, the cord would be hot to the touch if it were under a rug or had otherwise restricted air flow around it. It would certainly be considered unsafe by standard electrical safety codes.

Even though this wire is carrying a large amount of current for its size, the drift velocity of the electrons is tiny (less than one millimeter per second). This reflects the fact that there is a huge number of free (mobile) electrons in the wire. Let us illustrate this fact with a calculation.

Part B

The population of the Earth is roughly six billion people. If all free electrons contained in this extension cord are evenly split among the humans, how many free electrons (N_e) would *each* person get?

Part B.1 Find the volume first

Find the volume V of the wire.

Express your answer in cubic meters. Use two significant figures.

ANSWER: V = $5.3 \cdot 10^{-6}$ m^3

Use two significant figures.

ANSWER: N_e = $7.5 \cdot 10^{13}$

These free electrons undergo frequent collisions with atoms, slowing down and generating heat. How many collisions occur in such a conductor? Let us find out.

Part C

Find the *total* number of collisions (N_c) that *all free electrons* in this extension cord undergo in one second.

Hint C.1 **Consider a single electron**

How many collisions per second does *each* electron undergo per second?

Part C.2 **Find the time between collisions**

Find the algebraic expression for the mean time between collisions τ.

Express your answer in terms of ρ, the mass of the electron m, the charge of the electron e and the concentration of the electrons n.

ANSWER: $\tau = \dfrac{m}{\rho n e^2}$

ANSWER: N_c = $1.8 \cdot 10^{37}$

Note that τ does *not* depend on the applied electric field. The drift speed, however, does.

Problem 28.47

Description: The electric field in a current-carrying wire can be modeled as the electric field at the midpoint between two charged rings. Model a d aluminum wire as two d rings x apart. (a) What is the current in the wire after N electrons are transferred from...

The electric field in a current-carrying wire can be modeled as the electric field at the midpoint between two charged rings. Model a 3.60 $-\mathrm{mm}-\mathrm{diameter}$ aluminum wire as two 3.60 $-\mathrm{mm}-\mathrm{diameter}$ rings 1.70 mm apart.

Part A

What is the current in the wire after 30.0 electrons are transferred from one ring to the other?

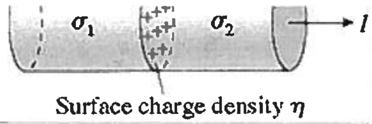
ANSWER: $\dfrac{9 \cdot 1.6 \cdot 2xN\pi \cdot 3.5d^2}{1000\left((x^2)+d^2\right)^{\frac{3}{2}}}$ A

Problem 28.57

Description: The figure shows a wire that is made of two equal-diameter segments with conductivities sigma_1 and sigma_2. When current I passes through the wire, a thin layer of charge appears at the boundary between the segments. (a) Find an expression for the...

The figure
shows a wire that is made of two equal-diameter segments with conductivities σ_1 and

σ_2. When current I passes through the wire, a thin layer of charge appears at the boundary between the segments.



Part A

Find an expression for the surface charge density η on the boundary. Give your result in terms of I, σ_1, σ_2, and the wire's cross-section area A.

ANSWER:

- $\eta = \frac{A}{\epsilon_0 I}(\frac{1}{\sigma_2} - \frac{1}{\sigma_1})$
- $\eta = \frac{\epsilon_0}{A}(\frac{1}{\sigma_2} - \frac{1}{\sigma_1})$
- (selected) $\eta = \frac{\epsilon_0 I}{A}(\frac{1}{\sigma_2} - \frac{1}{\sigma_1})$
- $\eta = \frac{\epsilon_0 I}{A}(\sigma_2 - \sigma_1)$

Part B

A 1.0-mm-diameter wire made of copper and iron segments carries a 3.00 A current. How much charge accumulates at the boundary between the segments?

ANSWER: $\frac{I_0 \cdot 5 \cdot 8.85}{6} \cdot 10^{-19}$ C

Are Coulomb Forces Conservative?

Description: Several questions requiring the student to find the work done by a uniform electrostatic field on a charged particle moving through different paths. Calculations are fairly straightforward and serve to verify the fact that Coulomb force is a conservative force.

