Summaries of chapters for final exam (cannot be brought to the exam)

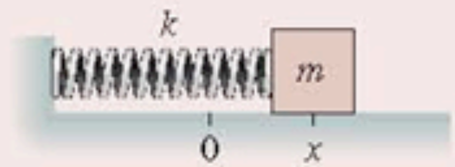
Chapter 14 (Oscillations)

GENERAL PRINCIPLES

Dynamics

SHM occurs when a linear restoring force acts to return a system to an equilibrium position.

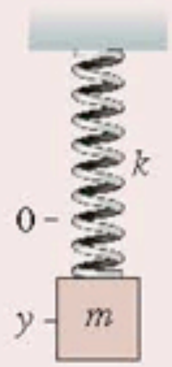
Horizontal spring



$(F_{net})_x = -kx$

Vertical spring

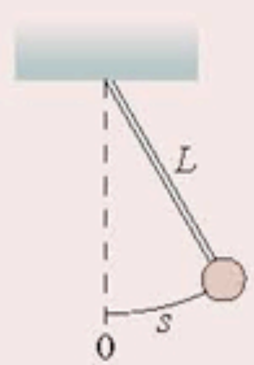
The origin is at the equilibrium position $\Delta L = mg/k$.



$(F_{net})_y = -ky$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Pendulum

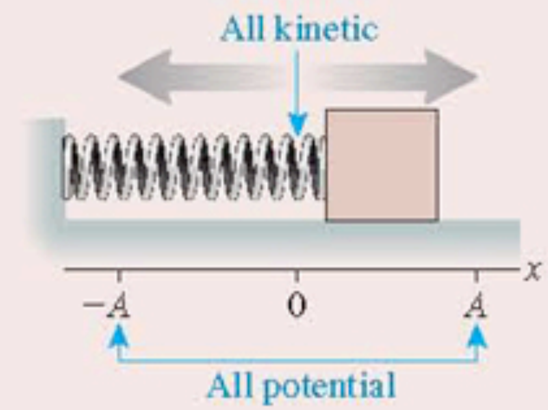


$$(F_{net})_t = -\left(\frac{mg}{L}\right)s$$

$$\omega = \sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

Energy

If there is **no friction** or dissipation, kinetic and potential energy are alternately transformed into each other, but the total mechanical energy $E = K + U$ is conserved.



$$E = \frac{1}{2}mv_x^{\,2} + \frac{1}{2}kx^2$$

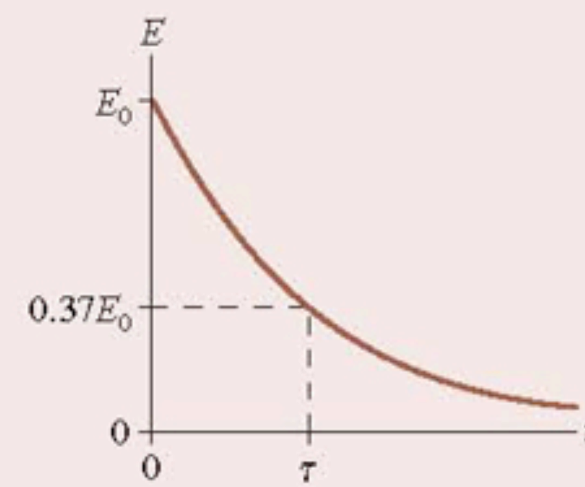
$$= \frac{1}{2}m(v_{max})^2$$

$$= \frac{1}{2}kA^2$$

In a **damped system,** the energy decays exponentially

$$E = E_0 e^{-t/\tau}$$

where τ is the time constant.



Chapter 14 (Oscillations) (contd.)

IMPORTANT CONCEPTS

Simple harmonic motion (SHM) is a sinusoidal oscillation with period T and amplitude A.

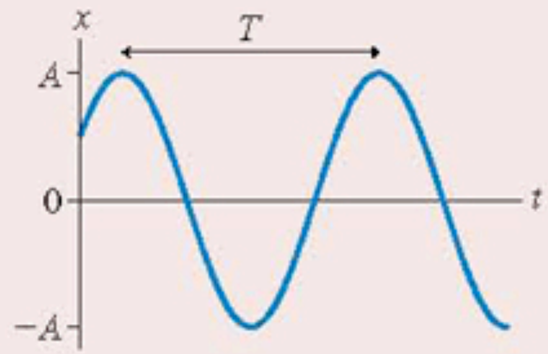
Frequency $f = \dfrac{1}{T}$

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Position $x(t) = A\cos(\omega t + \phi_0)$

$$= A\cos\left(\frac{2\pi t}{T} + \phi_0\right)$$



Velocity $v_x(t) = -v_{max}\sin(\omega t + \phi_0)$ with maximum speed $v_{max} = \omega A$

Acceleration $a_x = -\omega^2 x$

SHM is the projection onto the x-axis of **uniform circular motion.**

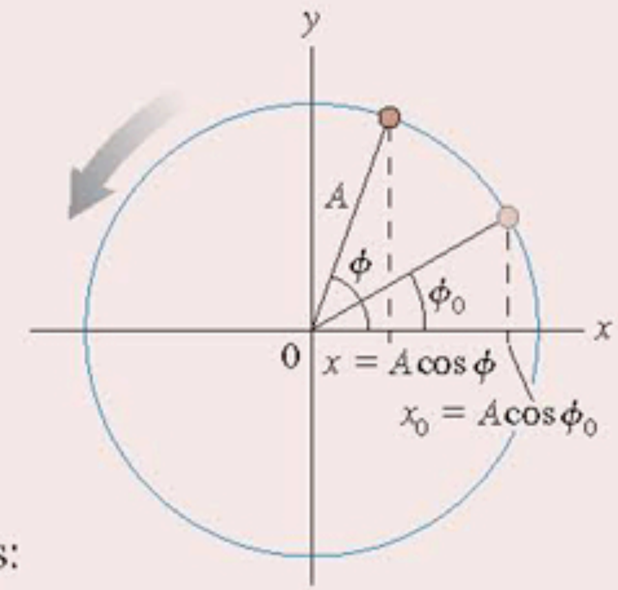
$\phi = \omega t + \phi_0$ is the **phase**

The position at time t is

$$x(t) = A\cos\phi$$
$$= A\cos(\omega t + \phi_0)$$

The **phase constant** ϕ_0 determines the initial conditions:

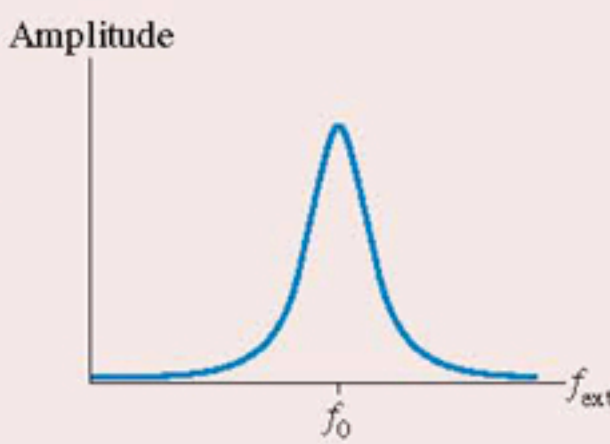
$x_0 = A\cos\phi_0 \qquad v_{0x} = -\omega A\sin\phi_0$



APPLICATIONS

Resonance

When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if $f_{ext} \approx f_0$ where f_0 is the system's natural oscillation frequency, or **resonant frequency.**

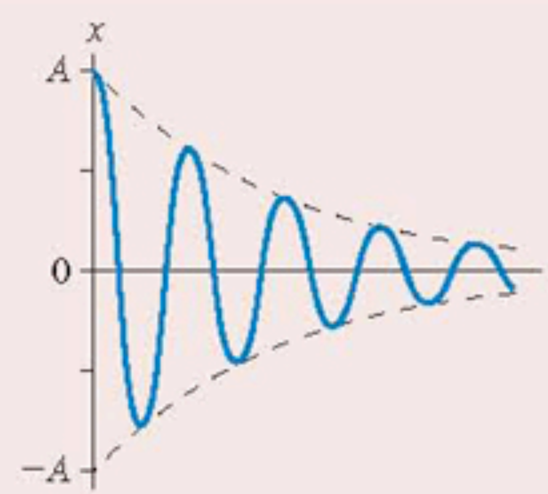


Damping

If there is a drag force $\vec{D} = -b\vec{v}$, where b is the damping constant, then (for lightly damped systems)

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0)$$

The time constant for energy loss is $\tau = m/b$.



