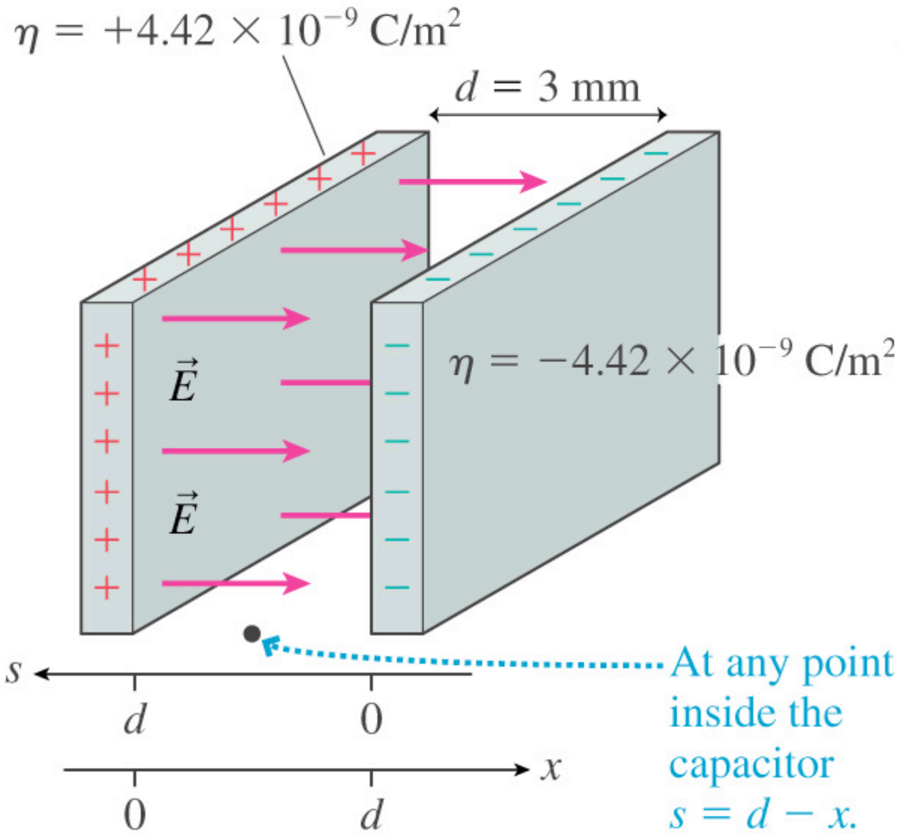
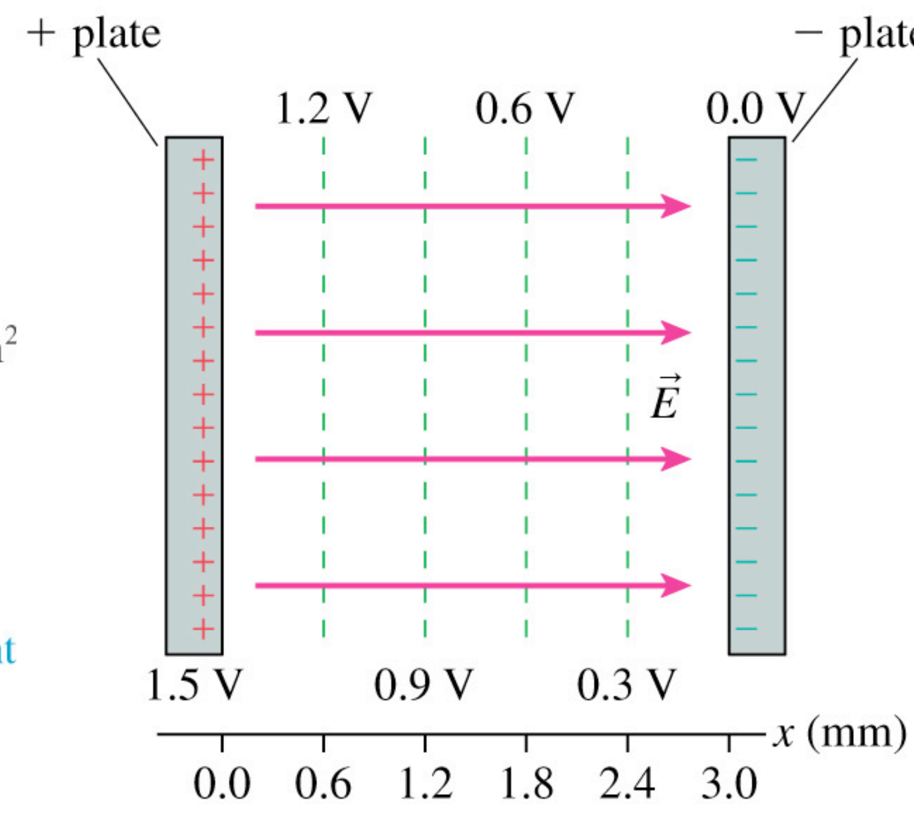
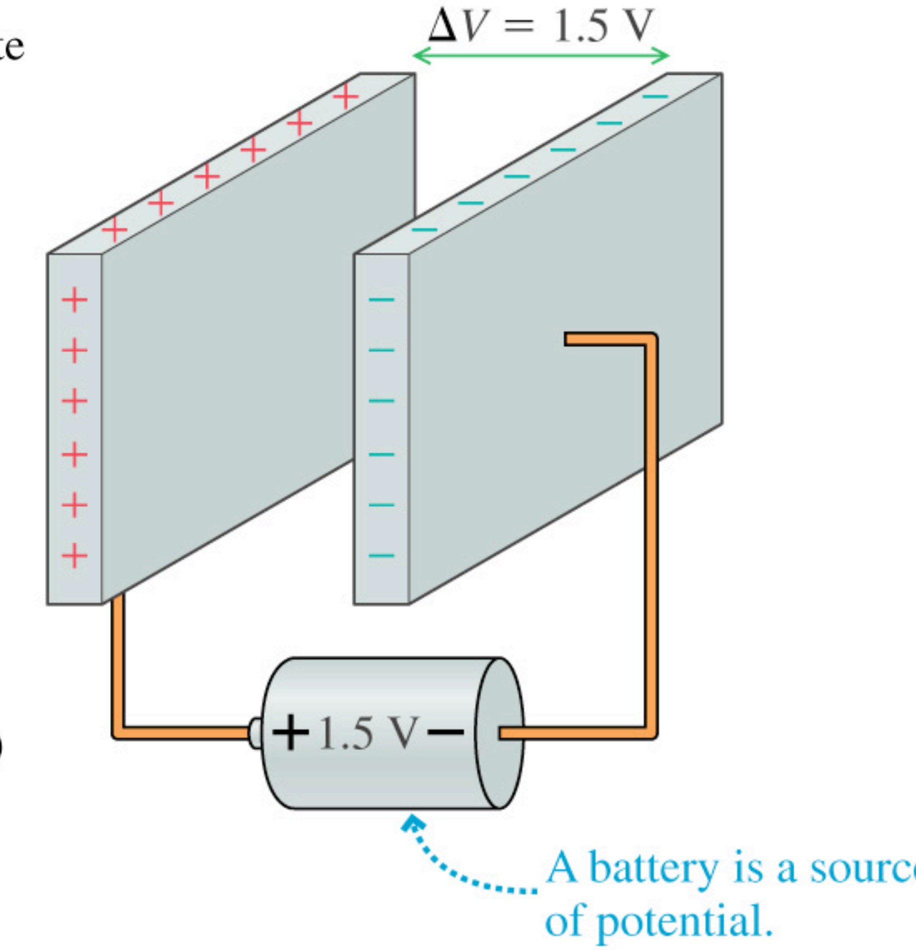
Lecture 22

