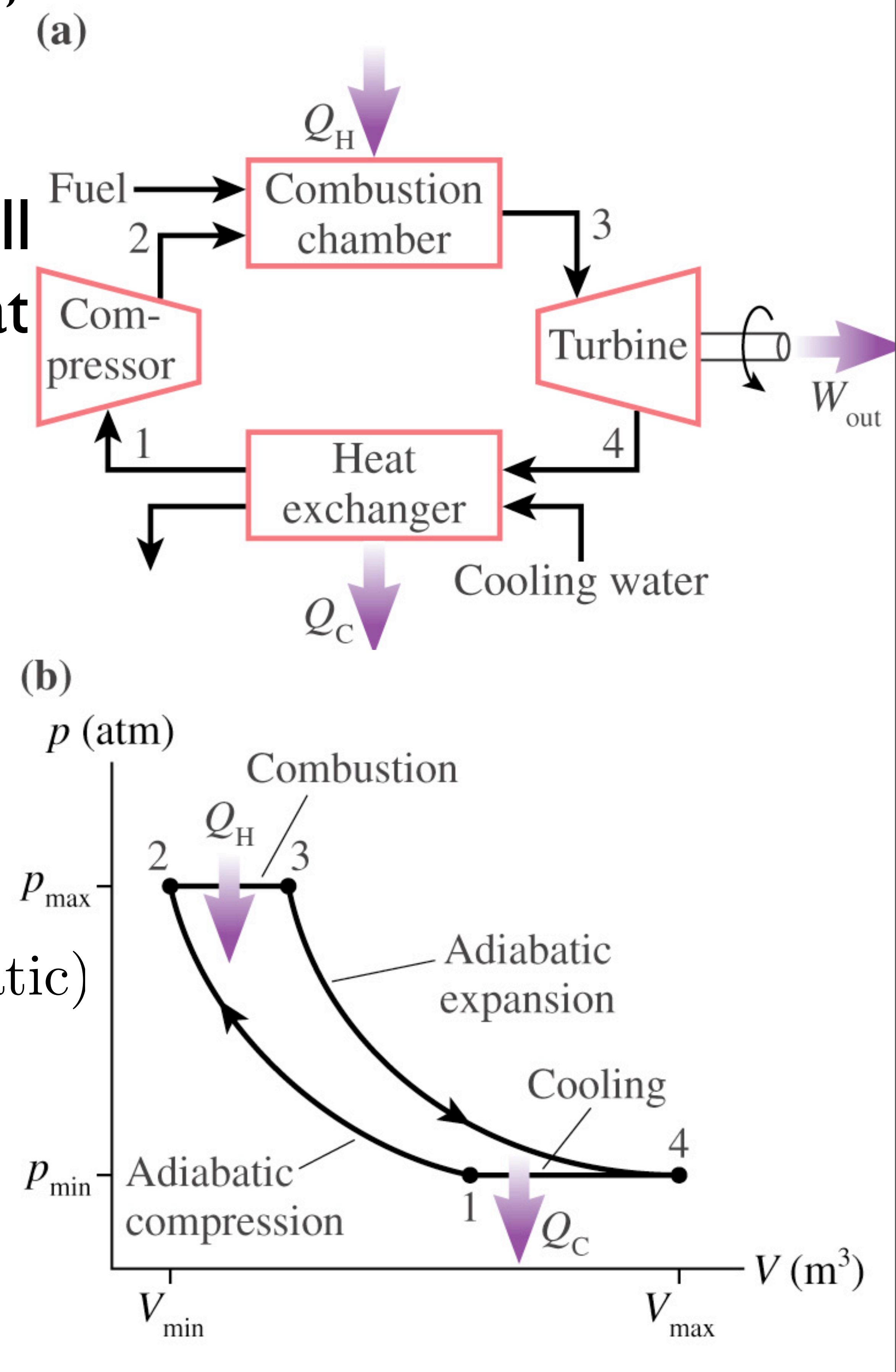
Lecture 16

- Brayton cycle
- Maximum efficiency for a perfectly reversible engine
- conditions for perfectly reversible engine
- efficiency for Carnot cycle

Brayton cycle (heat engine)

- adiabatic compression (1→2): raises T; isobaric expansion (2→3): raises T further, heat by fuel; adiabatic expansion (3 →4): spins turbine, T still high; isobaric compression (4→1): heat transferred to cooling fluid
  $T_H \geq T_3$; $T_C \leq T_1$
- Thermal efficiency: $\eta = 1 - \frac{Q_C}{Q_H}$



Process $2 \rightarrow 3$ (isobaric):

$Q_H = nC_P\,(T_3 - T_2)$

Process $4 \rightarrow 1$ (isobaric):