Chapter 15 (Fluids)

GENERAL PRINCIPLES

Fluid Statics

Gases

- Freely moving particles
- Compressible
- Pressure primarily thermal
- Pressure constant in a laboratory-size container

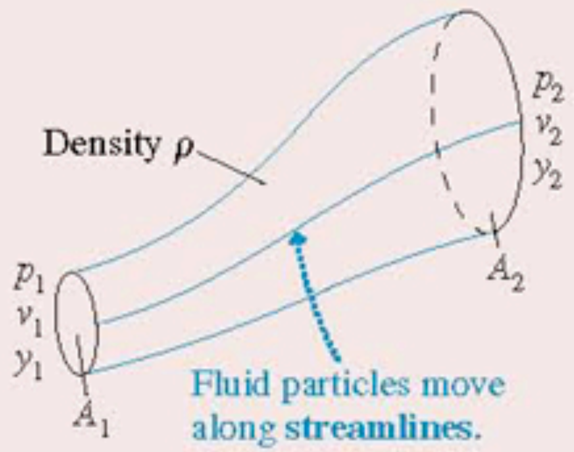
Liquids

- Loosely bound particles
- Incompressible
- Pressure primarily gravitational
- Hydrostatic pressure at depth d is $p = p_0 + \rho g d$

Fluid Dynamics

Ideal-fluid model

- Incompressible
- Smooth, laminar flow
- Nonviscous
- Irrotational



Equation of continuity

$$v_1 A_1 = v_2 A_2$$

Bernoulli's equation

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli's equation is a statement of energy conservation.

IMPORTANT CONCEPTS

Density $\rho = m/V$, where m is mass and V is volume.

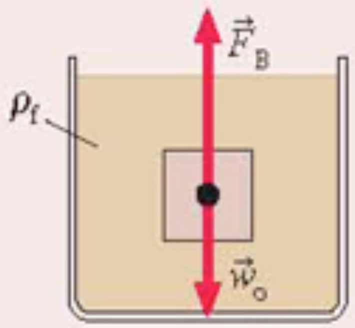
Pressure $p = F/A$, where F is the magnitude of the fluid force and A is the area on which the force acts.

- Exists at all points in a fluid
- Pushes equally in all directions
- Constant along a horizontal line
- Gauge pressure $p_g = p - 1$ atm

Chapter 15 (Fluids) (contd.)

APPLICATIONS

Buoyancy is the upward force of a fluid on an object.



Archimedes' principle

The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

Sink	$\rho_{avg} > \rho_f$	$F_B < w_o$
Rise to surface	$\rho_{avg} < \rho_f$	$F_B > w_o$
Neutrally buoyant	$\rho_{avg} = \rho_f$	$F_B = w_o$

Chapter 20 (Traveling Waves)

GENERAL PRINCIPLES

The Wave Model

This model is based on the idea of a traveling wave, which is an organized disturbance traveling at a well-defined **wave speed** v.

- In transverse waves the particles of the medium move perpendicular to the direction in which the wave travels.

- In longitudinal waves the particles of the medium move parallel to the direction in which the wave travels.

A wave transfers **energy**, but no material or substance is transferred outward from the source.

Three basic types of waves:

- Mechanical waves travel through a material medium such as water or air.
- Electromagnetic waves require no material medium and can travel through a vacuum.
- Matter waves describe the wavelike characteristics of atomic-level particles.

For mechanical waves, the speed of the wave is a property of the medium. Speed does not depend on the size or shape of the wave.

IMPORTANT CONCEPTS

The **displacement** D of a wave is a function of both position (where) and time (when).

- A snapshot graph shows the wave's displacement as a function of position at a single instant of time.

- A history graph shows the wave's displacement as a function of time at a single point in space.

A wave traveling in the positive x-direction with speed v must be a function of the form $D(x - vt)$.

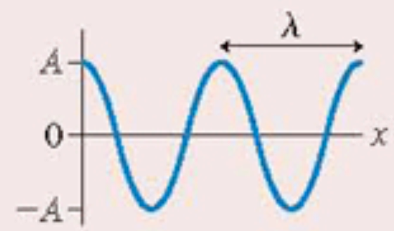
A wave traveling in the negative x-direction with speed v must be a function of the form $D(x + vt)$.

Sinusoidal waves are periodic in both time (period T) and space (wavelength λ).

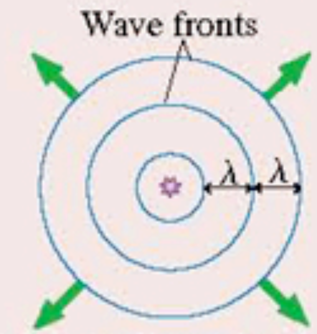
$$D(x, t) = A\sin[2\pi(x/\lambda - t/T) + \phi_0]$$
$$= A\sin(kx - \omega t + \phi_0)$$

where A is the **amplitude,** $k = 2\pi/\lambda$ is the **wave number,** $\omega = 2\pi f = 2\pi/T$ is the **angular frequency,** and ϕ_0 is the **phase constant** that describes initial conditions.

The fundamental relationship for any sinusoidal wave is $v = \lambda f$.



One-dimensional waves



Two- and three-dimensional waves

Chapter 20 (Traveling Waves) (contd.)

APPLICATIONS

Wave speeds for some specific waves:

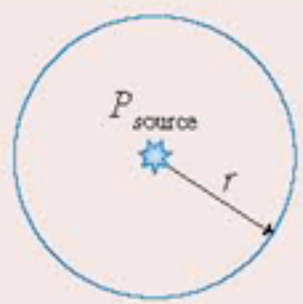
- **String** (transverse): $v = \sqrt{T_s/\mu}$
- **Sound** (longitudinal): $v = 343$ m/s in 20°C air
- **Light** (transverse): $v = c/n$, where $c = 3.00 \times 10^8$ m/s is the speed of light in a vacuum and n is the material's **index of refraction.**

The wave intensity is the power-to-area ratio

$$I = P/A$$

For a circular or spherical wave

$$I = P_{source}/4\pi r^2$$



The Doppler effect occurs when a wave source and detector are moving with respect to each other: the frequency detected differs from the frequency f_0 emitted.

Approaching source

$$f_+ = \frac{f_0}{1 - v_s/v}$$

Observer approaching a source

$$f_+ = (1 + v_o/v)f_0$$

Receding source

$$f_- = \frac{f_0}{1 + v_s/v}$$

Observer receding from a source

$$f_- = (1 - v_o/v)f_0$$

The Doppler effect for light uses a result derived from the theory of relativity.

Chapter 21(Superposition)

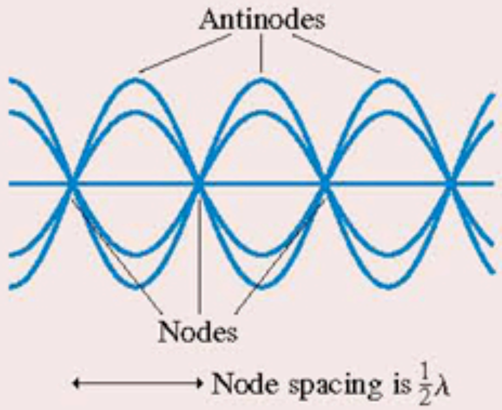
GENERAL PRINCIPLES

Principle of Superposition

The displacement of a medium when more than one wave is present is the sum of the displacements due to each individual wave.

IMPORTANT CONCEPTS

Standing waves are due to the superposition of two traveling waves moving in opposite directions.



The amplitude at position x is

$$A(x) = 2a \sin kx$$

where a is the amplitude of each wave.

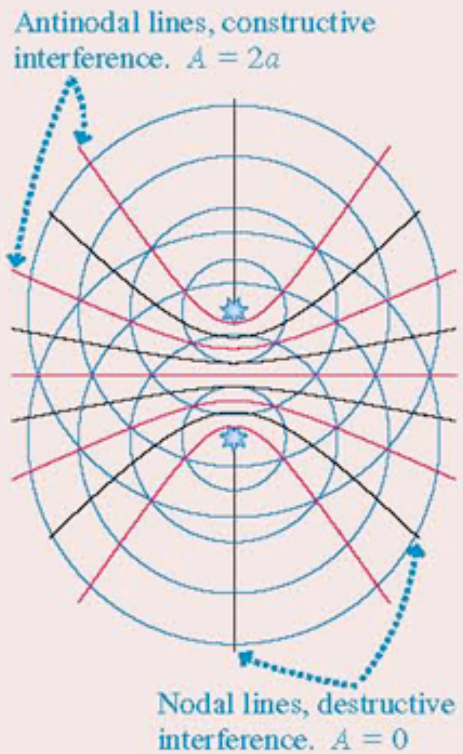
The boundary conditions determine which standing wave frequencies and wavelengths are allowed.