Learning Goal: To review the concept of conservative forces and to understand that electrostatic forces are, in fact, conservative.

As you may recall from mechanics, some forces have a very special property, namely, that the work done on an object does not depend on the object's trajectory; rather, it depends only on the initial and the final positions of the object.

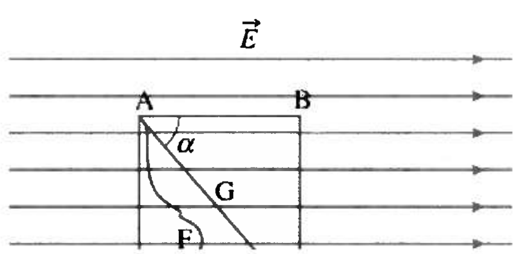
Such forces are called *conservative forces*. If only conservative forces act within a closed system, the total amount of mechanical energy is conserved within the system (hence the term "conservative"). Such forces have a number of properties that simplify the solution of many problems.

You may also recall that a *potential energy function* can be defined with respect to a conservative force. This property of conservative forces will be of particular interest of us.

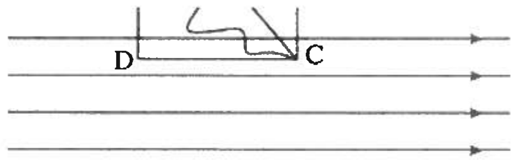
Not all forces that we deal with are conservative, of course. For instance, the amount of work done by a frictional force very much depends on the object's trajectory. Friction, therefore, is not a conservative force. In contrast, the gravitational force and the normal force are examples of conservative forces. What about electrostatic (Coulomb) forces? Are they conservative, and is there a potential energy function associated with them?

In this problem, you will be asked to use the given diagram to calculate the work done by the electric field $\vec{E}$ on a particle of charge q and see for yourself whether that work appears to be trajectory-independent. Recall that the force acting on a charged particle in an electric field is given by $\vec{F} = \vec{E}q$.

Recall that the work W done on an object by a constant force is



$$W = Fd\cos\theta,$$



where F is the magnitude of the force acting on the object, d is the magnitude of the displacement that the object undergoes, and θ is the angle between the vectors $\vec{F}$ and $\vec{d}$.

Consider a uniform electric field $\vec{E}$ and a rectangle ABCD, as shown in the figure. Sides AB and CD are parallel to $\vec{E}$ and have length L; let α be angle BAC.

Part A

Calculate the work W_{AB} done by the electrostatic force on a particle of charge q as it moves from A to B.

Part A.1 Find the angle

With reference to the given expression for the work done by a constant force, what value of θ should you use here?

ANSWER:

- (•) 0
- ( ) α
- ( ) $90^\circ - \alpha$
- ( ) 90°
- ( ) $180^\circ - \alpha$
- ( ) 180°

Express your answer in terms of some or all the variables E, q, L, and α.

ANSWER: $W_{AB} = EqL$

The angle θ between the force and the displacement is zero here, so $\cos\theta = 1$, and the general formula for work becomes $W = Fd$.

Part B

Calculate the work W_{BC} done by the electrostatic force on the charged particle as it moves from B to C.

Express your answer in terms of some or all the variables E, q, L, and α.

ANSWER: $W_{BC} = 0$

Now the angle θ between the force and the displacement is 90°, so $\cos\theta = 0$, and the work done is zero.

Part C

Calculate the total amount of work W_{ABC} done by the electrostatic force on the charged particle as it moves from A to B to C.

Express your answer in terms of some or all the variables E, q, L, and α.

ANSWER: $W_{ABC} = EqL$

Part D

Now assume that the particle "chooses" a different way of traveling. Calculate the total amount of work W_{ADC} done by the electrostatic force on the charged particle as it moves from A to D to C.

Express your answer in terms of some or all the variables E, q, L and α.

ANSWER: $W_{ADC} = EqL$

Since $W_{AB} = W_{DC}$ and $W_{BC} = W_{AD}$, it is clear that $W_{ABC} = W_{ADC}$. It appears that the work done by the electrostatic force on the particle is the same for both paths that begin at point A and end at point C. We now have a reasonable suspicion that this force may, in fact, be conservative. Let us check some more.