- Electric potential: inside capacitor; of point charge; of many charges
- chapter 30 (Potential and Field)

Electric Potential inside a Parallel Plate Capacitor







- $E = \frac{\eta}{\epsilon_0}$ due to source charges on plates

$U_{elec} = U_{q+sources} = qEs \Rightarrow V = Es$ (electric potential inside a parallel-plate capacitor)

- potential difference: $\Delta V_c = V_+ - V_- = Ed$; $E = \frac{\Delta V_c}{d}$ ($1\ \mathrm{N/C} = 1\ \mathrm{V/m}$)

  $V = \frac{\Delta V_C}{d}(d - x)$ (decreases from positive to negative plate)

- electric field vectors $\perp$ to (imaginary) equipotential surfaces/ contour lines; potential decreases along direction of E

- choice of zero of potential (V_+ or $V_- = 0$ or...): no physical difference

Electric Potential of Point Charge; Charged Sphere

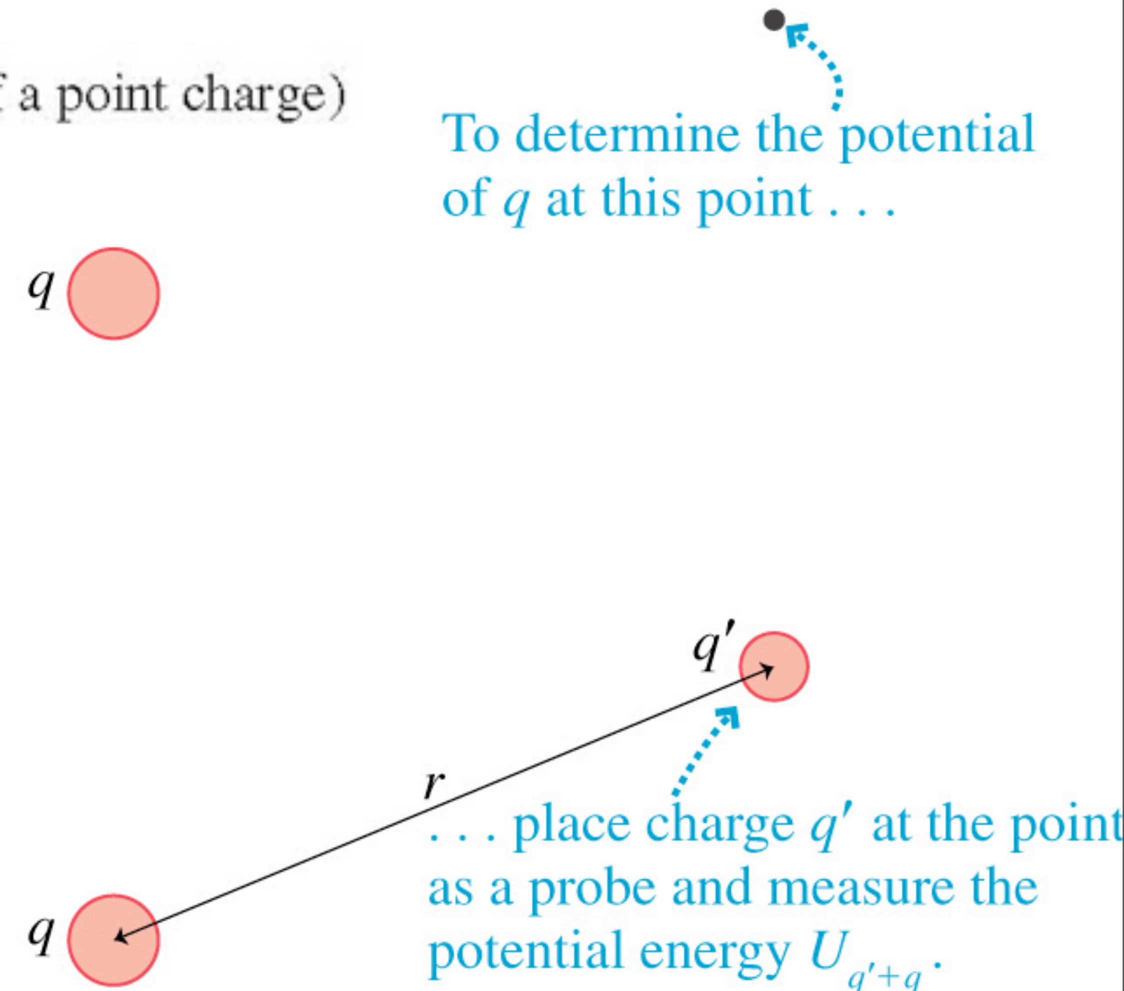
$U_{q+q'} = \frac{qq'}{4\pi\epsilon_0 r} \Rightarrow \quad V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ (electric potential of a point charge)

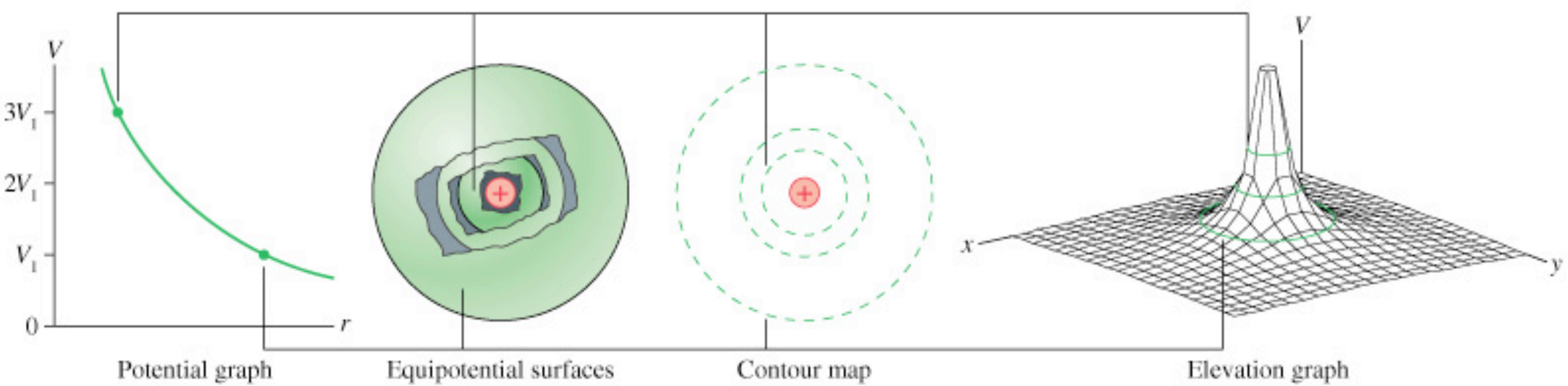
(scalar vs. vector $E \propto 1/r^2$)

