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phy260S08

Homework 10

Due at 11:00pm on Monday, April 21, 2008

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A Proton between Oppositely Charged Plates

Description: Find the electric field between two oppositely charged parallel plates needed to move a proton a given distance in a given amount of time, then find the final speed of the proton at the end.

A uniform electric field exists in the region between two oppositely charged parallel plates 1.62 cm apart. A proton is released from rest at the surface of the positively charged plate and strikes the surface of the opposite plate in a time interval 1.53×10^{-6} s .

Part A

Find the magnitude of the electric field.

Hint A.1 How to approach the problem

Use the equation for the motion over time of a particle under constant acceleration, and use the acceleration you calculate to find the electric field needed to create that acceleration for the proton.

Hint A.2 A relationship between electric force and electric field

Recall that a charged particle in an electric field will have an electric force on it equal to $\vec{F} = q\vec{E}$. This means that the force will point in the same direction as $\vec{E}$ if the particle is positively charged, and in the opposite direction as $\vec{E}$ if the particle is negatively charged.

Part A.3 Calculate the acceleration of the proton

Calculate the acceleration a of the proton, given the time taken to get from one plate to the other and the separation of the plates.

ANSWER: $a = \frac{2s}{\Delta t^2}$

Part A.4 Calculate the force on the proton

Calculate the force F on the proton needed to create the acceleration calculated in Part A.3.

ANSWER: $F = \frac{2sm_p}{\Delta t^2}$ N

Use 1.60×10^{-19} C for the magnitude of the charge on an electron and 1.67×10^{-27} kg for the mass of a proton.

ANSWER: $\frac{2sm_p}{e\Delta t^2}$ N/C

Remember that the electric field will point from the positively charged plate to the negatively charged plate, and that the positively charged proton moves in the same direction as the electric field.

Part B

Find the speed of the proton at the moment it strikes the negatively charged plate.

Hint B.1 How to approach the problem

Use the acceleration calculated in Part A and the equations of motion to find the final velocity.

ANSWER: $\frac{2s}{\Delta t}$ m/s

Electric Field near a Long Wire

Description: Determine the distance from a very long straight charged wire to a point where the electric field is equal to a given value.

A very long straight wire has charge per unit length 1.51×10^{-10} $\mathrm{C/m}$.

Part A

At what distance from the wire is the magnitude of the electric field equal to 2.55 $\mathrm{N/C}$?

Hint A.1 Equation for the electric field from a long wire

A very long wire can usually be approximated as an infinitely long wire. More specifically, you can approximate the wire as infinitely long if you are computing the field at a point whose distance from the axis of the wire is much smaller than the length of the wire, and you are computing the field at a point whose distance from the axis of the wire is much smaller than its distance from the end of the wire. The magnitude of the electric field due to an infinite wire is given by $E = \frac{\lambda}{2\pi\epsilon_0 r}$.

Use 8.85×10^{-12} $\mathrm{C^2/(N * m^2)}$ for the permittivity of free space, and use $\pi = 3.14159$.

ANSWER: $\frac{\lambda}{2\cdot3.14159\epsilon_0 E}$ m

Problem 26.15

Description: Two d-cm-diameter charged disks face each other, s apart. Both disks are charged to - Q. What is the electric field strength (a) at the midpoint between the two disks? (b) at the center of the left disk?

Two 9.00-cm-diameter charged disks face each other, 24.0 cm apart. Both disks are charged to - 20.0 nC. What is the electric field strength

Part A

at the midpoint between the two disks?

ANSWER: 0 N/C

Part B

at the center of the left disk?

ANSWER: $\frac{Q}{2\pi\left(\frac{d}{2}\right)^2\cdot 8.85\cdot10^{-12}\sqrt{1+\left(\frac{d}{2s}\right)^2}}$ N/C

Calculating Flux for Hemispheres of Different Radii

Description: Compute the electric flux through three surfaces surrounding a point charge: two hemispheres of unequal radii and the circular annulus connecting them.

Learning Goal: To understand the definition of electric flux, and how to calculate it.