Part E

Calculate the work W_{AGC} done by the electrostatic force on the charged particle as it moves from A straight to C.

Part E.1 Find the distance between A and C

Find the distance $|AC|$.

Express your answer in terms of L and α.

ANSWER: $|AC| = \frac{L}{\cos(\alpha)}$

Part E.2 Find the angle

What value of θ should you use in calculating the work done to move the charged particle from A straight to C?

ANSWER:
- 0
- (•) α
- $90^\circ - \alpha$
- 90°
- $180^\circ - \alpha$
- 180°

Express your answer in terms of some or all the variables E, q, L, and α.

ANSWER: $W_{AGC} = EqL$

Though we have not proved it, it can be shown that the Coulomb force is indeed conservative. This implies that the amount of work W_{AFC} done by the electrostatic force on the charged particle as it moves in a curved path from A to F to C is also equal to qEL.

With the knowledge that the Coulomb force is conservative, and again referring to the diagram, answer the following questions. These questions are meant to highlight some important properties of conservative forces.

Part F

Find the amount of work W_{BA} done by the electrostatic force on the charged particle as it moves along the straight path from B to A.

Part F.1 Find the angle

What value of θ should you use here?

ANSWER:
- 0
- α
- $90^\circ - \alpha$
- 90°

- $180^\circ - \alpha$
- 180°

Express your answer in terms of some or all the variables E, q, L, and α.

ANSWER: $W_{BA} = -EqL$

The angle θ between the force and the displacement is 180° here, so $\cos\theta = -1$, and the general formula for work becomes $W = -Fd$.

Note that $W_{BA} = -W_{AB}$.

The amount of work W_{ABA} done by the electrostatic force on the charged particle as it moves from A to B to A is equal to

$$\begin{aligned} W_{ABA} &= W_{AB} + W_{BA} \\ &= W_{AB} + (-W_{AB}) \\ &= 0. \end{aligned}$$

Part G

Find the amount of work W_{ABCDA} done by the electrostatic force on the charged particle as it moves from A to B to C to D to A.

Express your answer in terms of some or all the variables E, q, L and α.

ANSWER: $W_{ABCDA} = 0$

Another important property of conservative forces, which can be very helpful in problem solving, is that the total work done by a conservative force over a closed path is zero.

PSS 29.2: The Hot Rod

Description: Find the electric potential of a uniformly charged rod at a point along the rod axis some distance away from end of the rod. (PSS 29.2: The electric potential of a continuous distribution of charge)

Learning Goal: To practice Problem-Solving Strategy 29.2 for problems involving the electric potential due to continuous charge distributions.

A straight rod of length L has a positive charge Q distributed along its length. Find the electric potential due to the rod at a point located a distance d from one end of the rod along the line extending from the rod.

MODEL: Model the charges as a simple shape, such as a line or a disk. Assume the charge is uniformly distributed.

VISUALIZE: For the pictorial representation:

1. Draw a picture and establish a coordinate system.
2. Identify the point P at which you want to calculate the electric potential.
3. Divide the total charge Q into small pieces of charge ΔQ using shapes for which you already know how to determine V. This division is often, but not always, into point charges.
4. Identify distances that need to be calculated.

SOLVE: The mathematical representation is $V = \Sigma V_i$.

- Use superposition to form an algebraic expression for the potential at P.
- Let the (x, y, z) coordinates remain as variables.
- Replace the small charge ΔQ with an equivalent expression involving a charge density and a coordinate, such as dx, that describes the shape of charge ΔQ. *This is the critical step in making the transition from a sum to an integral* because you need a coordinate to serve as an integration variable.
- Express all distances in terms of the coordinates.
- Let the sum become an integral. The integration will be over the coordinate variable that is related to ΔQ. The integration limits for this variable will depend on the coordinate system you have chosen. Carry out the integration and simplify the result.

ASSESS: Check that your result is consistent with any limits for which you know what the potential should be.

Model

Start by making simplifying assumptions appropriate to the situation.

Part A

A rod can be modeled as a *line of charge* only if which of the following is true?