Interference

In general, the superposition of two or more waves into a single wave is called interference.

Maximum constructive interference occurs where crests are aligned with crests and troughs with troughs. These waves are in phase. The maximum displacement is $A = 2a$.

Perfect destructive interference occurs where crests are aligned with troughs. These waves are out of phase. The amplitude is $A = 0$.



Interference depends on the phase difference $\Delta\phi$ between the two waves.

$$\text{Constructive: } \Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = 2m\pi$$

$$\text{Destructive: } \Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = 2(m + \tfrac{1}{2})\pi$$

Δr is the path-length difference of the two waves and $\Delta\phi_0$ is any phase difference between the sources. For identical sources (in phase, $\Delta\phi_0 = 0$):

Interference is constructive if the path-length difference $\Delta r = m\lambda$.

Interference is destructive if the path-length difference $\Delta r = (m + \frac{1}{2})\lambda$.

The amplitude at a point where the phase difference is $\Delta\phi$ is $A = \left|2a\cos\left(\frac{\Delta\phi}{2}\right)\right|$

Chapter 21 (Superposition) (contd.)

APPLICATIONS

Boundary conditions

Strings, electromagnetic waves, and sound waves in closed-closed tubes must have nodes at both ends.

$$\lambda_m = \frac{2L}{m} \qquad f_m = m\frac{v}{2L} = mf_1$$

where $m = 1, 2, 3, \ldots$

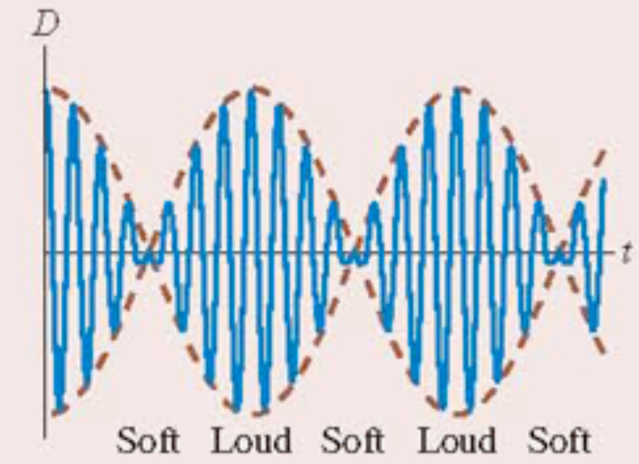
The frequencies and wavelengths are the same for a sound wave in an open-open tube, which has antinodes at both ends.

A sound wave in an open-closed tube must have a node at the closed end but an antinode at the open end. This leads to

$$\lambda_m = \frac{4L}{m} \qquad f_m = m\frac{v}{4L} = mf_1$$

where $m = 1, 3, 5, 7, \ldots$

Beats (loud-soft-loud-soft modulations of intensity) occur when two waves of slightly different frequency are superimposed.



The beat frequency between waves of frequencies f_1 and f_2 is

$$f_{beat} = f_1 - f_2$$

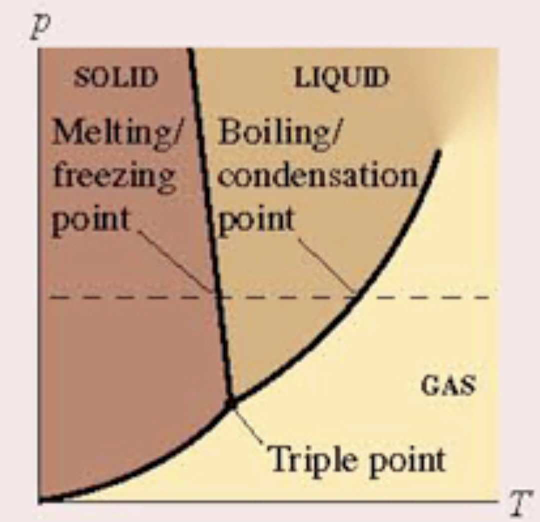
Chapter 16 (Macroscopic Description of Matter)

GENERAL PRINCIPLES

Three Phases of Matter

Solid	Rigid, definite shape. Nearly incompressible.
Liquid	Molecules loosely held together by molecular bonds, but able to move around. Nearly incompressible.
Gas	Molecules move freely through space. Compressible.

The different phases exist for different conditions of temperature T and pressure p. The boundaries separating the regions of a phase diagram are lines of phase equilibrium. Any amounts of the two phases can coexist in equilibrium. The **triple point** is the one value of temperature and pressure at which all three phases can coexist in equilibrium.



APPLICATIONS

Temperature scales

$$T_F = \frac{9}{5}T_C + 32° \qquad T_K = T_C + 273$$

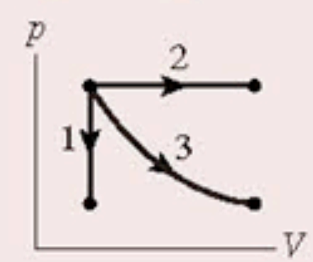
The Kelvin temperature scale is based on:

- Absolute zero at $T_0 = 0\text{ K}$
- The triple point of water at $T_3 = 273.16\text{ K}$

Three basic gas processes

1. **Isochoric,** or constant volume
2. **Isobaric,** or constant pressure
3. **Isothermal,** or constant temperature

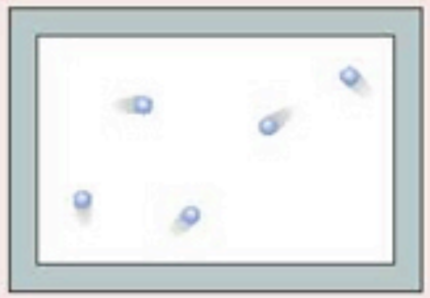
pV diagram



Chapter 16 (contd.)

IMPORTANT CONCEPTS

Ideal-Gas Model



- Atoms and molecules are small, hard spheres that travel freely through space except for occasional collisions with each other or the walls.
- The molecules have a distribution of speeds.
- The model is valid when the density is low and the temperature well above the condensation point.

Ideal-Gas Law

The **state variables** of an ideal gas are related by the ideal-gas law

$$pV = nRT \quad \text{or} \quad pV = Nk_BT$$

where $R = 8.31$ J/mol K is the universal gas constant and $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant.

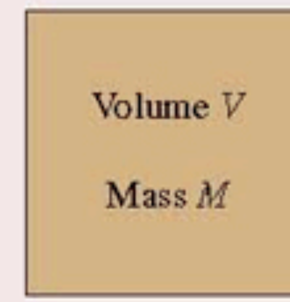
p, V, and T *must* be in SI units of Pa, m^3, and K. For a gas in a sealed container, with constant n:

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1}$$

Counting atoms and moles

A macroscopic sample of matter consists of N atoms (or molecules), each of mass m (the **atomic** or **molecular mass**):

$$N = \frac{M}{m}$$



Alternatively, we can state that the sample consists of n **moles**

$$n = \frac{N}{N_A} \quad \text{or} \quad \frac{M(\text{in grams})}{M_{mol}}$$

$N_A = 6.02 \times 10^{23}$ mol^{-1} is **Avogadro's number.**

The numerical value of the molar mass M_{mol}, in g/mol, equals the numerical value of the atomic or molecular mass m in u. The atomic or molecular mass m, in atomic mass units u, is well approximated by the **atomic mass number** A.

$$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$$

The **number density** of the sample is $\frac{N}{V}$.

Chapter 17 (Work, Heat and 1st Law of Thermodynamics)

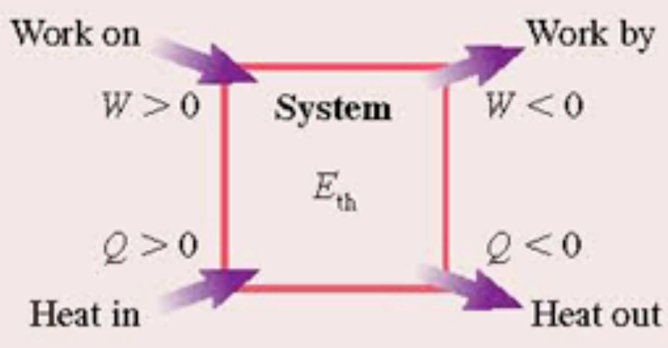
GENERAL PRINCIPLES

First Law of Thermodynamics

$\Delta E_{th} = W + Q$

The first law is a general statement of energy conservation.