$Q_C = |Q_{41}| = nC_P\,(T_4 - T_1)$

$\Rightarrow \eta_B = 1 - \frac{T_4 - T_1}{T_3 - T_2}$

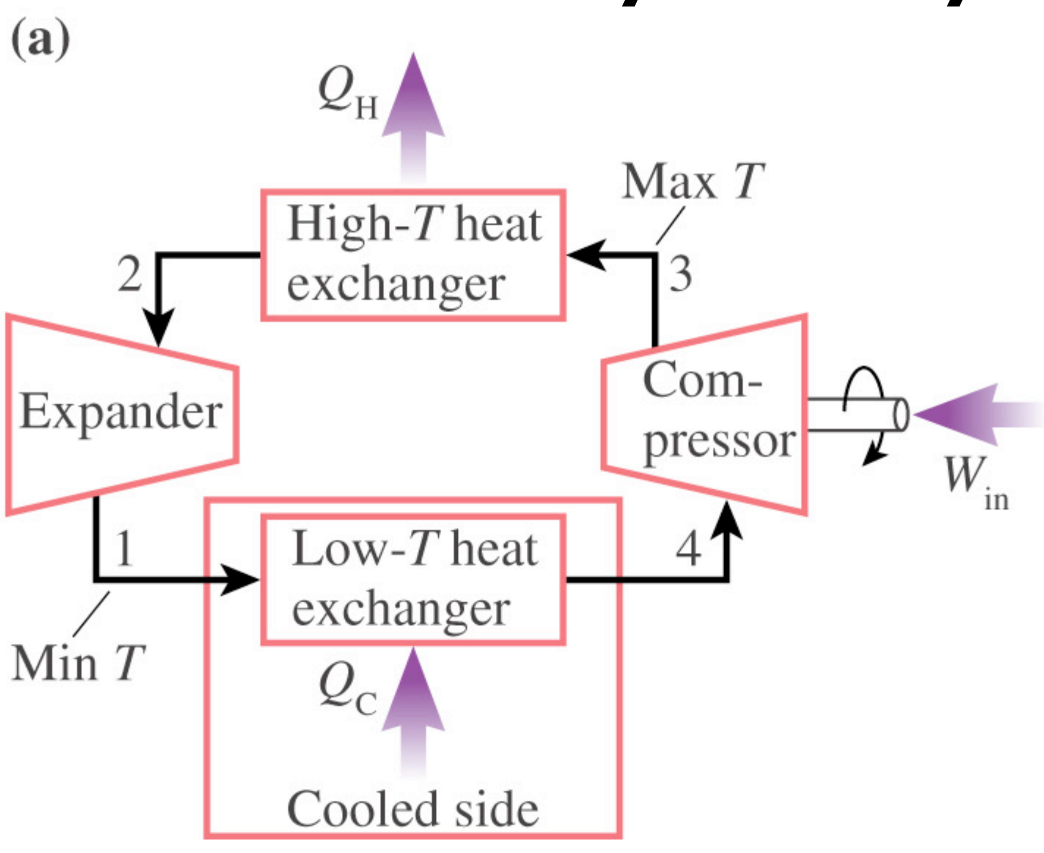
Use $pV = nRT$ and $pV^\gamma =$ constant (adiabatic) to give $p^{(1-\gamma)/\gamma}T =$ constant