- 4 graphical representations
- Outside sphere (same as point charge at center): $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r};\ r \geq R$
- In terms of potential at surface,

$$V_0 = V(\text{at } r = R) = \frac{Q}{4\pi\epsilon_0 R}:$$

$$Q = 4\pi\epsilon_0 R V_0 \text{ and } V = \frac{R}{r}V_0$$





Electric Potential of Many Charges

- Principle of superposition (like for E): $V = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i}$
- Continuous distribution of charge (like for E, easier due to scalar)

model as simple shape, uniform charge distribution

draw picture...identify P where V to be calculated...

divide Q into ΔQ (shapes for which V known)

$\mathrm{V} = \sum_i V_i$

$\Delta Q \rightarrow$ charge density $\times dx$

Sum $\rightarrow$ integral

Potential of Ring of Charge

r_i same for all charge segments

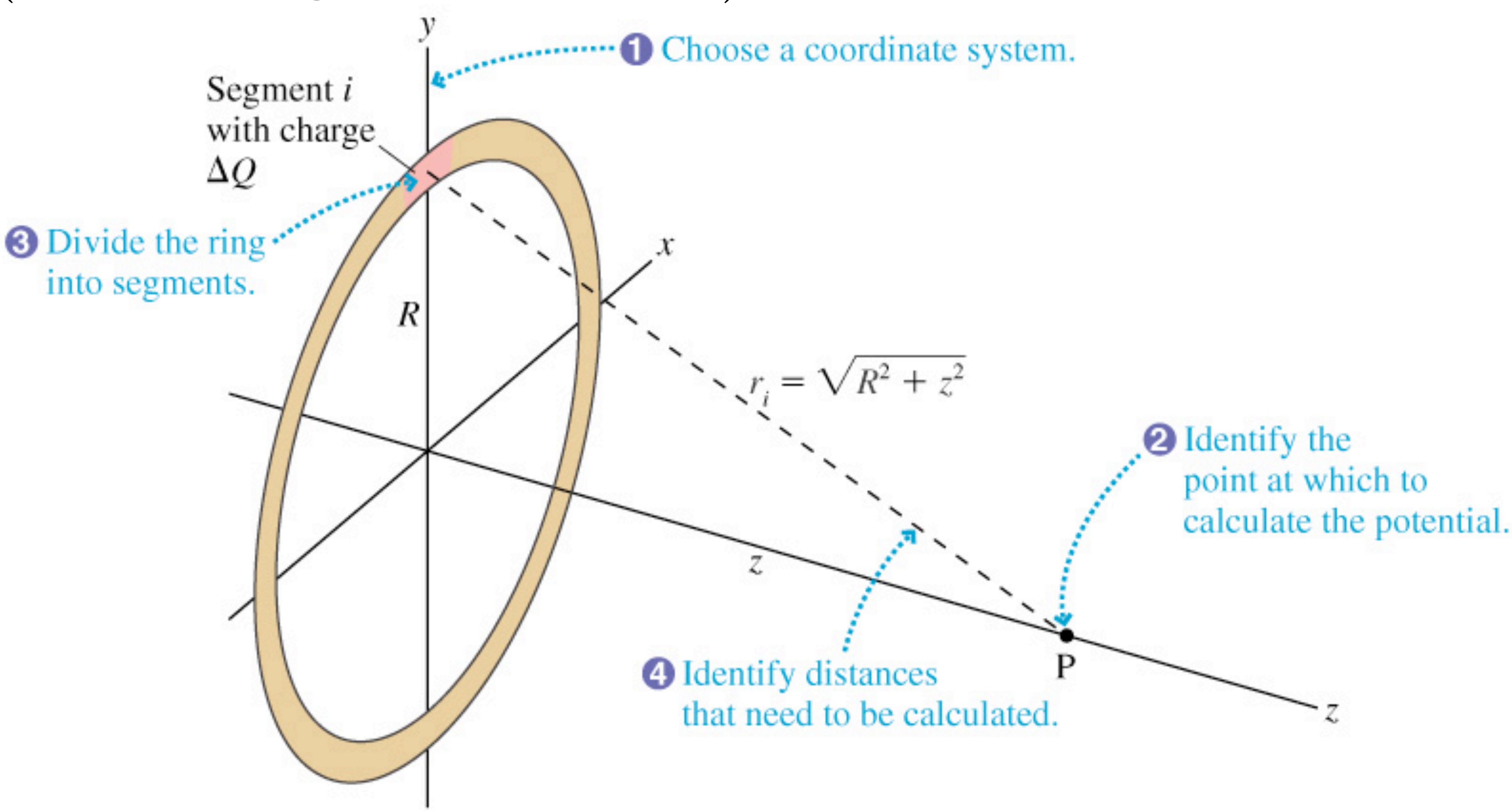
$$V = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{r_i} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{R^2+z^2}}\sum_{i=1}^{N} \Delta Q$$

No need for integral: $\sum_{i=1}^{N} \Delta Q = Q \Rightarrow$

$$V_{\text{ring on axis}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+z^2}}$$

Check: $z \gg R \Rightarrow V_{ring} \approx Q/\left(4\pi\epsilon_0 z\right)$

(point charge, as expected)