ANSWER:

- ○ The diameter of the cross section is constant throughout the rod.
- ◉ The diameter of the cross section is much smaller than the length of the rod.
- ○ The diameter of the cross section has the same order of magnitude as the length of the rod.
- ○ The cross section has a circular shape.
- ○ The cross section has a highly symmetrical though not necessarily circular shape.

Part B

It is reasonable to assume that the linear charge density along the rod is

ANSWER:

- ○ increasing toward each end.
- ○ increasing toward the middle of the rod.
- ◉ constant throughout the length of the rod.

Visualize

Consider the following questions as you construct the pictorial representation. To be consistent with these questions, choose the traditional directions of the coordinate axes (+*x* to the right, +*y* up), choose the origin of your coordinate system to be at the left end of the rod, and choose point P to be located beyond the right end of the rod.

Part C

To most efficiently solve this problem, you should divide the rod into pieces of charge that consist of

ANSWER:

- ○ thin lines of charge of length L and a very small cross section.

- thin "slices" of the rod cut parallel to the axis of the rod.
- thin "slices" of the rod cut perpendicular to the axis of the rod.

A piece of the rod of short length Δx can be modeled as a *point charge*. The potential due to such a point charge can be determined in a relatively straightforward manner.

Part D

What is the distance r_i between point P and a piece of charge located at position x_i?

Part D.1 Find the *x* coordinate of point P

What is the *x* coordinate of point P?

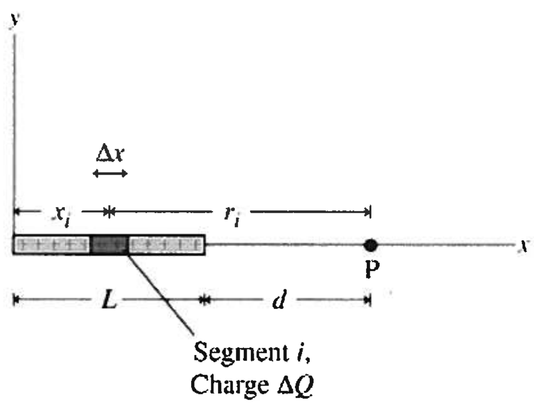
Express your answer in terms of L and d.

ANSWER: $x_P = L + d$

Express your answer in terms of x_i, L, and d.

ANSWER: $r_i = L + d - x_i$

An example of what a pictorial representation for this problem might look like is shown in the figure: .



Solve

Now use the information and the insights that you have accumulated to construct the necessary mathematical expressions and to derive the solution.

Part E

Find the electric potential at point P.

Part E.1 Find the expression for ΔQ

Because you will integrate with respect to coordinate *x*, it is important to express the charge on a segment, ΔQ, in terms of its length Δx.

Find the mathematical expression for ΔQ.

Part E.1.a Find the linear charge density

$\Delta Q = \lambda \Delta x$, where λ is the linear charge density. Find the mathematical expression for λ.

Express your answer in terms of the known quantities. You may or may not need all of them.

ANSWER: $\lambda = \frac{Q}{L}$

Express your answer in terms of Δx and the known quantities. You may or may not need all of them.

ANSWER: $\Delta Q = \frac{\Delta x Q}{L}$

Part E.2 Find the potential due to ΔQ

Find the electric potential V_i at point P due to the charge segment located at x position x_i.

Hint E.2.a Getting the distance right

The formula for electric potential of a point charge involves the distance from the charge to the point where the potential is to be computed.

Recall that, in the visualize step, you obtained an expression relating the distance between point P and charge segment i to the x coordinate of the charge segment.

Express your answer in terms of variables given in the problem introduction, x_i, ΔQ, ϵ_0, and π.

ANSWER: $V_i = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{L+d-x_i}$

Part E.3 Find the limits of integration

When you change the sum over charge segments into an integral, the variable of integration will be the coordinate x. What should the limits of integration be?

ANSWER:

- ○ 0 to infinity
- ○ 0 to d
- ○ d to L
- ○ d to $L+d$
- ◉ 0 to L

Hint E.4 Help with integration

You can simplify the integral by changing the variable of integration from x to $x' = L + d - x$. Remember to adjust the limits of integration appropriately when you make this change. The following formula will also be helpful:

$$\int \frac{dx}{x} = \ln(x).$$

Express your answer in terms of the given quantities along with the constants ϵ_0 and/or π.