Flux is the amount of a vector field that "flows" through a surface. We now discuss the electric flux through a surface (a quantity needed in Gauss's law): $\Phi_E = \int \vec{E} \cdot d\vec{A}$, where Φ_E is the flux through a surface with differential area element $d\vec{A}$, and $\vec{E}$ is the electric field in which the surface lies. There are several important points to consider in this expression:

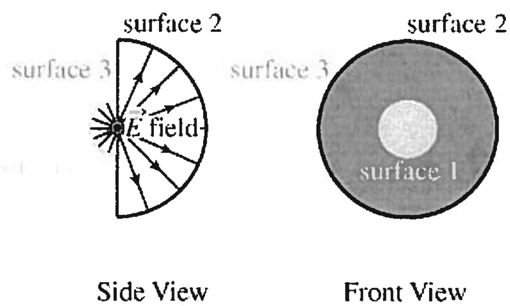
1. It is an integral over a surface, involving the electric field at the surface.
2. $d\vec{A}$ is a vector with magnitude equal to the area of an infinitesmal surface element and pointing in a direction normal (and usually outward) to the infinitesmal surface element.
3. The scalar (dot) product $\vec{E} \cdot d\vec{A}$ implies that only the component of $\vec{E}$ *normal* to the surface contributes to the integral. That is, $\vec{E} \cdot d\vec{A} = |\vec{E}|\,|d\vec{A}|\cos(\theta)$, where θ is the angle between $\vec{E}$ and $d\vec{A}$.

When you compute flux, try to pick a surface that is either parallel or perpendicular to $\vec{E}$, so that the dot product is easy to compute.

Two hemispherical surfaces, 1 and 2, of respective radii r_1 and r_2, are centered at a point charge and are facing each other so that their edges define an annular ring (surface 3), as shown. The field at position $\vec{r}$ due to the point charge is:

$$\vec{E}(\vec{r}) = \frac{C}{r^2}\hat{r}$$

where C is a constant proportional to the charge, $r = |\vec{r}|$, and $\hat{r} = \vec{r}/r$ is the unit vector in the radial direction.



Part A

What is the electric flux Φ_3 through the annular ring, surface 3?

Hint A.1 Apply the definition of electric flux

The integrand in the equation defining flux (in the problem introduction) can be calculated by noting the following:

$$\vec{E}(\vec{r}) \cdot d\vec{A} = E(r)|d\vec{A}|\cos\theta.$$

Since the electric field is everywhere parallel to the surface of the annular ring, the element $d\vec{A}$ is normal to the electric field, and thus $\theta = \pi/2$. Therefore, $\cos(\theta) = \cos(\pi/2) = 0$.

Express your answer in terms of C, r_1, r_2, and any constants.

ANSWER: $\Phi_3 = 0$

Part B

What is the electric flux Φ_1 through surface 1?

Hint B.1 Apply the definition of electric flux

The integrand in the equation defining flux (in the problem introduction) can be calculated by noting the following:

$$\vec{E}(\vec{r}) \cdot d\vec{A} = E(r)|d\vec{A}|\cos\theta$$

Since the electric field is everywhere perpendicular to surface 2, $\cos(\theta) = \cos(0) = 1$. Therefore, the integral simply becomes the magnitude of the electric field multiplied by the surface area of surface 2.

Part B.2 **Find the area of surface 1**

Find the area A_1 of the hemisphere that is surface 1.

Express A_1 in terms of r_1 and other given or known quantities.

ANSWER: $A_1 = 2\pi r_1^2$

Express Φ_1 in terms of C, r_1, r_2, and any needed constants.

ANSWER: $\Phi_1 = 2\pi C$

Part C

What is the electric flux Φ_2 passing outward through surface 2?

Hint C.1 **Apply the definition of electric flux**

The integrand in the equation defining flux (in the problem introduction) can be calculated by noting the following:

$$\vec{E}(\vec{r}) \cdot d\vec{A} = E(r)|d\vec{A}|\cos\theta$$

Since the electric field is everywhere perpendicular to surface 2, $\cos(\theta) = \cos(0) = 1$. Therefore, the integral simply becomes the magnitude of the electric field multiplied by the surface area of surface 2.

Part C.2 **Find the area of surface 2**

Find the area A_2 of the hemisphere that is surface 2.

Express your answer in terms of r_2, and any needed constants.

ANSWER: $A_2 = 2\pi r_2^2$

Express Φ_2 in terms of r_1, r_2, C, and any constants or other known quantities.

ANSWER: $\Phi_2 = 2\pi C$

Observe that the electric flux through surface 1 is the same as that through surface 2, despite the fact that surface 2 has a larger area. If you think in terms of field lines, this means that there is the same number of field lines passing through both surfaces. This is because of the inverse square, $\frac{1}{r^2}$, behavior of the electric field surrounding a point particle. A good rule of thumb is that the flux through a surface is proportional to the number of field lines that pass through the surface.

The Electric Field of a Ball of Uniform Charge Density

Description: Use Gauss's law to find the electric field inside and outside a uniformly charged ball.

A solid ball of radius r_b has a uniform charge density ρ.