Potential of Disk of Charge

uniform surface charge density $\eta = Q/A = Q/\left(\pi R^2\right)$

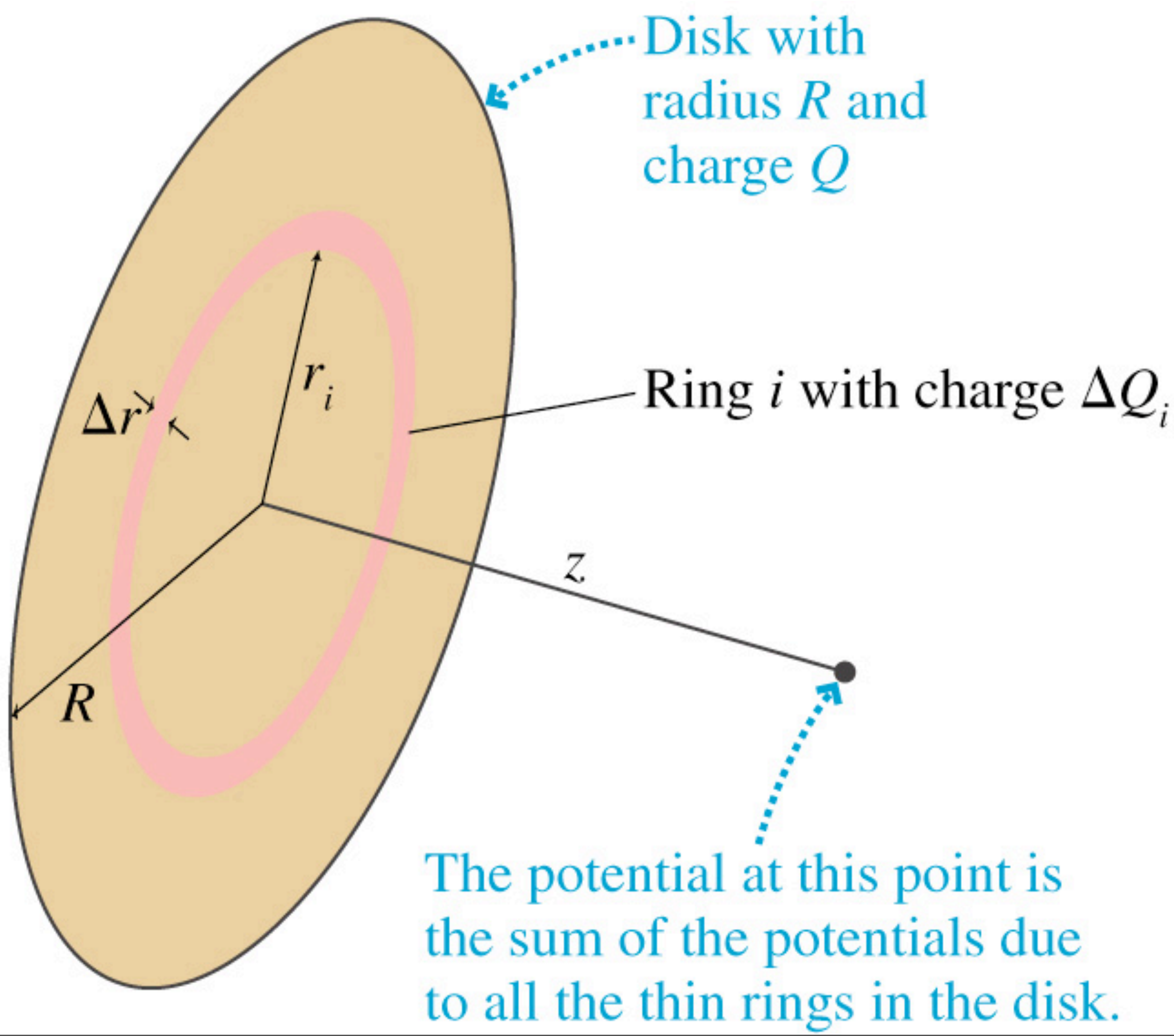
Use potential of ring $V_i = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q_i}{\sqrt{r_i^2+z^2}}$

$$V = \sum_{i=1} V_i = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{\Delta Q}{\sqrt{r_i^2+z^2}}$$

$\Delta Q_i = \eta\Delta A_i$; $\Delta A_i = 2\pi r_i \Delta r$

$$V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{2Q}{R^2}\frac{r_i\Delta r_i}{\sqrt{r_i^2+z^2}} \rightarrow \frac{Q}{2\pi\epsilon_0 R^2}\int_0^R \frac{rdr}{\sqrt{r^2+z^2}}$$

Charge of variables: $u = r^2 + z^2$... $V_{\text{disk on axis}} = \frac{Q}{2\pi\epsilon_0 R^2}\left(\sqrt{R^2+z^2} - z\right)$



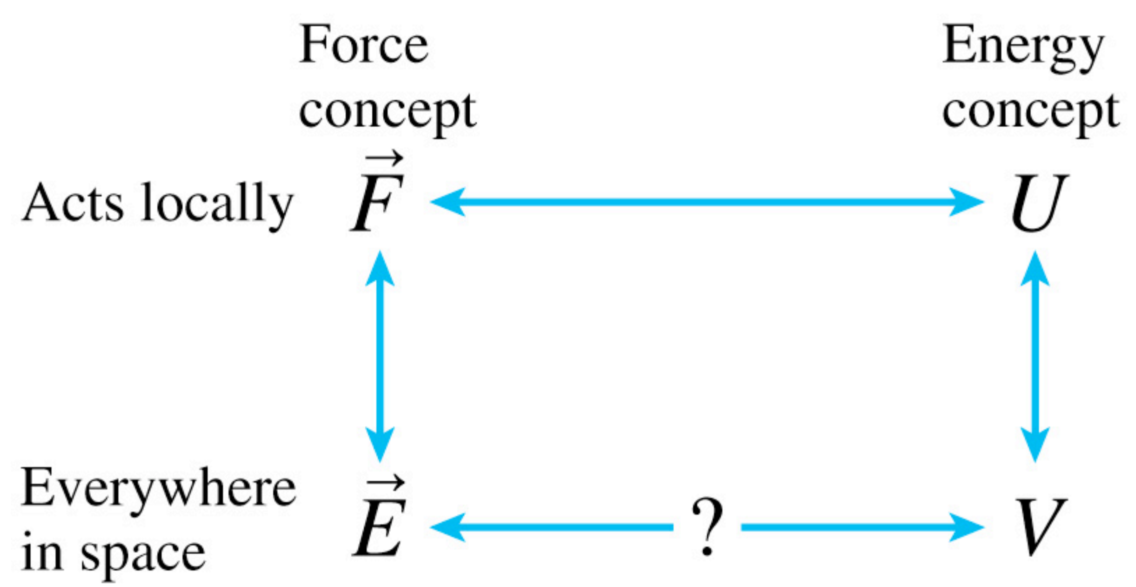
Chapter 30 (Potential and Field)

- calculate V from E and vice versa (E and V not independent, two different mathematical representations of how source charges alter space around them); geometry of E and V

- sources of potential (batteries); capacitors; currents in wires (chapter 31) and Electric Circuits (chapter 32)

- **Finding Potential from Electric Field**

Using (i) $V = U_{q+sources}/q$;
(ii) $\Delta U = -W(i \rightarrow f) = -\int_{s_i}^{s_f} F_s ds$ and
(iii) $\bar{F} = q\bar{E} \Rightarrow \Delta V = V(s_f) - V(s_i) = -\int_{s_i}^{s_f} E_s ds$



- Uniform E: $\Delta V = -E_s \Delta s$

- Choose zero point of potential to assign V (often at ∞)