Work W and heat Q depend on the process by which the system is changed.



The change in the system depends only on the total energy exchanged $W + Q$, not on the process.

Energy

Thermal energy E_{th} Microscopic energy of moving molecules and stretched molecular bonds. ΔE_{th} depends on the initial/final states but is independent of the process.

Work W Energy transferred to the system by forces in a mechanical interaction.

Heat Q Energy transferred to the system via atomic-level collisions when there is a temperature difference. A thermal interaction.

SUMMARY OF BASIC GAS PROCESSES

Process	Definition	Stays constant	Work	Heat
Isochoric	$\Delta V = 0$	V and p/T	$W = 0$	$Q = nC_V\Delta T$
Isobaric	$\Delta p = 0$	p and V/T	$W = -p\Delta V$	$Q = nC_P\Delta T$
Isothermal	$\Delta T = 0$	T and pV	$W = -nRT\ln(V_f/V_i)$	$\Delta E_{th} = 0$
Adiabatic	$Q = 0$	pV^γ	$W = \Delta E_{th}$	$Q = 0$
All gas processes		Ideal-gas law	$pV = nRT$	
		First law	$\Delta E_{th} = W + Q = nC_V\Delta T$	

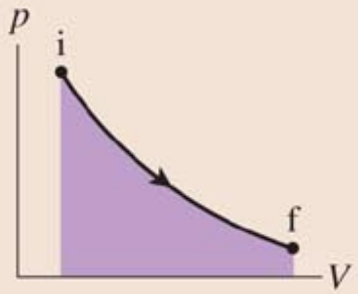
Chapter 17 (contd.)

Important Concepts

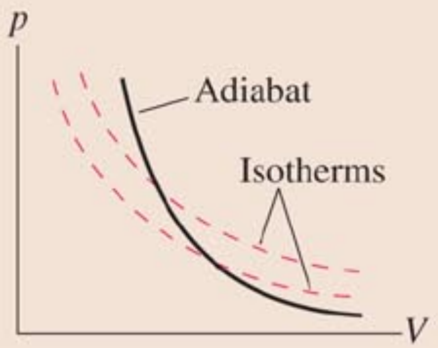
The **work** done on a gas is

$$W = -\int_{V_i}^{V_f} p\,dV$$

$$= -(\text{area under the } pV \text{ curve})$$



An **adiabatic process** is one for which $Q = 0$. Gases move along an **adiabat** for which $pV^{\gamma} = \text{constant}$, where $\gamma = C_P/C_V$ is the **specific heat ratio.** An adiabatic process changes the temperature of the gas without heating it.



Calorimetry When two or more systems interact thermally, they come to a common final temperature determined by

$$Q_{net} = Q_1 + Q_2 + \cdots = 0$$

The heat of transformation L is the energy needed to cause 1 kg of substance to undergo a phase change

$$Q = \pm ML$$

The specific heat c of a substance is the energy needed to raise the temperature of 1 kg by 1 K:

$$Q = Mc\Delta T$$

The molar specific heat C is the energy needed to raise the temperature of 1 mol by 1 K:

$$Q = nC\Delta T$$

The molar specific heat of gases depends on the *process* by which the temperature is changed:

C_V = molar specific heat at **constant volume**
$C_P = C_V + R$ = molar specific heat at **constant pressure**

Heat is transferred by **conduction, convection, radiation,** and **evaporation.**

Conduction: $Q/\Delta t = (kA/L)\Delta T$

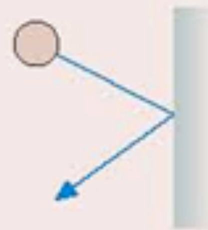
Radiation: $Q/\Delta t = e\sigma AT^4$

Chapter 18 (The Micro/Macro Connection)

IMPORTANT CONCEPTS

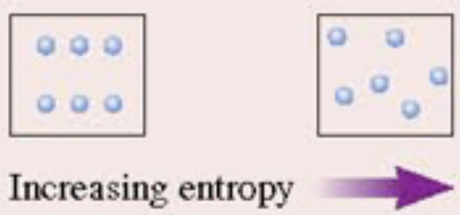
Pressure is due to the force of the molecules colliding with the walls.

$$p = \frac{1}{3}\frac{N}{V} m{v_{rms}}^2 = \frac{2}{3}\frac{N}{V}\epsilon_{avg}$$



The average translational kinetic energy of a molecule is $\epsilon_{avg} = \frac{3}{2}k_B T$. The temperature of the gas $T = \frac{2}{3k_B}\epsilon_{avg}$ measures the average translational kinetic energy.

Entropy measures the probability that a macroscopic state will occur or, equivalently, the amount of disorder in a system.

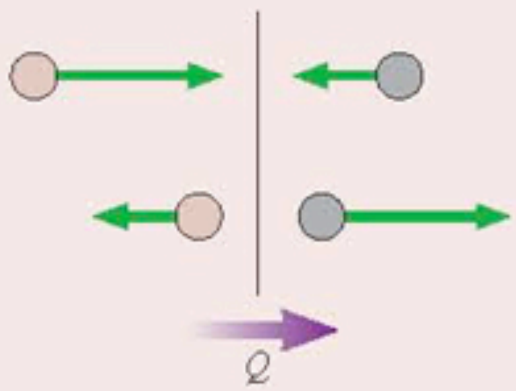


The **thermal energy** of a system is

E_{th} = translational kinetic energy + rotational kinetic energy + vibrational energy

- **Monatomic gas** $E_{th} = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$
- **Diatomic gas** $E_{th} = \frac{5}{2}Nk_B T = \frac{5}{2}nRT$
- **Elemental solid** $E_{th} = 3Nk_B T = 3nRT$

Heat is energy transferred via collisions from more-energetic molecules on one side to less-energetic molecules on the other. Equilibrium is reached when $(\epsilon_1)_{avg} = (\epsilon_2)_{avg}$, which implies $T_{1f} = T_{2f}$.



Chapter 18 (contd.)

GENERAL PRINCIPLES

Kinetic theory, the **micro/macro connection**, relates the macroscopic properties of a system to the motion and collisions of its atoms and molecules.

The Equipartition Theorem

Tells us how collisions distribute the energy in the system. The energy stored in each mode of the system (each **degree of freedom**) is $\frac{1}{2}Nk_BT$ or, in terms of moles, $\frac{1}{2}nRT$.

The Second Law of Thermodynamics

Tells us how collisions move a system toward equilibrium. The entropy of an isolated system can only increase or, in equilibrium, stay the same.

- Order turns into disorder and randomness.
- Systems run down.
- Heat energy is transferred spontaneously from the hotter to the colder system, never from colder to hotter.

APPLICATIONS

The root-mean-square speed v_{rms} is the square root of the average of the squares of the molecular speeds:

$$v_{rms} = \sqrt{(v^2)_{avg}}$$

For molecules of mass m at temperature T,

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

Molar specific heats can be predicted from the thermal energy because $\Delta E_{th} = nC\Delta T$.

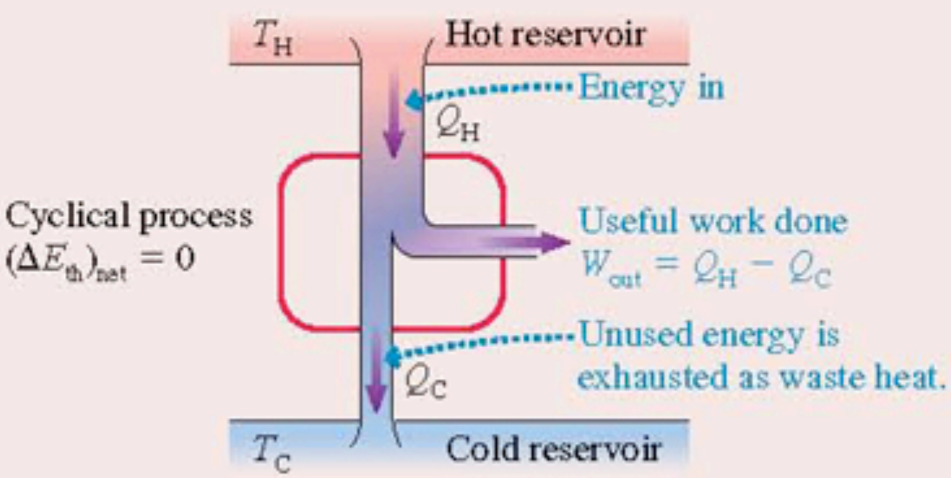
- **Monatomic gas** $C_V = \frac{3}{2}R$
- **Diatomic gas** $C_V = \frac{5}{2}R$
- **Elemental solid** $C = 3R$

Chapter 19 (Heat Engines and Refrigerators)

GENERAL PRINCIPLES

Heat Engines

Devices which transform heat into work. They require two energy reservoirs at different temperatures.