Part A

What is the magnitude of the electric field $E(r)$ at a distance $r > r_b$ from the center of the ball?

Hint A.1 **Gauss's law**

Gauss's law can be written as

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0},$$

where $d\vec{A}$ refers to an infinitesimal element of an imaginary Gaussian surface, q_{encl} is the net charge enclosed in the Gaussian surface, and ϵ_0 is the permittivity of free space. Always choose a Gaussian surface that matches the symmetry of the problem. Implicit in the question "what is $E(r)$?" is the assumption that the electric field depends only on distance from the origin, which is also the center of the charged ball. Also, the ball has uniform charge density. Therefore, the electric field must point either radially outward or radially inward, since by symmetry there is no possibility for the electric field to point in any other direction. Given the symmetry of this problem, the best Gaussian surface to use is a sphere centered at the origin. Since the electric field is the same at all points on this surface, the constant $E(r)$ can be "pulled out" of the integrand. The left side of Gauss's law reduces to $E(r)A(r)$, where $A(r)$ is the surface area of a sphere with radius r.

Part A.2 **Find q_{encl}**

The q_{encl} in Gauss's law refers to the net charge enclosed inside the Gaussian surface. What is q_{encl} here?

Part A.2.a **What is the volume of the sphere?**

If a body has uniform charge density ρ, the charge in a volume V is ρV (this formula is the same as that for the mass of a sphere of uniform mass density). What is the volume of a sphere with radius r?

Express your answer in terms of π and r.

ANSWER: $V = \frac{4}{3}\pi r^3$

Express your answer in terms of ρ, π, and r_b.

ANSWER: $q_{encl} = \frac{4}{3}\pi r_b^3 \rho$

Express your answer in terms of ρ, r_b, r, and ϵ_0.

ANSWER: $E(r) = \frac{\rho r_b^3}{3\epsilon_0 r^2}$

Notice that this result is identical to that reached by applying Coulomb's law to a point charge centered at the origin with $q = \rho V$. The field outside of a uniformly charged sphere does not depend on the size of the sphere, only on its charge. A uniformly charged sphere generates an electric field as if all the charge were concentrated at its center.

Part B

What is the magnitude of the electric field $E(r)$ at a distance $r < r_b$ from the center of the ball?

Part B.1 **How does this situation compare to that of the field outside the ball?**

Now you are asked to find the electric field inside the ball, as opposed to outside the ball. What is different in the physical situation when you move inside the ball?

ANSWER: ○ The electric field now depends on spatial variables besides the radius.

- The direction of E is different.
- The shape of the appropriate Gaussian surface is no longer a sphere.
- (•) The net charge q_{encl} enclosed by the Gaussian surface is different.
- The charge density ρ is different.

Since the Gaussian surface is inside the ball, the surface encloses only a fraction of the ball's charge. This is why Gauss's law is so useful: As long as it is symmetrically distributed, the charge outside the Gaussian surface is irrelevant in calculating the field!

Express your answer in terms of ρ, r, r_b, and ϵ_0.

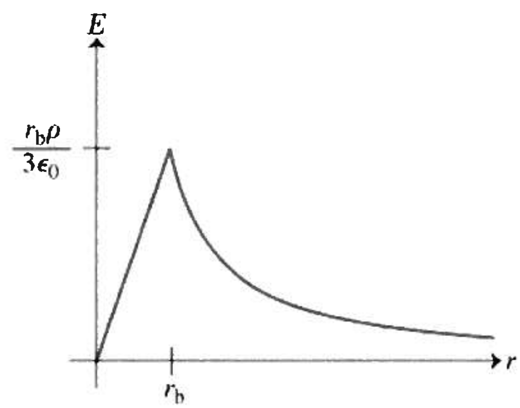
ANSWER: $E(r) = \frac{\rho r}{3\epsilon_0}$

Part C

Let $E(r)$ represent the electric field due to the charged ball throughout all of space. Which of the following statements about the electric field are true?

Hint C.1 **Plot the electric field**

The figure shows a plot of the electric field as a function of r.



Check all that apply.

ANSWER:

- [x] $E(0) = 0$.
- [ ] $E(r_b) = 0$.
- [x] $\lim_{r\to\infty} E(r) = 0$.
- [ ] The maximum electric field occurs when $r = 0$.
- [x] The maximum electric field occurs when $r = r_b$.
- [ ] The maximum electric field occurs as $r \to \infty$.

Problem 27.18

Description: A Q point charge is at the center of a a * a * a cube. (a) What is the electric flux through the top surface of the cube?

A 15.0 nC point charge is at the center of a 5.00 m × 5.00 m × 5.00 m cube.