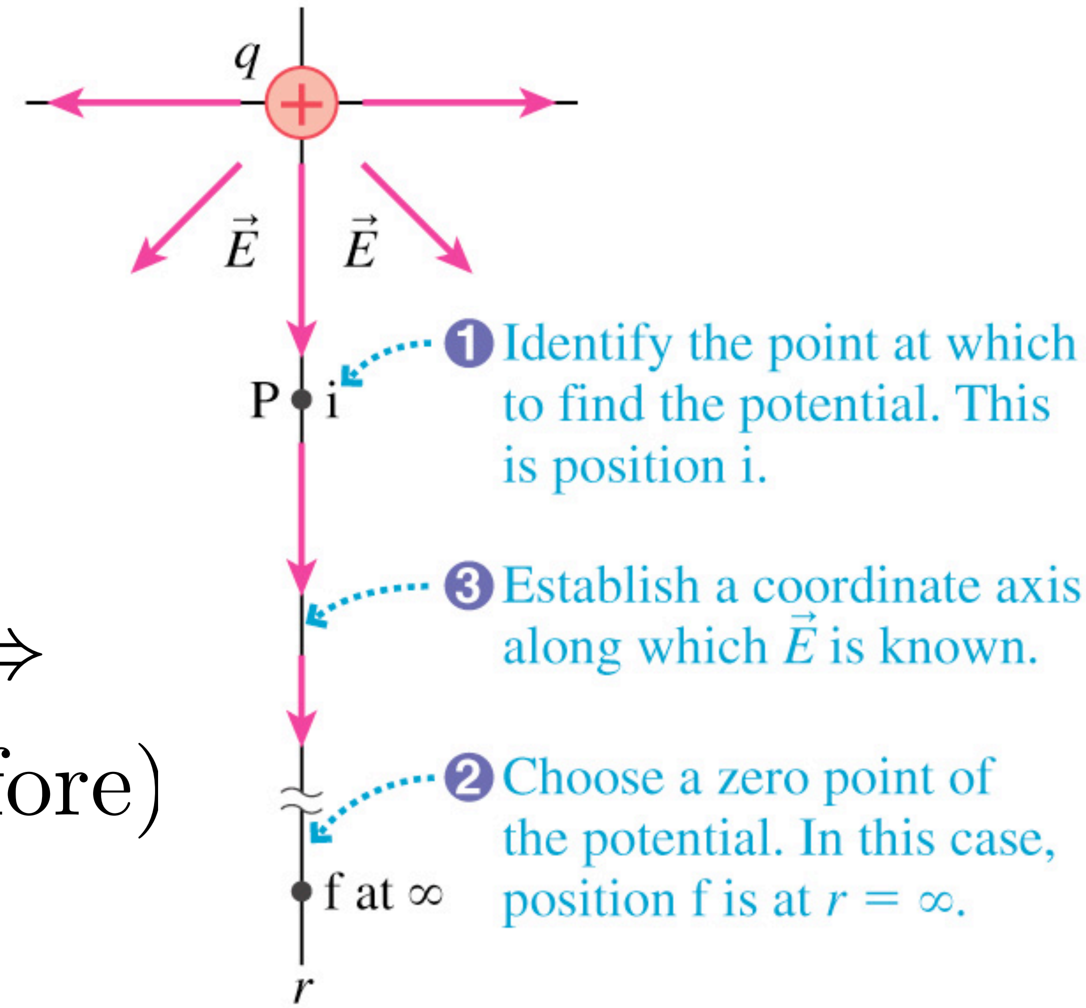
Finding Potential from Electric Field

- Point Charge

$$\Delta V = V(\infty) - V(r) = -\int_r^\infty E_r dr$$

with $E_r = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ and $V(\infty) = 0 \Rightarrow$

$$V(r) = \frac{q}{4\pi\epsilon_0}\int_r^\infty \frac{dr}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}\ \text{(as before)}$$



- Disk of Charge

$$E = \frac{Q}{2\pi R^2\epsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right]$$

with $V(\infty) = 0 \Rightarrow$

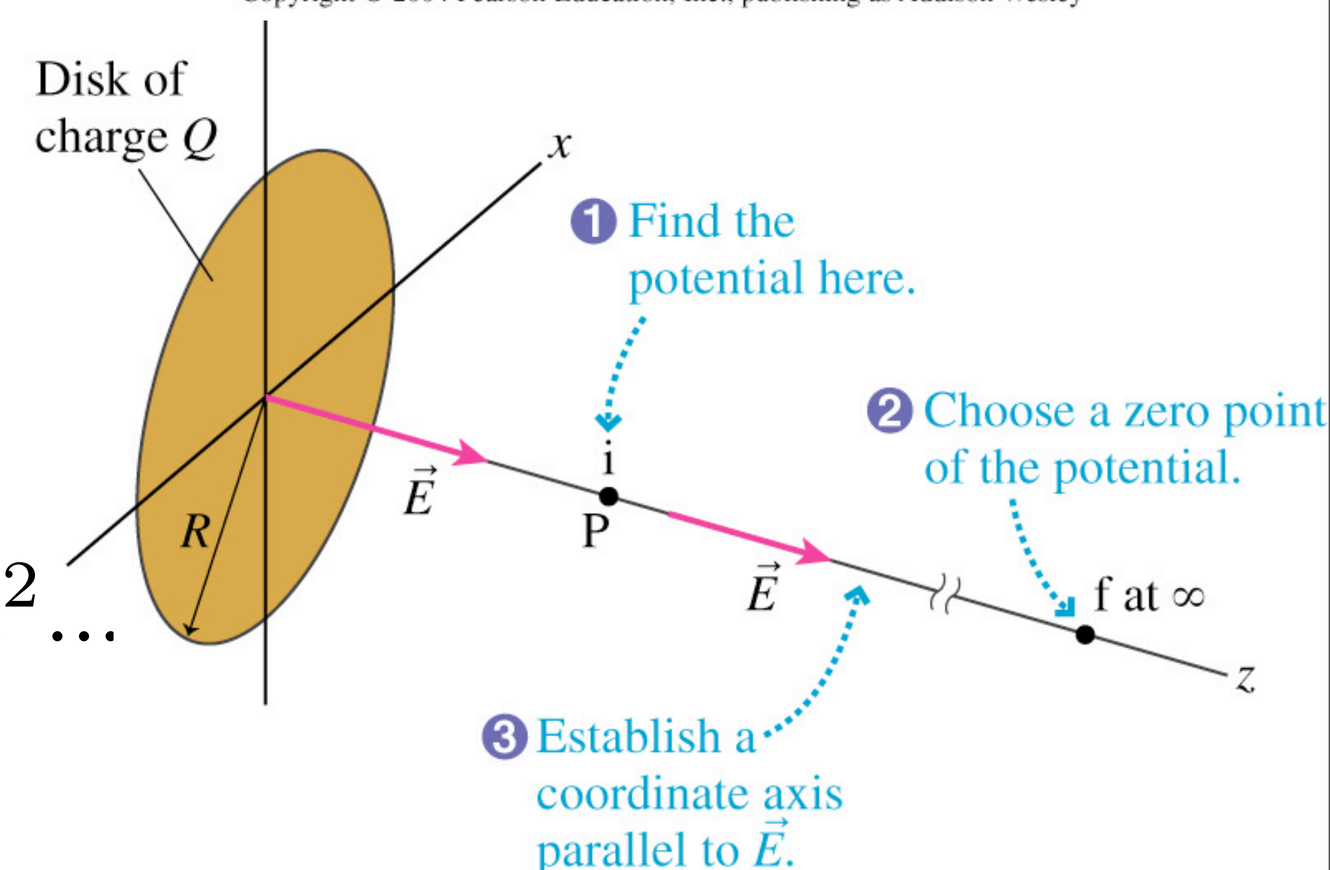
$$V(z) = \int_z^\infty E_z(z)dz$$

$$= \frac{Q}{2\pi R^2\epsilon_0}\int_z^\infty \left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right]dz$$