Thermal efficiency

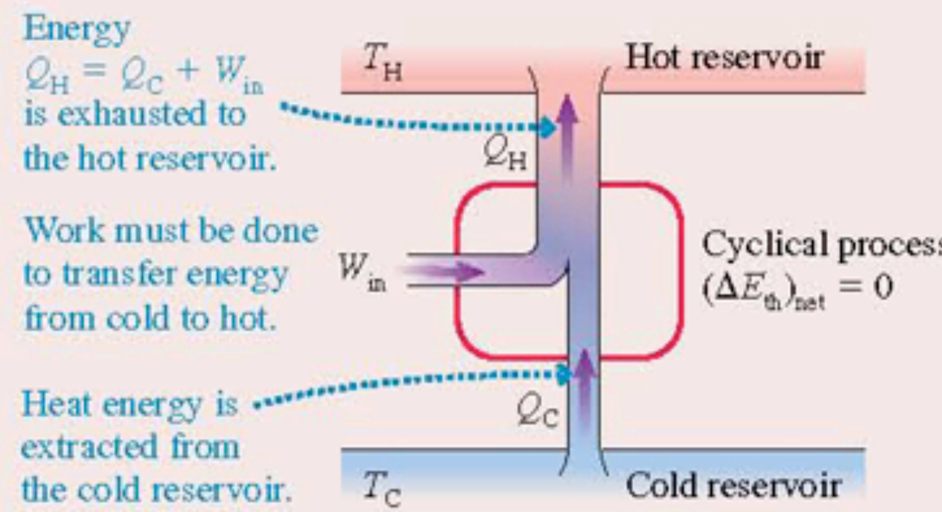
$$\eta = \frac{W_{out}}{Q_H} = \frac{\text{what you get}}{\text{what you pay}}$$

Second law limit:

$$\eta \leq 1 - \frac{T_C}{T_H}$$

Refrigerators

Devices which use work to transfer heat from a colder object to a hotter object.



Coefficient of performance

$$K = \frac{Q_C}{W_{in}} = \frac{\text{what you get}}{\text{what you pay}}$$

Second law limit:

$$K \leq \frac{T_C}{T_H - T_C}$$

Chapter 19 (contd.)

IMPORTANT CONCEPTS

A perfectly reversible engine (a **Carnot engine**) can be operated as either a heat engine or a refrigerator between the same two energy reservoirs by reversing the cycle and with no other changes.

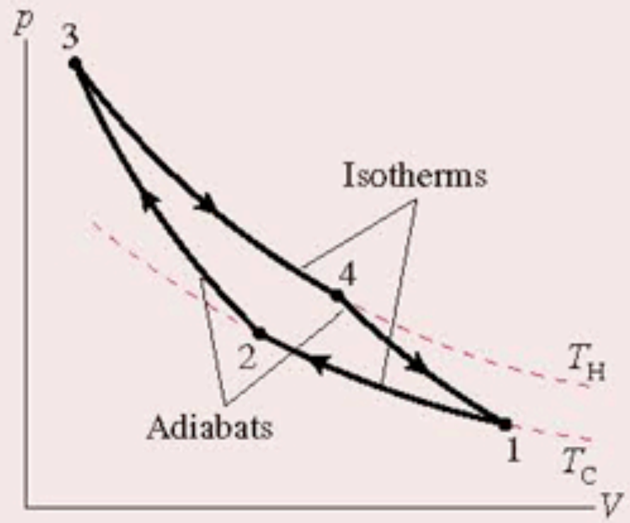
- A **Carnot heat engine** has the maximum possible thermal efficiency of any heat engine operating between T_H and T_C.

$$\eta_{Carnot} = 1 - \frac{T_C}{T_H}$$

- A **Carnot refrigerator** has the maximum possible coefficient of performance of any refrigerator operating between T_H and T_C.

$$K_{Carnot} = \frac{T_C}{T_H - T_C}$$

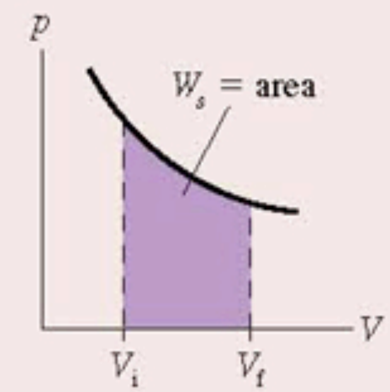
The Carnot cycle for a gas engine consists of two isothermal processes and two adiabatic processes.



An energy reservoir is a part of the environment so large in comparison to the system that its temperature doesn't change as the system extracts heat energy from or exhausts heat energy to the reservoir. All heat engines and refrigerators operate between two energy reservoirs at different temperatures T_H and T_C.

The **work** W_s done *by* the system has the opposite sign to the work done *on* the system.

W_s = area under pV curve



APPLICATIONS

To analyze a heat engine or refrigerator:

MODEL Identify each process in the cycle.

VISUALIZE Draw the pV diagram of the cycle.

SOLVE There are several steps:

- Determine p, V, and T at the beginning and end of each process.
- Calculate ΔE_{th}, W_s, and Q for each process.
- Determine W_{in} or W_{out}, Q_H, and Q_C.
- Calculate $\eta = W_{out}/Q_H$ or $K = Q_C/W_{in}$.

ASSESS Verify $(\Delta E_{th})_{net} = 0$. Check signs.

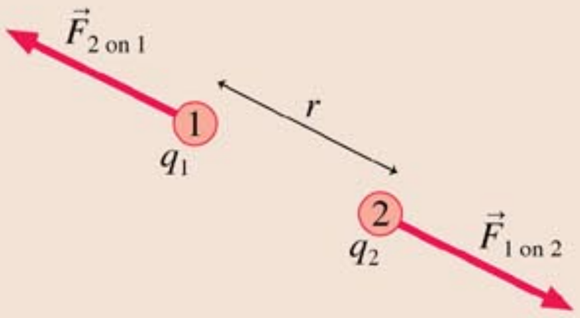
Chapter 26 (Electric Charges and Forces)

General Principles

Coulomb's Law

The forces between two charged particles q_1 and q_2 separated by distance r are

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{K|q_1||q_2|}{r^2}$$



These forces are an action/reaction pair directed along the line joining the particles.

- The forces are repulsive for two like charges, attractive for two opposite charges.
- The net force on a charge is the sum of the forces from all other charges.
- The unit of charge is the coulomb (C).
- The electrostatic constant is $K = 9.0 \times 10^9\ \text{N m}^2/\text{C}^2$.

Chapter 26 (continued)

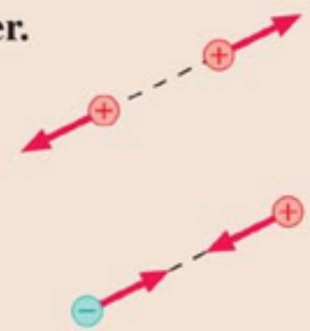
Important Concepts

The Charge Model

There are two kinds of charge, positive and negative.

- Fundamental charges are protons and electrons, with charge $\pm e$ where $e = 1.60 \times 10^{-19}$ C.
- Objects are charged by adding or removing electrons.
- The amount of charge is $q = (N_p - N_e)e$.
- An object with an equal number of protons and electrons is neutral, meaning no *net* charge.

Charged objects exert electric forces on each other.



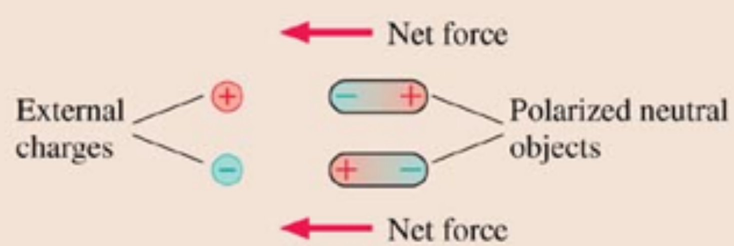
- Like charges repel, opposite charges attract.
- The force increases as the charge increases.
- The force decreases as the distance increases.

There are two types of material, insulators **and** conductors.

- Charge remains fixed in or on an insulator.
- Charge moves easily through or along conductors.
- Charge is transferred by contact between objects.

Charged objects attract neutral objects.

- Charge polarizes metal by shifting the electron sea.
- Charge polarizes atoms, creating electric dipoles.
- The polarization force is always an attractive force.