ANSWER: $V_P = \frac{Q}{L \cdot 4\pi\epsilon_0}\ln\left(\frac{L+d}{d}\right)$

Assess

When you work on a problem on your own, without the computer-provided feedback, only you can assess whether your answer seems right. The following questions will help you practice the skills necessary for such an assessment.

Part F

Imagine that distance d is much greater than the length of the rod. Intuitively, the potential should be approximately the same as the potential at a distance d from which of the following charge distributions?

ANSWER:
- an infinitely long wire with total charge Q
- an infinitely long wire with total charge Qd/L
- a point charge of magnitude Q
- an electric dipole with moment QL

From far away, a short rod looks very much like a simple point charge. Not surprisingly, the mathematical expression you obtained for the potential does reduce to that of a point charge if $L/d \ll 1$:

$$\ln\left(\frac{L+d}{d}\right) = \ln\left(1+\frac{L}{d}\right) \approx \frac{L}{d},$$

so

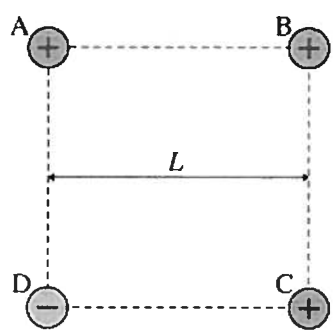
$$\frac{1}{4\pi\epsilon_0}\frac{Q}{L}\ln\left(\frac{L+d}{d}\right) \approx \frac{1}{4\pi\epsilon_0}\frac{Q}{d}.$$

Energy Stored in a Charge Configuration

Description: Find the work required to assemble four charges of the same magnitude--three positive, one negative--at the corners of a square.

Four point charges, A, B, C, and D, are placed at the corners of a square with side length L. Charges A, B, and C have charge $+q$, and D has charge $-q$.

Throughout this problem, use k in place of $\frac{1}{4\pi\epsilon_0}$.



Part A

If you calculate W, the amount of work it took to assemble this charge configuration if the point charges were initially infinitely far apart, you will find that the contribution for each charge is proportional to $\frac{kq^2}{L}$. In the space provided, enter the numeric value that multiplies the above factor, in W.

Hint A.1 How to approach the problem

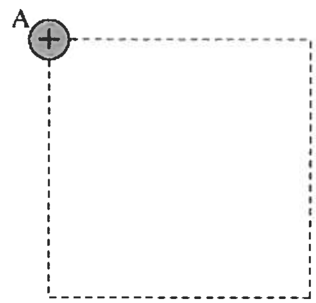
The Coulomb force is conservative. If we define the potential energy of the system to be zero when the charges are infinitely far apart, the amount of work needed to place any one charge in a configuration is equal to its electric potential energy. Imagine moving charge A, then B, then C, and finally D into place. Find the work required to add each charge to the configuration by calculating the potential energy of each just after it is added. Add the work required for each charge to find the total work required.

Hint A.2 Electric potential and potential energy

Recall that the electric potential at a point at distance r from a charge q is $V = k\frac{q}{r}$, where $k = \frac{1}{4\pi\epsilon_0}$. Note that this equation implicitly defines the electric potential to be zero at $r = \infty$. The electric *potential energy* of a charge q is equal to qV, where V is the electric *potential* at the position of the charge *before* the charge is placed there. To find the potential at a point due to multiple charges, sum the potentials at that point due to each charge.

Part A.3 Work required to place charge A

What is W_A, the work required to assemble the charge distribution shown in the figure?



Part A.3.a Find the potential at the location of charge A

What is V_A, the electric potential at the upper left corner of the square *before* charge A is placed there?

Express your answer in terms of some or all of the variables k, q, and L.

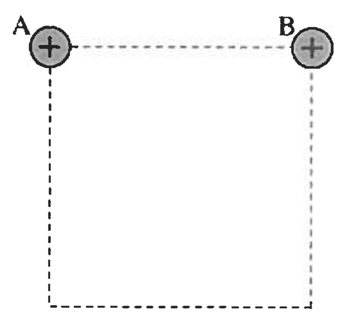
ANSWER: $V_A = 0$

Express your answer in terms of some or all of the variables k, q, and L.