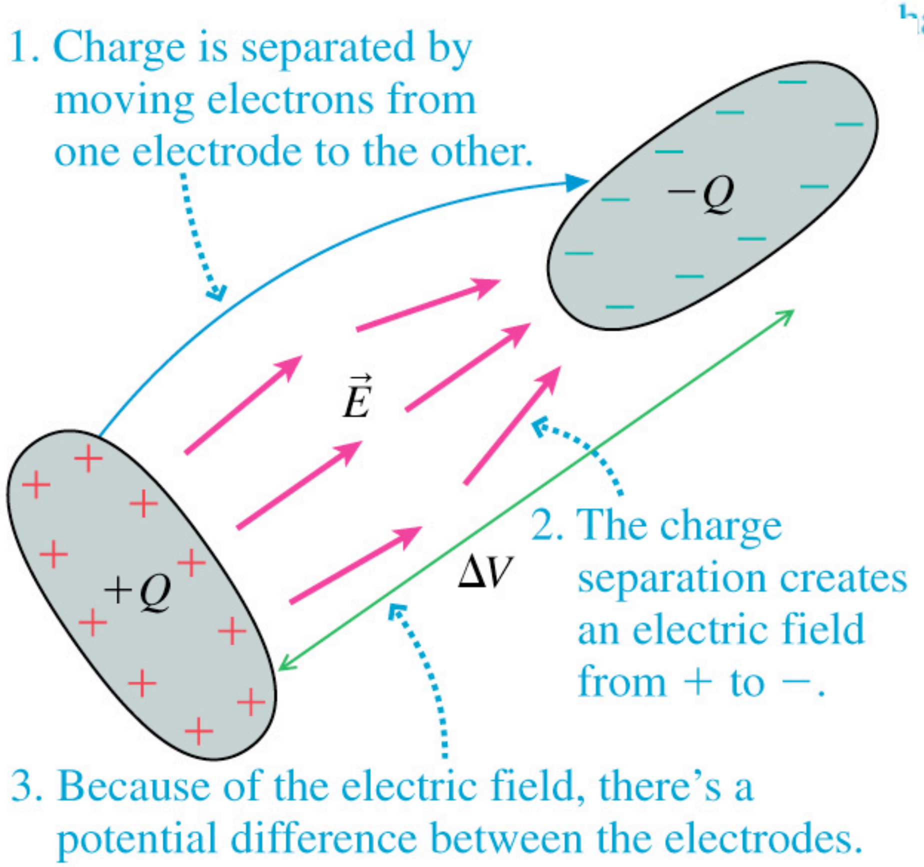
Charge of variables; $u = z^2 + R^2 \ldots$

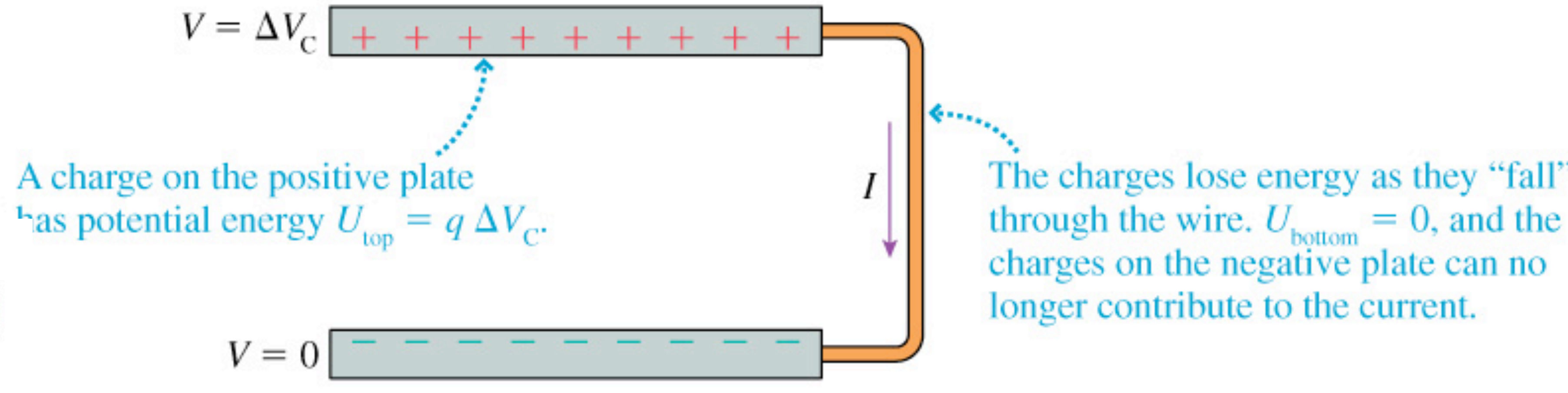
$$V_{disk} = \frac{Q}{2\pi R^2\epsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

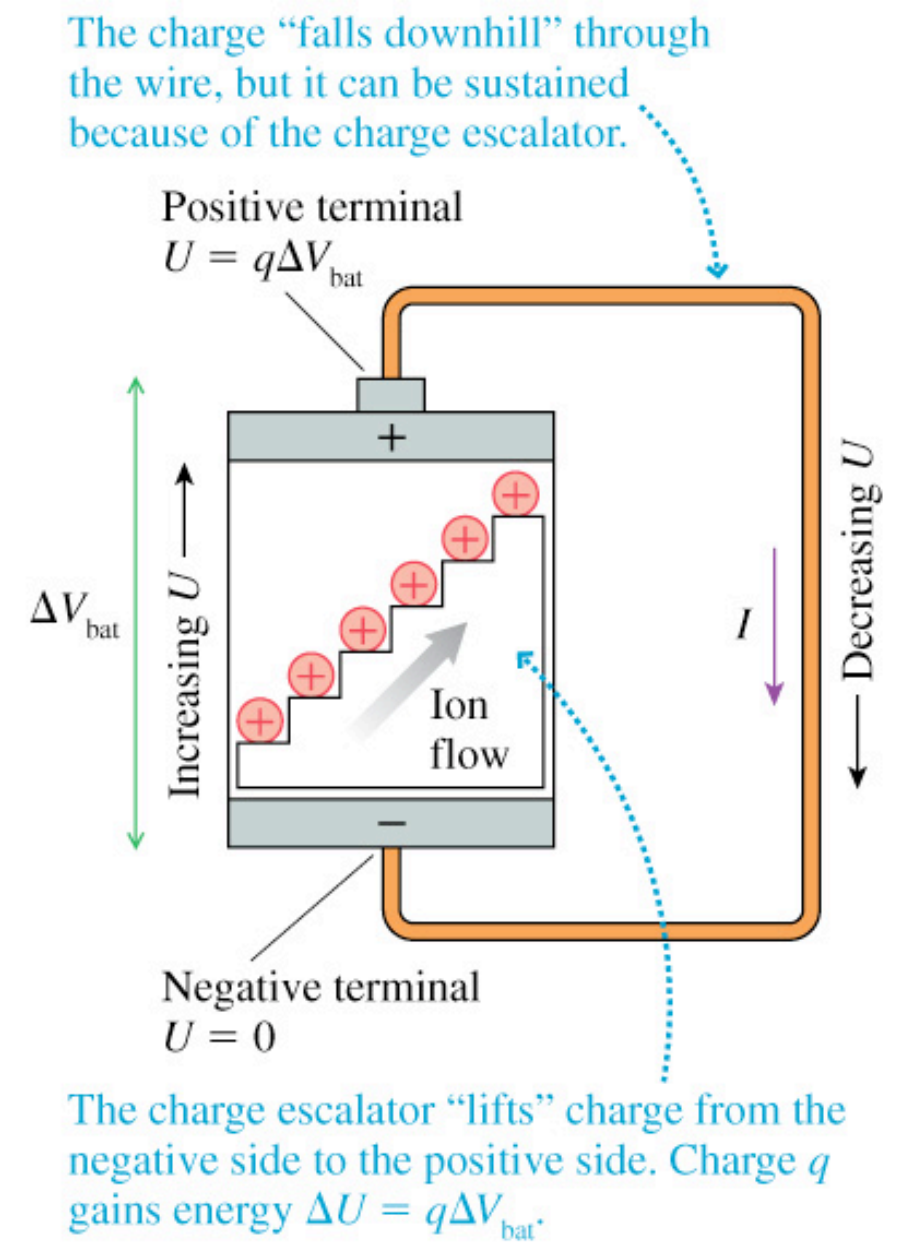
(as before)