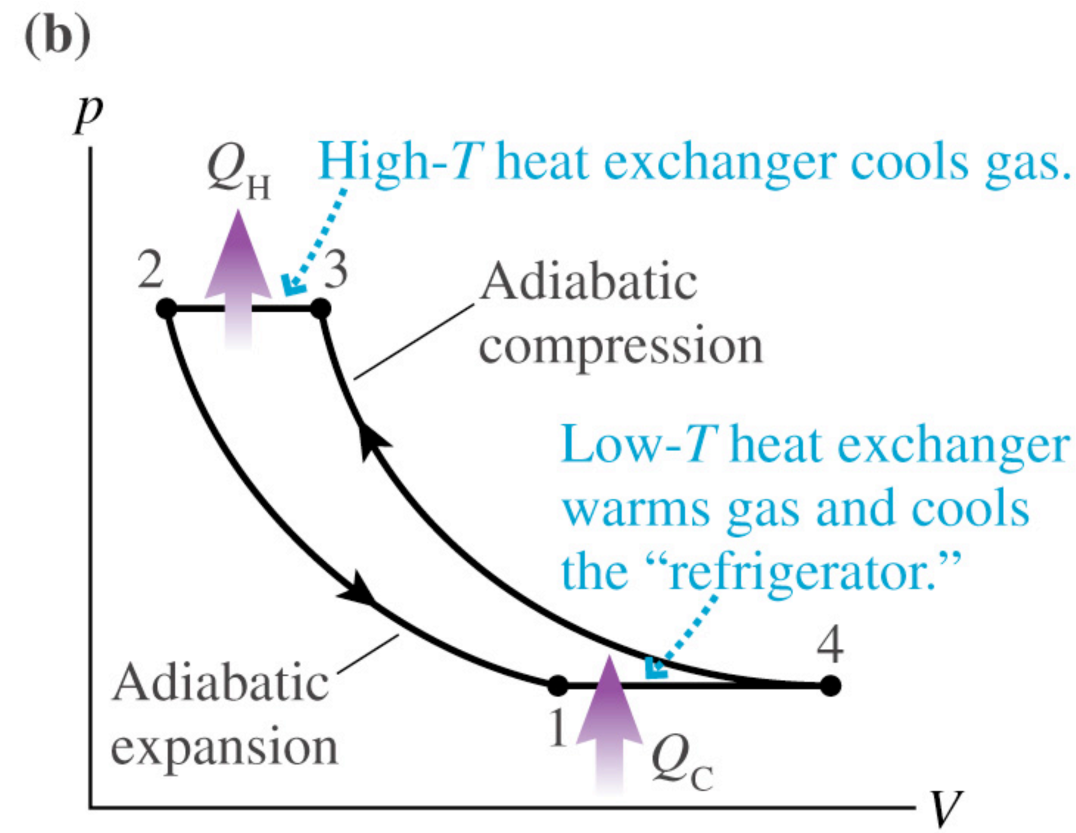
$$\Rightarrow T_1 = T_2 \left(\frac{p_2}{p_1}\right)^{(1-\gamma)/\gamma} = T_2 r_p^{(1-\gamma)/\gamma},$$

where $r_p \equiv \frac{p_{max}}{p_{min}}$ and $T_4 = T_3 r_p^{(1-\gamma)/\gamma}$

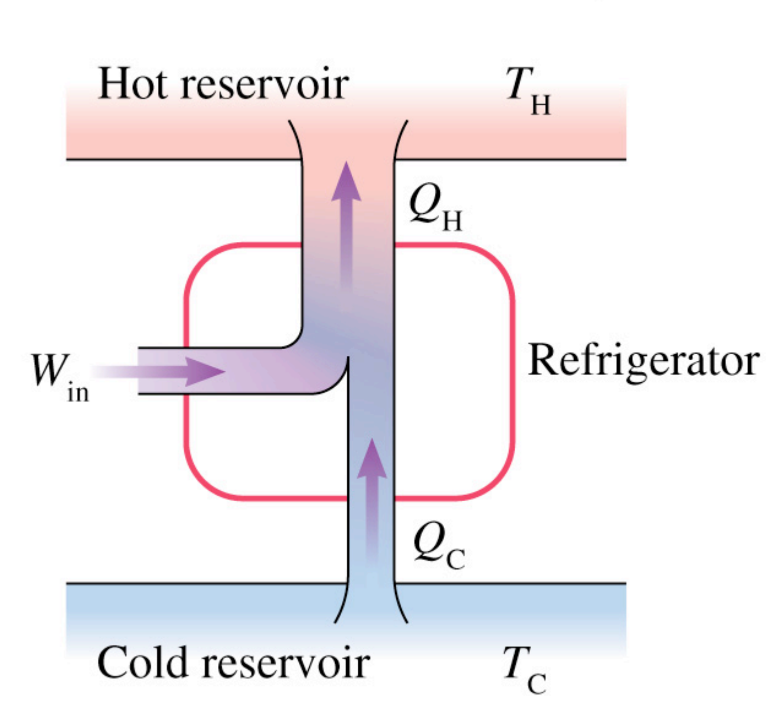
$\Rightarrow \eta_B = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}$ (increases with r_p)

Brayton cycle (refrigerator)