The Field Model

Charges interact with each other via the electric field $\vec{E}$.

- Charge A alters the space around it by creating an electric field.

- The field is the agent that exerts a force. The force on charge q_B is $\vec{F}_{\text{on B}} = q_B\vec{E}$.

An electric field is identified and measured in terms of the force on a **probe charge** q:

$$\vec{E} = \vec{F}_{\text{on } q}/q$$

- The electric field exists at all points in space.
- An electric field vector shows the field only at one point, the point at the tail of the vector.

The electric field of a **point charge** is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

GENERAL PRINCIPLES

Sources of $\vec{E}$

Electric fields are created by charges.

Two major tools for calculating $\vec{E}$ are

- The field of a point charge

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

- The principle of superposition

Multiple point charges

Use superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$

Continuous distribution of charge

- Divide the charge into point-like ΔQ
- Find the field of each ΔQ
- Find $\vec{E}$ by summing the fields of all ΔQ

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a **charge density** (λ or η) and an integration coordinate.

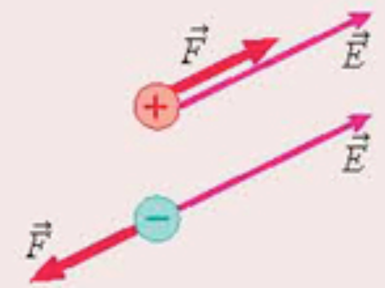
Consequences of $\vec{E}$

The electric field exerts a force on a charged particle.

$$\vec{F} = q\vec{E}$$

The force causes acceleration

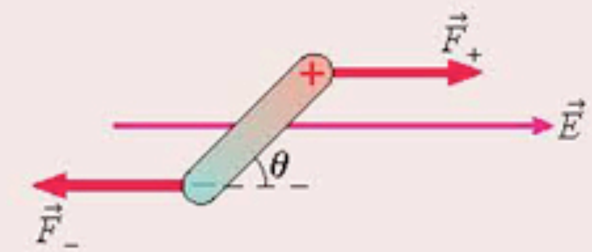
$$\vec{a} = (q/m)\vec{E}$$



Trajectories of charged particles are calculated with kinematics.

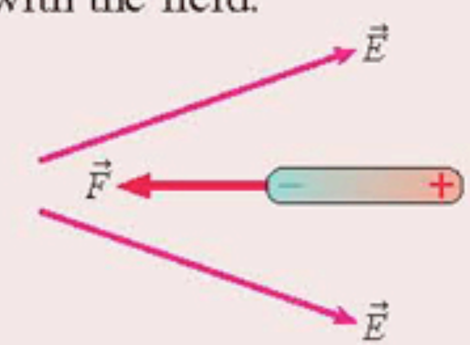
The electric field exerts a torque on a dipole.

$$\tau = pE\sin\theta$$



The torque tends to align the dipoles with the field.

In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.

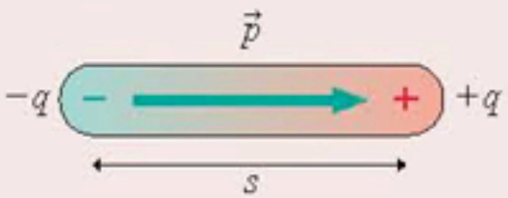


Chapter 27 (continued)

APPLICATIONS

The following fields are important models of the electric field:

Electric dipole



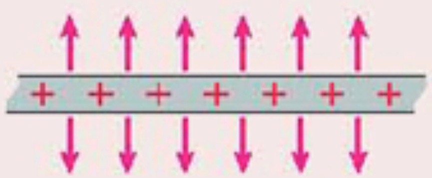
The electric dipole moment is

$\vec{p} = (qs, \text{from negative to positive})$

Field on axis $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{2\vec{p}}{r^3}$

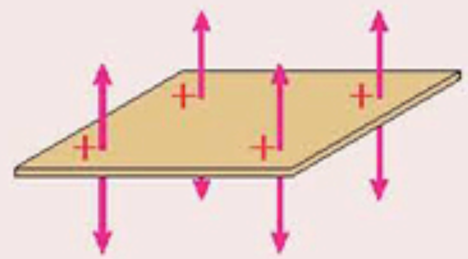
Field in bisecting plane $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{\vec{p}}{r^3}$

Infinite line of charge with linear charge density λ



$\vec{E} = \left(\dfrac{1}{4\pi\epsilon_0}\dfrac{2\lambda}{r}, \text{perpendicular to line}\right)$

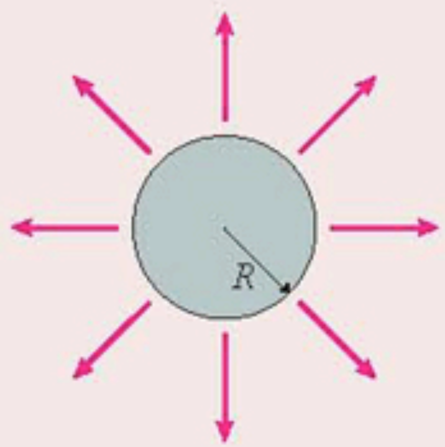
Infinite plane of charge with surface charge density η



$\vec{E} = \left(\dfrac{\eta}{2\epsilon_0}, \text{perpendicular to plane}\right)$

Sphere of charge

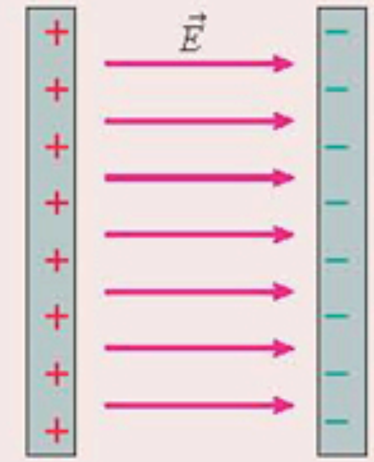
Same as a point charge Q for $r > R$



Parallel-plate capacitor

The electric field inside an ideal capacitor is a **uniform electric field**

$\vec{E} = \left(\dfrac{\eta}{\epsilon_0}, \text{from positive to negative}\right)$



A real capacitor has a weak **fringe field** around it.

Chapter 28 (Gauss' Law)

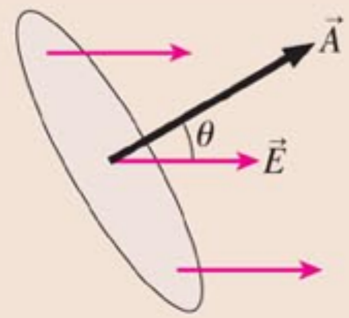
Important Concepts

Charge creates the electric field that is responsible for the electric flux.

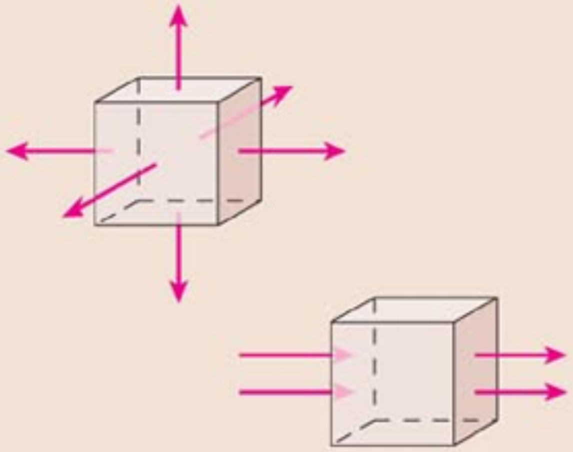
Flux is the amount of electric field passing through a surface of area A:

$$\Phi_e = \vec{E} \cdot \vec{A}$$

where $\vec{A}$ is the **area vector.**



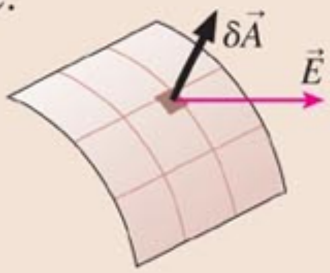
For closed surfaces:
A net flux in or out indicates that the surface encloses a net charge. Field lines through but with no *net* flux mean that the surface encloses no *net* charge.