Sources of Potential







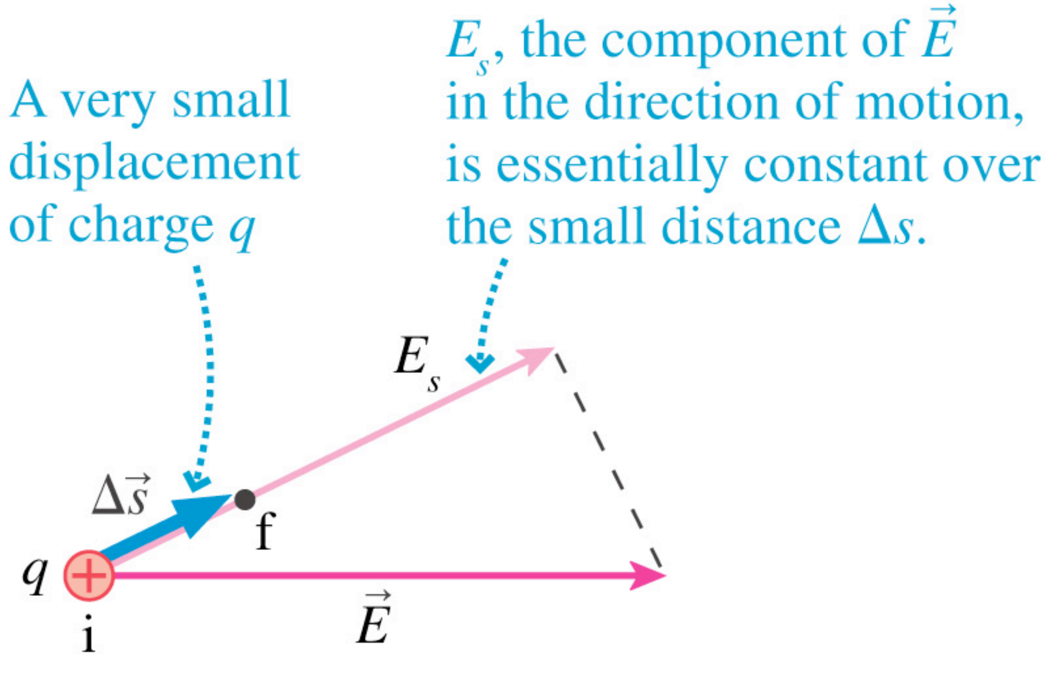
- Need to "lift" charge to sustain ΔV and current

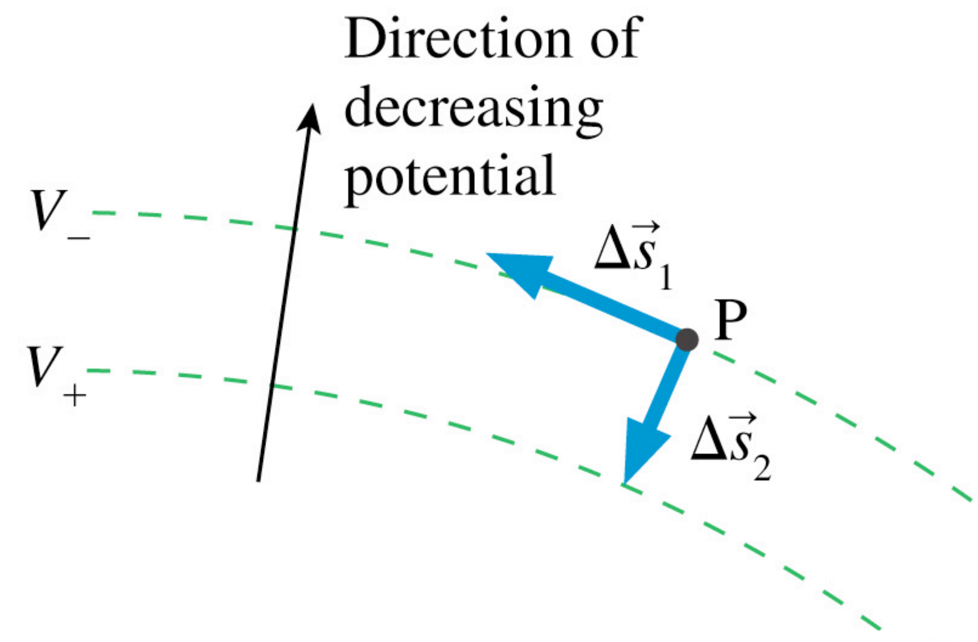
- van de Graff generator (mechanical)
- Batteries (electrodes with chemicals in-between): work done by chemical reactions per unit charge (emf) $\mathcal{E} = \frac{W_{chem}}{q}$

ideal battery: $\Delta U = W_{chem} \Rightarrow \Delta V_{bat} = \mathcal{E}$

$$\Delta V_{series} = \Delta V_1 + \Delta V_2 + ...$$

Finding E from V





- Work done by E $= F_s\Delta s = qE_s\Delta s \Rightarrow \ldots \Delta V = -E_s\Delta s \ldots E_s = -\frac{dV}{ds}$