ANSWER: $W_A = 0$

Part A.4 Work required to place charge B

What is W_B, the amount of work required to add charge B to the configuration, as shown in the figure?



Part A.4.a Find the potential at the location of charge B

What is V_B, the potential at the upper right corner due to charge A, before charge B is placed there?

Express your answer in terms of some or all of the variables k, q, and L.

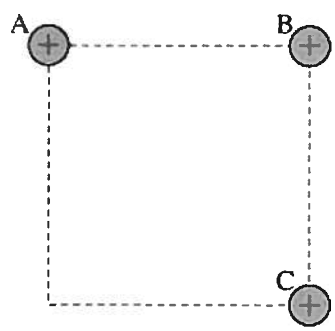
ANSWER: $V_B = \frac{kq}{L}$

Express your answer in terms of some or all of the variables k, q, and L.

ANSWER: $W_B = \frac{kq^2}{L}$

Part A.5 **Work required to place charge C**

What is W_C, the amount of work required to add charge C to the configuration, as shown in the figure?



Part A.5.a **Find the potential at the location of charge C**

What is V_C, the potential at the lower right corner of the square *before* charge C is placed there?

Hint A.5.a.i **How to approach this part**

The potential at C is the sum of the individual potentials due to the charges at B and A.

Part A.5.a.ii **Find the potential at C due to the charge at B**

What is the potential at C due to the charge at B?

Express your answer in terms of some or all of the variables k, q, and L.

ANSWER: $V_{CB} = \frac{kq}{L}$

Part A.5.a.iii **Find the potential at C due to the charge at A**

What is the potential at C due to the charge at A?

Express your answer in terms of some or all of the variables k, q, and L.

ANSWER: $V_{CA} = \frac{kq}{\sqrt{2}L}$

$V_C = V_{CB} + V_{CA}$.

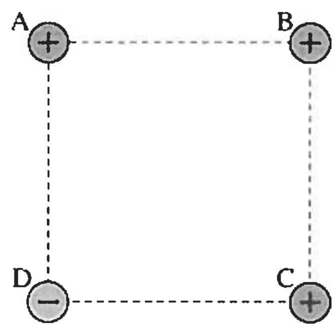
ANSWER: $V_C = \frac{kq}{L}\left(1+\frac{1}{\sqrt{2}}\right)$

Express your answer in terms of some or all of the variables k, q, and L.

ANSWER: $W_C = \frac{kq^2}{L}\left(1+\frac{1}{\sqrt{2}}\right)$

Part A.6 Find the work required to place charge D

What is W_D, the amount of work required to add charge D to the configuration?



Part A.6.a Find the potential at the position of charge D

What is V_D, the potential at the lower left corner of the square *before* charge D is placed there?

Express your answer in terms of some or all of the variables k, q, and L.

ANSWER: $V_D = \frac{kq}{L}\left(2+\frac{1}{\sqrt{2}}\right)$

Express your answer in terms of some or all of the variables k, q, and L.

ANSWER: $W_D = -\frac{kq^2}{L}\left(2+\frac{1}{\sqrt{2}}\right)$

Because D has a negative charge, it is attracted to charges A, B, and C. This accounts for the negative sign on the work. We would have to do positive work to *remove* charge D from the configuration.

ANSWER: $W = 0 \quad \times \left(\frac{kq^2}{L}\right)$

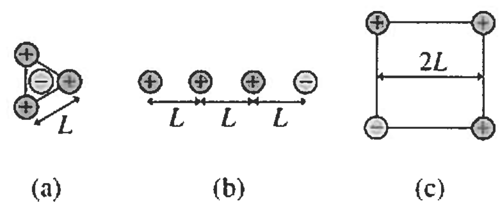
The hints led you through the problem by adding one charge at a time. A little thought shows that this is equivalent to simply adding the energies of all possible pairs:

$$W = \frac{q^2}{4\pi\epsilon_0}\left(\frac{1}{R_{BA}}+\frac{1}{R_{BC}}+\frac{1}{R_{CA}}-\frac{1}{R_{DA}}-\frac{1}{R_{DB}}-\frac{1}{R_{DC}}\right).$$

Note that this is *not* equivalent to adding the potential energies of each charge. Adding the potential energies will give you double the correct answer because you will be counting each charge twice.