Part A

What is the electric flux through the top surface of the cube?

Hint A.1 **Symmetry**

Find the total flux through the cube, then simply divide by the number of faces.

ANSWER: $\frac{\frac{\frac{q}{6}}{8.85}}{10^{-12}}$ $\mathrm{Nm^2/C}$

Problem 27.34

Description: A thin, horizontal L * L copper plate is charged with n electrons. Consider the electrons are uniformly distributed on the surface. (a) What is the strength of the electric field 0.1 mm above the center of the top surface of the plate? (b)...

A thin, horizontal 11.0 cm × 11.0 cm copper plate is charged with 1.80×10^{10} electrons. Consider the electrons are uniformly distributed on the surface.

Part A

What is the strength of the electric field 0.1 mm above the center of the top surface of the plate?

ANSWER: $\frac{0.5n \cdot 1.6 \cdot 1000}{L^2 \cdot 8.85}$ N/C

Part B

What is the strength of the electric field at the plate's center of mass?

ANSWER: 0 N/C

Part C

What is the strength of the electric field 0.1 mm below the center of the bottom surface of the plate?

ANSWER: $\frac{0.5n \cdot 1.6 \cdot 1000}{L^2 \cdot 8.85}$ N/C

Problem 27.45

Description: A uniformly charged ball of radius a and charge - Q is at the center of a hollow metal shell with inner radius b and outer radius c. The hollow sphere has net charge + 2 Q. (a) Determine the electric field strength in the region r <= a. (b) ...

A uniformly charged ball of radius a and charge - Q is at the center of a hollow metal shell with inner radius b and outer radius c. The hollow sphere has net charge + $2Q$.

Part A

Determine the electric field strength in the region $r \leq a$.

ANSWER:

$$\frac{-Qr}{a^3}$$

$$\frac{Qr}{a^3} \cdot \frac{1}{4\pi\epsilon_0}$$

Part B

Determine the electric field strength in the region $a < r < b$.

ANSWER:

$\frac{-Q}{r^2}$

$\frac{Q}{r^2}$ $\cdot \frac{1}{4\pi\epsilon_0}$

Part C

Determine the electric field strength in the region $b \leq r \leq c$.

ANSWER: $0 \cdot \frac{1}{4\pi\epsilon_0}$

Part D

Determine the electric field strength in the region $r > c$.

ANSWER:

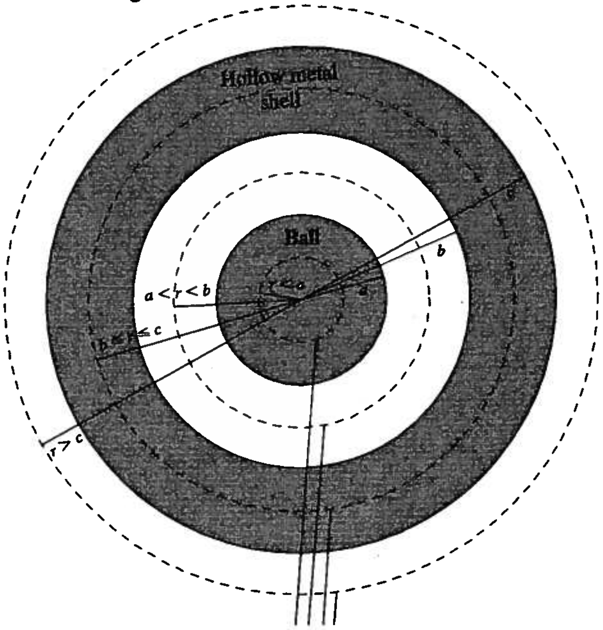
$\frac{Q}{r^2}$

$\frac{-Q}{r^2}$ $\cdot \frac{1}{4\pi\epsilon_0}$

Summary	0 of 8 items complete (0% avg. score) 0 of 80 points

27.45. Model: The charge distributions of the ball and the metal shell are assumed to have spherical symmetry.

Visualize:



Spherical Gaussian surfaces

The spherical symmetry of the charge distribution tells us that the electric field points radially inward or outward. We will therefore choose Gaussian surfaces to match the spherical symmetry of the charge distribution and the field. The figure shows four Gaussian surfaces in the four regions: $r \le a$, $a < r < b$, $b \le r \le c$ and $r > c$.