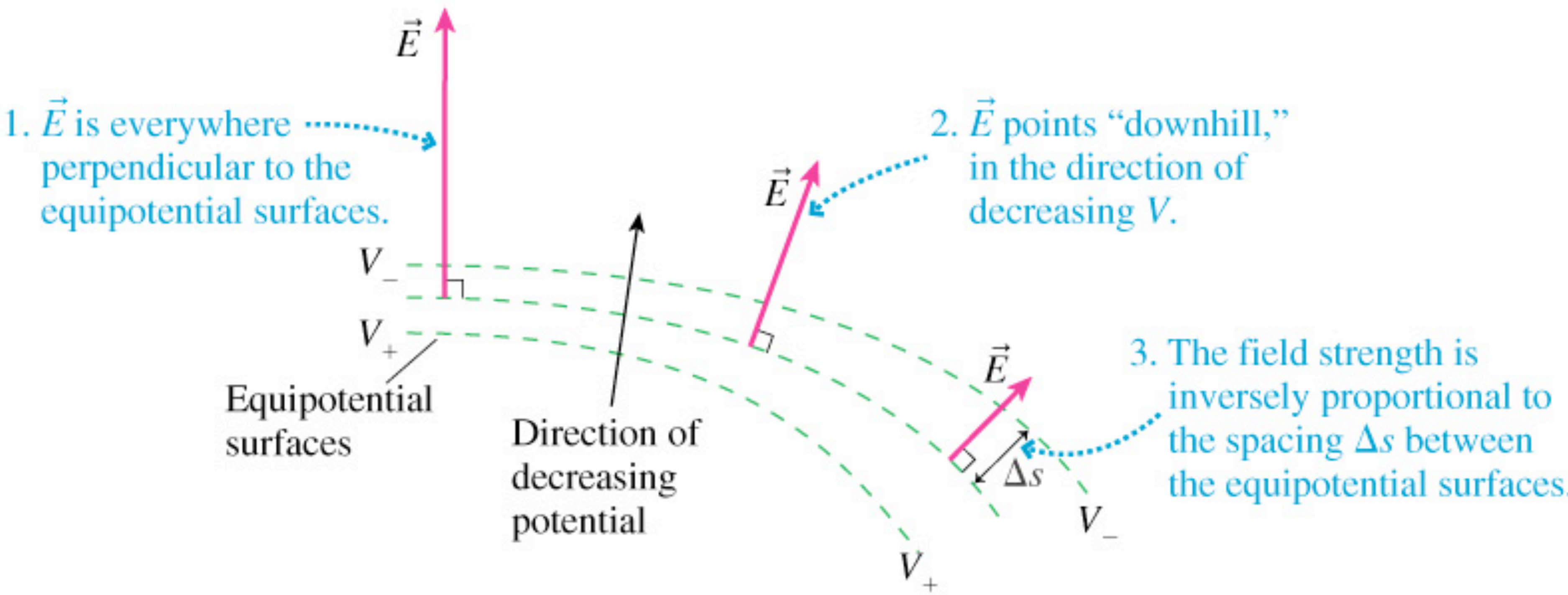
- Use symmetry to select coordinate axis parallel to E : e.g. point charge $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \Rightarrow E = E_r = -\frac{dV}{dr} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$
- Useful for continuous distribution: easier to calculate V (scalar)

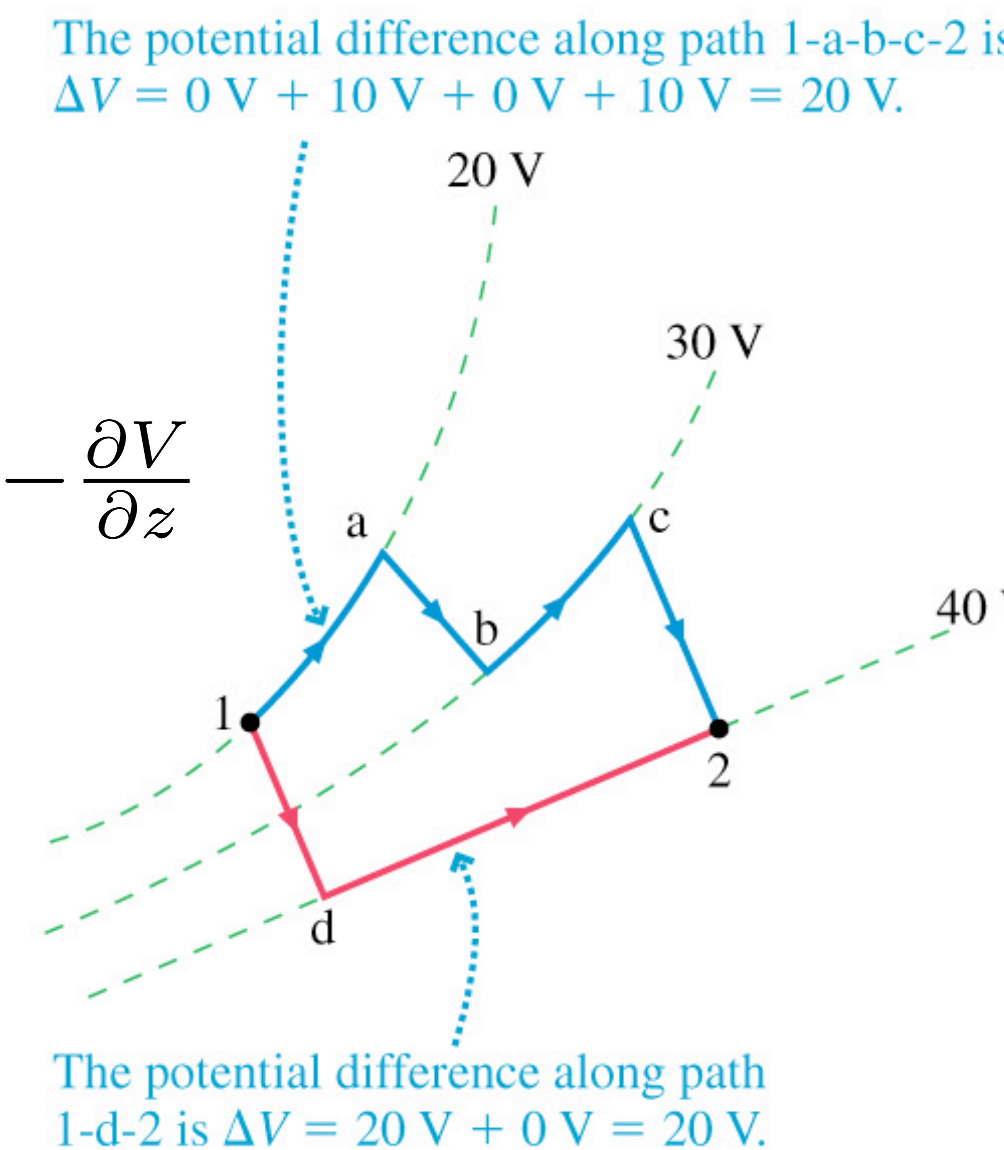
$$V_{ring,\ on\ axis} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{z^2+R^2}} \Rightarrow E = E_z = -\frac{dV}{dz} = \frac{Q}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}}$$

- $\bar{E}$ is slope of V-vs.-s graph: like $F = -\frac{dU}{ds}$ (divide by q to obtain V, E)
- Geometry of E and V: $\bar{E}$ parallel to equipotential surface = 0

$$E_\perp \approx -\frac{\Delta V}{\Delta s} = -\frac{V_+ - V_-}{\Delta s}$$

Finding E from V





- In 3 D: $E_x = -\frac{\partial V}{\partial x};\ E_y = -\frac{\partial V}{\partial y};\ E_z = -\frac{\partial V}{\partial z}$
- Kirchhoff's loop law: ΔV independent of path → sum of ΔV`s around a loop is zero (conservation of energy, $\Delta U = q\Delta V = 0$)

$$\Delta V_{\text{loop}} = \sum_i (\Delta V)_i = 0$$