Part B

Which of the following figures depicts a charge configuration that requires less work to assemble than the configuration in the problem introduction? Assume that all charges have the same magnitude q.

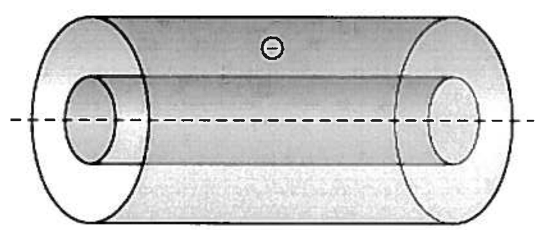


ANSWER:
- figure a
- figure b
- figure c

An Electron in a Diode

Description: Calculate the final speed of an electron (initially at rest) that travels from the cathode to the anode in a cylindrical diode, given the potential difference between the electrodes.

Before the advent of solid-state electronics, vacuum tubes were widely used in radios and other devices. A simple type of vacuum tube known as a *diode* consists essentially of two electrodes within a highly evacuated enclosure. One electrode, the *cathode*, is maintained at a high temperature and emits electrons from its surface. A potential difference of a few hundred volts is maintained between the cathode and the other electrode, known as the *anode*, with the anode at the higher potential.



Part A

Suppose a diode consists of a cylindrical cathode with a radius of 6.200×10^{-2} cm, mounted coaxially within a cylindrical anode with a radius of 0.5580 cm. The potential difference between the anode and cathode is 355 V. An electron leaves the surface of the cathode with zero initial speed ($v_{initial} = 0$). Find its speed v_{final} when it strikes the anode.

Hint A.1 How to approach the problem

Try to draw a simple diagram of the diode, with the path of the electron going from the central cathode to the outer anode. Since the diode is a cylinder, the symmetry implies that only radial motion needs to be considered for the electron, since any motion of the electron around the center will not change its potential energy. Note that only a potential difference is given between the plates, so you will need to be careful about your definitions. Use the equation for the conservation of energy to find the final speed of the electron.

Part A.2 Calculate the change in potential energy of the electron

Calculate the change in the potential energy ΔU of the electron as it moves from the inner cathode to the outer anode.

Hint A.2.a Potential energy and potential

Recall that the potential energy U_q of a charge q in an electric field is related to the potential V_0 of that field, evaluated at the position of the charge, by the equation $U_q = qV_0$. In this problem, the potential is not known at either the anode or cathode. However, the potential difference $\Delta V = V_{anode} - V_{cathode}$ is given, so use the relation above to find the change in the potential energy ΔU of the electron.

Express your answer numerically in joules.

ANSWER: $\Delta U = -eV$ J

Part A.3 Calculate the initial kinetic energy of the electron

Calculate the initial kinetic energy $K_{initial}$ of the electron at the cathode.

Hint A.3.a Initial velocity of the electron

Recall that the electron is emitted from the surface of the cathode with zero initial speed.

Express your answer in joules.

ANSWER: $K_{initial} = 0$ J

Hint A.4 Putting it all together

The equation of conservation of energy, $K_{initial} + U_{initial} = K_{final} + U_{final}$, can be rewritten as $K_{final} = K_{initial} + U_{initial} - U_{final} = K_{initial} - \Delta U$. Since $K = mv^2/2$, you now have enough information to find the final speed of the electron.

Express your answer numerically in meters per second.

ANSWER: $v_{final} = \sqrt{\frac{2eV}{m_e}}$ m/s

Note that the size of the diode makes no difference, as long as the potential difference between the two electrodes is a known constant. Also, note that the potential at the surface of the anode and cathode are not known separately, but the potential difference is enough for these calculations. In general, the potential at a particular point is not physically important. Only potential differences are important, just as only the change in potential energy is important to mechanics problems.

Summary	0 of 8 items complete (0% avg. score) 0 of 80 points