Solve: (a) Gauss's law is $\Phi_e = \oint \vec{E} \cdot d\vec{A} = Q_{in}/\varepsilon_0$. Applying it to the region $r \le a$, where the charge is negative so $\vec{E}$ points inward, we get

$$-EA_{sphere} = -E(4\pi r^2) = \frac{+\rho\left(\frac{4\pi}{3}r^3\right)}{\varepsilon_0} \Rightarrow E = \frac{-\rho r}{3\varepsilon_0}$$

Here $\rho = -Q/\frac{4\pi}{3}a^3$ is the charge density (C/m^3). Thus

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Qr}{a^3}, \text{ inward}\right) = -\frac{1}{4\pi\varepsilon_0}\frac{Qr}{a^3}\hat{r}$$

Applying Gauss's law to the region $a < r < b$,

$$-EA_{sphere} = \frac{-Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \Rightarrow \vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

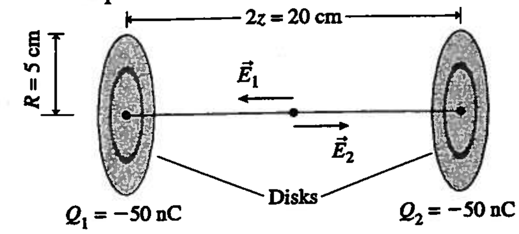
$\vec{E} = \vec{0}$ in the region $b \le r \le c$ because this is a conductor in electrostatic equilibrium.

To apply Gauss's to the region $r > c$, we use $Q_{in} = -Q + 2Q = +Q$. Thus,

$$EA_{sphere} = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

26.15. Model: Model each disk as a uniformly charged disk. When the disk is charged negatively, the on-axis electric field of the disk points toward the disk.

Visualize:



Solve: (a) The surface charge density on the disk is

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{-50 \times 10^{-9}\ \text{C}}{\pi (0.05\ \text{m})^2} = -6.366 \times 10^{-6}\ \text{C/m}^2$$

From Equation 26.22, the electric field of the left disk at $z = 0.10$ m is

$$(E_1)_z = \frac{\eta}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{1 + R^2/z^2}}\right] = \frac{-6.366 \times 10^{-6}\ \text{C/m}^2}{2(8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)}\left[1 - \frac{1}{\sqrt{1 + (0.05\ \text{m}/0.10\ \text{m})^2}}\right] = -38{,}000\ \text{N/C}$$

Hence, $\vec{E}_1 = (38{,}000\ \text{N/C, left})$. Similarly, the electric field of the right disk at z = 0.10 m (to its left) is $\vec{E}_2 = (38{,}000\ \text{N/C, right})$. The net field at the midpoint between the two disks is $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0\ \text{N/C}$.

(b) The electric field of the left disk at $z = 0$ m is

$$(E_1)_z = \frac{\eta}{2\varepsilon_0}\left[1 - \frac{1}{\infty}\right] = \frac{\eta}{2\varepsilon_0} = \frac{-6.366 \times 10^{-6}\ \text{C/m}^2}{2(8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)} = -3.60 \times 10^5\ \text{N/C} \Rightarrow \vec{E}_1 = (3.60 \times 10^5\ \text{N/C, left})$$

Similarly, the electric field of the right disk at $z = 0.20$ m (to its left) is $\vec{E}_2 = (1.075 \times 10^4\ \text{N/C, right})$. The net field is thus

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (3.49 \times 10^5\ \text{N/C, left})$$

The field strength is 3.49×10^5 N/C.

27.18. Solve: For *any* closed surface that encloses a total charge Q_{in}, the net electric flux through the closed surface is $\Phi_e = Q_{in}/\varepsilon_0$. The flux through the top surface of the cube is one-sixth of the total:

$$\Phi_{e\ \text{surface}} = \frac{Q_{in}}{6\varepsilon_0} = \frac{10 \times 10^{-9}\ \text{C}}{6(8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)} = 188\ \text{N m}^2/\text{C}$$

27.34. Model: The copper plate is a conductor. The excess charge resides on the surface of the plate.

Solve: (a) One-half of the electrons are located on the top surface and one-half on the bottom surface of the copper plate, so the surface charge density is

$$\eta = \frac{(0.5 \times 10^{10})(1.60 \times 10^{-19}\ \text{C})}{0.1\ \text{m} \times 0.1\ \text{m}} = 80 \times 10^{-9}\ \text{C/m}^2$$

Thus, the electric field at the surface of the plate is

$$E = \frac{\eta}{\varepsilon_0} = \frac{80 \times 10^{-9}\ \text{C/m}^2}{8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2} = 9040\ \text{N/C}$$

Because the charge on the plate is negative, the direction of the electric field is toward the plate.

(b) $E = 0$ N/C because the electric field within a conductor is zero.

(c) The electric field $E = 9040$ N/C, toward the plate.