Surface integrals calculate the flux by summing the fluxes through many small pieces of the surface:

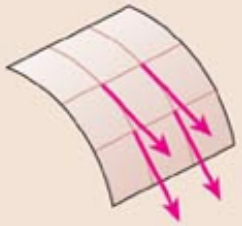
$$\Phi_e = \sum \vec{E} \cdot \delta\vec{A}$$

$$\to \int \vec{E} \cdot d\vec{A}$$



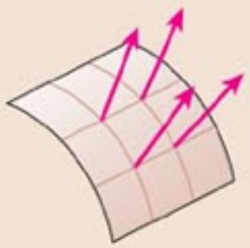
Two important situations:
If the electric field is everywhere tangent to the surface, then

$$\Phi_e = 0$$



If the electric field is everywhere perpendicular to the surface *and* has the same strength E at all points, then

$$\Phi_e = EA$$



Chapter 28 (contd.)

General Principles

Gauss's Law

For any *closed* surface enclosing net charge Q_{in}, the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

The electric flux Φ_e is the same for *any* closed surface enclosing charge Q_{in}.

Symmetry

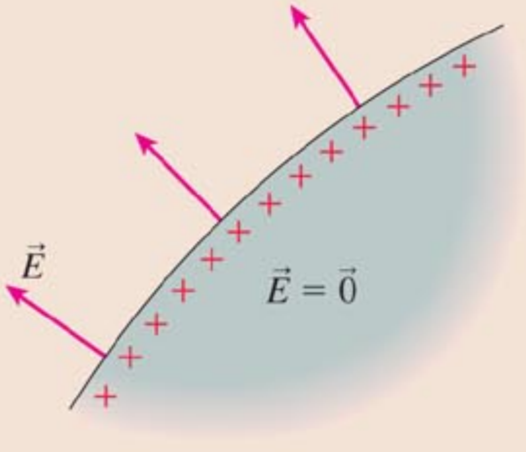
The symmetry of the electric field must match the symmetry of the charge distribution.

In practice, Φ_e is computable only if the symmetry of the Gaussian surface matches the symmetry of the charge distribution.

Applications

Conductors in electrostatic equilibrium

- The electric field is zero at all points within the conductor.
- Any excess charge resides entirely on the exterior surface.
- The external electric field is perpendicular to the surface and of magnitude η/ϵ_0, where η is the surface charge density.
- The electric field is zero inside any hole within a conductor unless there is a charge in the hole.



Chapter 29 (The Electric Potential)

GENERAL PRINCIPLES

Sources of V

The electric potential, like the electric field, is created by charges.

Two major tools for calculating V are

- The potential of a point charge $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$
- The principle of superposition

Multiple point charges

Use superposition: $V = V_1 + V_2 + V_3 + \ldots$

Continuous distribution of charge

- Divide the charge into point-like ΔQ.
- Find the potential of each ΔQ.
- Find V by summing the potentials of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a charge density and an integration coordinate. Calculating V is usually easier than calculating $\vec{E}$ because the potential is a scalar.

Consequences of V

A charged particle has potential energy

$$U = qV$$

at a point where source charges have created an electric potential V.

The electric force is a conservative force, so the mechanical energy is conserved for a charged particle in an electric potential:

$$K_f + U_f = K_i + U_i$$

The potential energy of **two point charges** separated by distance r is

$$U_{q_1+q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

The **zero point** of potential and potential energy is chosen to be convenient. For point charges, we let $U = 0$ when $r \to \infty$.

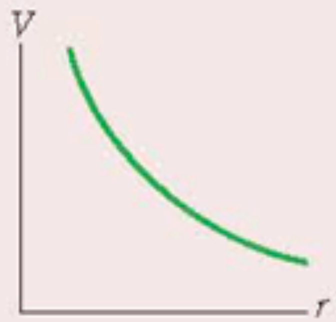
The potential energy in an electric field of an **electric dipole** with dipole moment $\vec{p}$ is

$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

Chapter 29 (continued)

APPLICATIONS

Graphical representations of the potential:



Potential graph

Equipotential surfaces

Contour map

Elevation graph

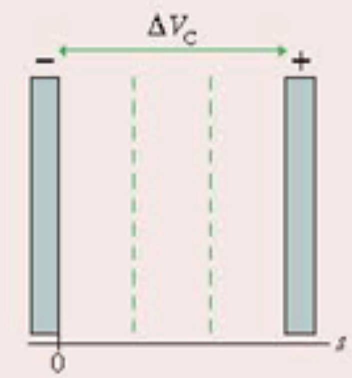
Sphere of charge Q

Same as a point charge if $r \geq R$.

Parallel-plate capacitor

$V = Es$, where s is measured from the negative plate. The electric field inside is

$$E = \Delta V_C / d$$



Units

Electric potential: $1\ \text{V} = 1\ \text{J/C}$

Electric field: $1\ \text{V/m} = 1\ \text{N/C}$

Chapter 30 (Potential and Field)

GENERAL PRINCIPLES

Connecting V and $\vec{E}$

The electric potential and the electric field are two different perspectives of how source charges alter the space around them. V and $\vec{E}$ are related by

$$\Delta V = V(s_f) - V(s_i) = -\int_{s_i}^{s_f} E_s\, ds$$

where s is measured from point i to point f and E_s is the component of $\vec{E}$ parallel to the line of integration.

Graphically

$\Delta V =$ the negative of the area under the E_s graph

and

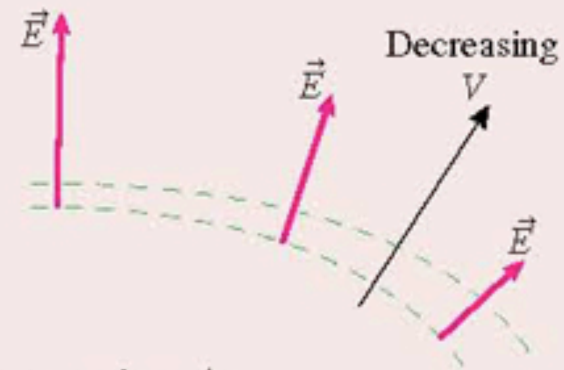
$$E_s = -\frac{dV}{ds}$$

$=$ the negative of the slope of the potential graph.

The Geometry of Potential and Field

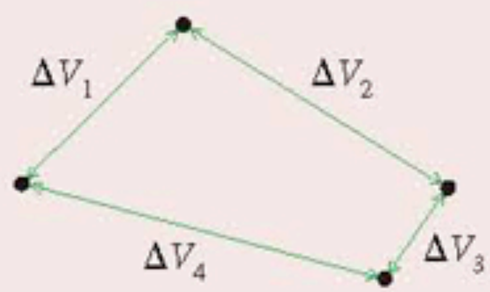
The electric field

- Is perpendicular to the equipotential surfaces.
- Points "downhill" in the direction of decreasing V.
- Is inversely proportional to the spacing Δs between the equipotential surfaces.



Conservation of Energy

The sum of all potential differences around a closed path is zero. $\sum(\Delta V)_i = 0$.

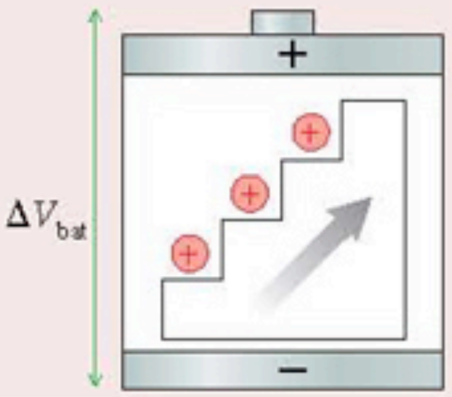


IMPORTANT CONCEPTS

A battery is a source of potential.
The charge escalator in a battery uses chemical reactions to move charges from the negative terminal to the positive terminal.

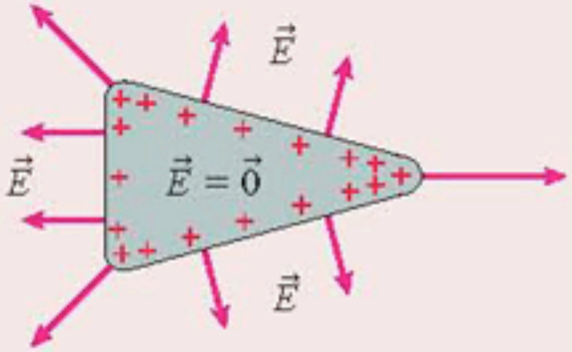
$$\Delta V_{bat} = \mathcal{E}$$

where the emf $\mathcal{E}$ is the work per charge done by the charge escalator.



For a conductor in electrostatic equilibrium

- The interior electric field is zero.
- The exterior electric field is perpendicular to the surface.
- The surface is an equipotential.
- The interior is at the same potential as the surface.

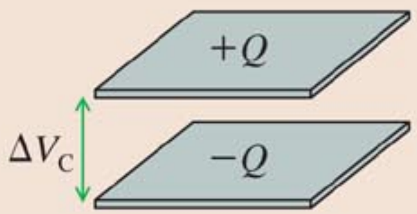


Applications

Capacitors

The **capacitance** of two conductors charged to $\pm Q$ is

$$C = \frac{Q}{\Delta V_C}$$



A parallel-plate capacitor has

$$C = \frac{\epsilon_0 A}{d}$$

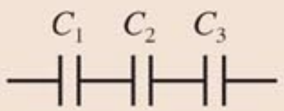
Filling the space between the plates with a dielectric of dielectric constant κ increases the capacitance to $C = \kappa C_0$

The energy stored in a capacitor is $u_C = \frac{1}{2}C(\Delta V_C)^2$

This energy is stored in the electric field at density $u_E = \frac{1}{2}\epsilon_0 E^2$.

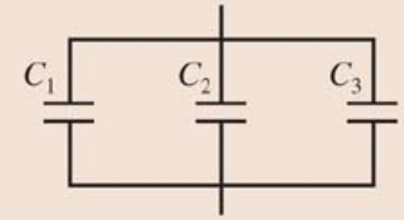
Combinations of capacitors

Series capacitors



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$

Parallel capacitors

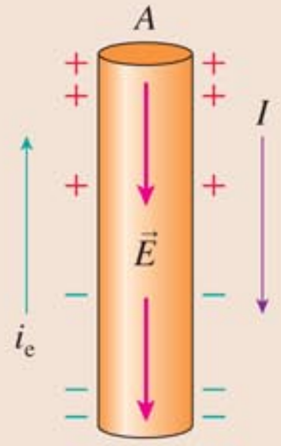


$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

Chapter 31 (Current and Resistance)

General Principles

Current is a nonequilibrium motion of charges sustained by an electric field. Nonuniform surface charge density creates an electric field in a wire. The electric field pushes the electron current i_e in a direction opposite to $\vec{E}$. The conventional current I is in the direction in which positive charge *seems* to move.

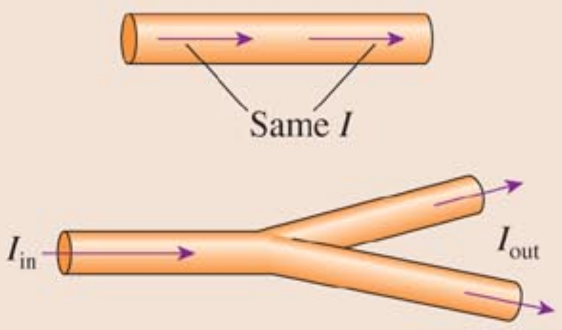


Conservation of Current

The current is the same at any two points in a wire.
At a junction,

$$\sum I_{in} = \sum I_{out}$$

This is **Kirchhoff's junction law.**



Electron current

i_e = rate of electron flow

$N_e = i_e \Delta t$

Conventional current

I = rate of charge flow $= ei_e$

$Q = I\Delta t$

Current density

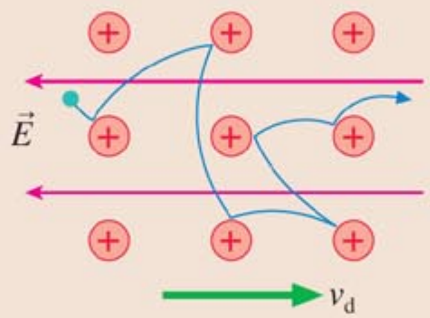
$J = I/A$

Chapter 31 (contd.)

Important Concepts

Sea of electrons

Conduction electrons move freely around the positive ions that form the atomic lattice.



Conduction

An electric field causes a slow drift at speed v_d to be superimposed on the rapid but random thermal motions of the electrons.

Collisions of electrons with the ions transfer energy to the atoms. This makes the wire warm and lightbulbs glow. More collisions mean a higher resistivity ρ and a lower conductivity σ.

The **drift speed** is $v_d = \frac{e\tau}{m}E$

where τ is the mean time between collisions. v_d is related to the electron current by

$$i_e = n_e A v_d$$

where n_e is the electron density

An electric field E in a conductor causes a current density $J = n_e e v_d = \sigma E$ where the **conductivity** is

$$\sigma = \frac{n_e e^2 \tau}{m}$$

The **resistivity** is $\rho = 1/\sigma$.

Applications

Resistors

A potential difference ΔV_{wire} between the ends of a wire creates an electric field inside the wire

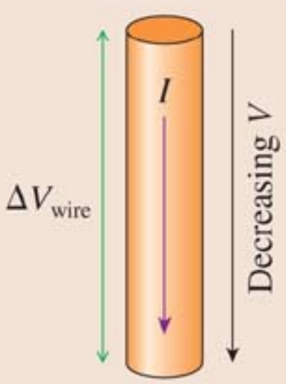
$$E_{wire} = \frac{\Delta V_{wire}}{L}$$

The electric field causes a current

$$I = \frac{\Delta V_{wire}}{R}$$

where $R = \frac{\rho L}{A}$ is the wire's **resistance.**

This is **Ohm's law.**



Chapter 32 (Fundamentals of Circuits)

GENERAL STRATEGY

MODEL Assume that wires and, where appropriate, batteries are ideal.

VISUALIZE Draw a circuit diagram. Label known and unknown quantities.

SOLVE The solution is based on Kirchhoff's laws.

- Reduce the circuit to the smallest possible number of equivalent resistors.
- Find the current and the potential difference.
- "Rebuild" the circuit to find I and ΔV for each resistor.

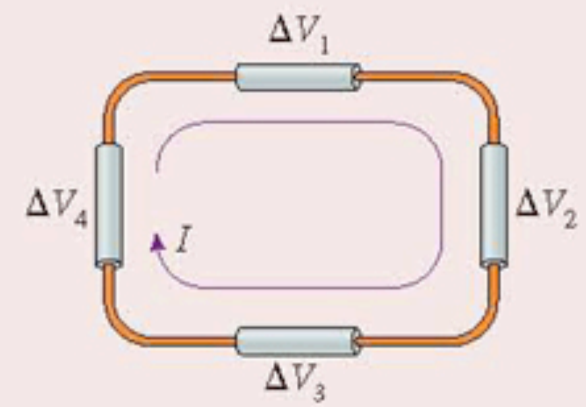
ASSESS Verify that

- The sum of potential differences across series resistors matches ΔV for the equivalent resistor.
- The sum of the currents through parallel resistors matches I for the equivalent resistor.

Kirchhoff's loop law

For a closed loop:

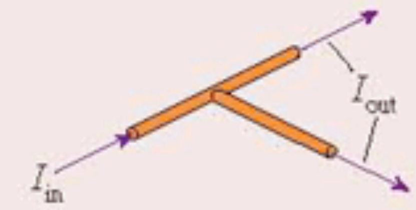
- Assign a direction to the current I.
- $\sum_i (\Delta V)_i = 0$



Kirchhoff's junction law

For a junction:

- $\sum I_{in} = \sum I_{out}$



Chapter 32 (continued)

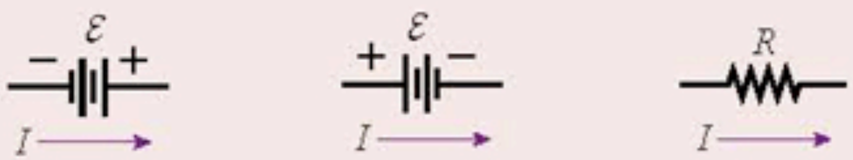
IMPORTANT CONCEPTS

Ohm's law

A potential difference ΔV between the ends of a conductor with resistance R creates a current

$$I = \frac{\Delta V}{R}$$

Signs of ΔV



$\Delta V_{\text{bat}} = +\mathcal{E}$ $\quad$ $\Delta V_{\text{bat}} = -\mathcal{E}$ $\quad$ $\Delta V_{\text{R}} = -IR$

The energy used by a circuit is supplied by the emf $\mathcal{E}$ of the battery through the energy transformations

$$E_{\text{chem}} \rightarrow U \rightarrow K \rightarrow E_{\text{th}}$$

The battery *supplies* energy at the rate

$$P_{\text{bat}} = I\mathcal{E}$$

The resistors *dissipate* energy at the rate

$$P_{\text{R}} = I\Delta V_{\text{R}} = I^2 R = \frac{(\Delta V_{\text{R}})^2}{R}$$

APPLICATIONS

Series resistors

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \ \ldots$$

Parallel resistors

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ \ldots\right)^{-1}$$