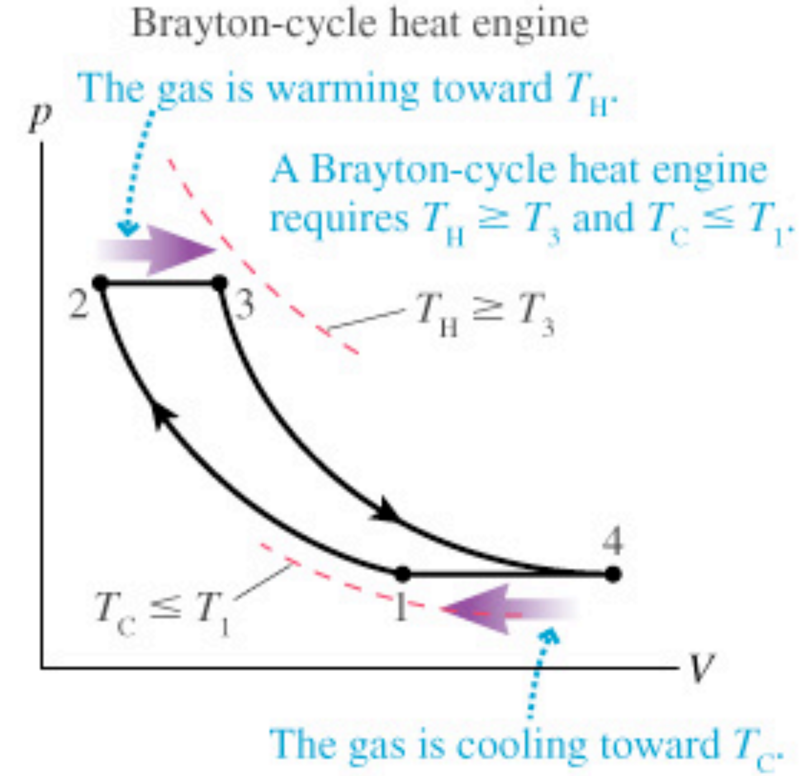


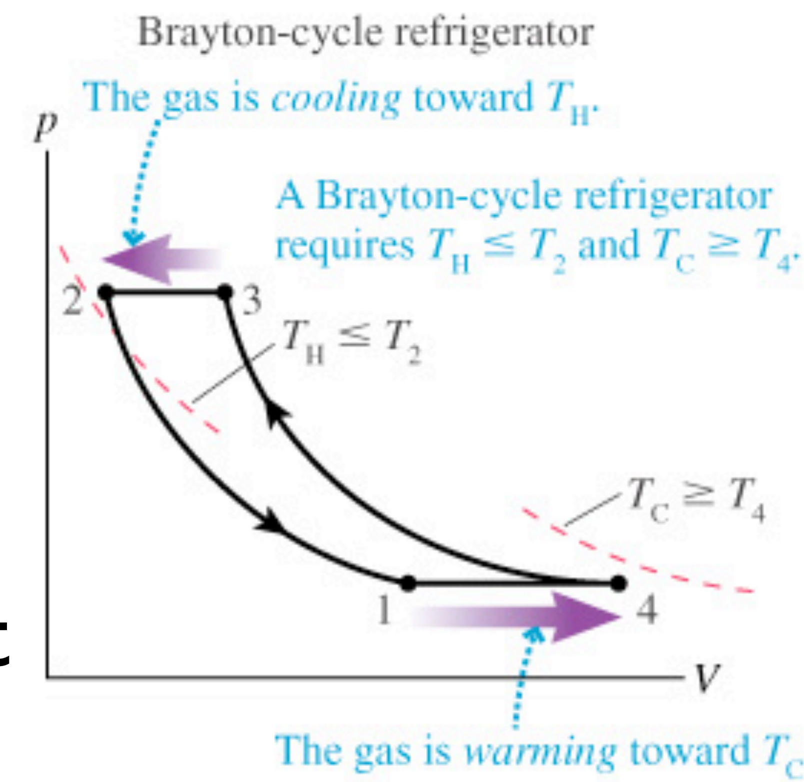
- heat engine backward, ccw in pV: low-T heat exchanger is "refrigerator"
- sign of W reversed, area inside curve is W_{in} used to extract Q_C from cold reservoir an exhaust Q_H to hot...
- gas T lower than T_C (1$\longrightarrow$ 4), higher than T_H (3$\rightarrow$2) $\Rightarrow$ gas must reach $T_1 (< T_C)$ by adiabatic expansion, $T_3 (> T_H)$ by adiabatic compression



Comparison of Brayton cycle heat engine and refrigerator

- Brayton cycle refrigerator is <u>not</u> simply heat engine run backward, must <u>change hot and cold reservoir</u>: heat transferred <u>into cold</u> reservoir for <u>heat engine</u> $(T_C \leq T_1)$, from cold reservoir in refrigerator $(T_C \geq T_4)$; heat transferred <u>from hot</u> reservoir for <u>heat engine</u> $(T_H \geq T_3)$, into hot reservoir for refrigerator $(T_H \leq T_2)$
- <u>heat engine</u>: heat transfer from hot to cold is spontaneous, extract useful work in this process via system...
- <u>refrigerator</u>: heat transfer from cold to hot not spontaneous, make it happen by doing work via system...





Reversible Engine

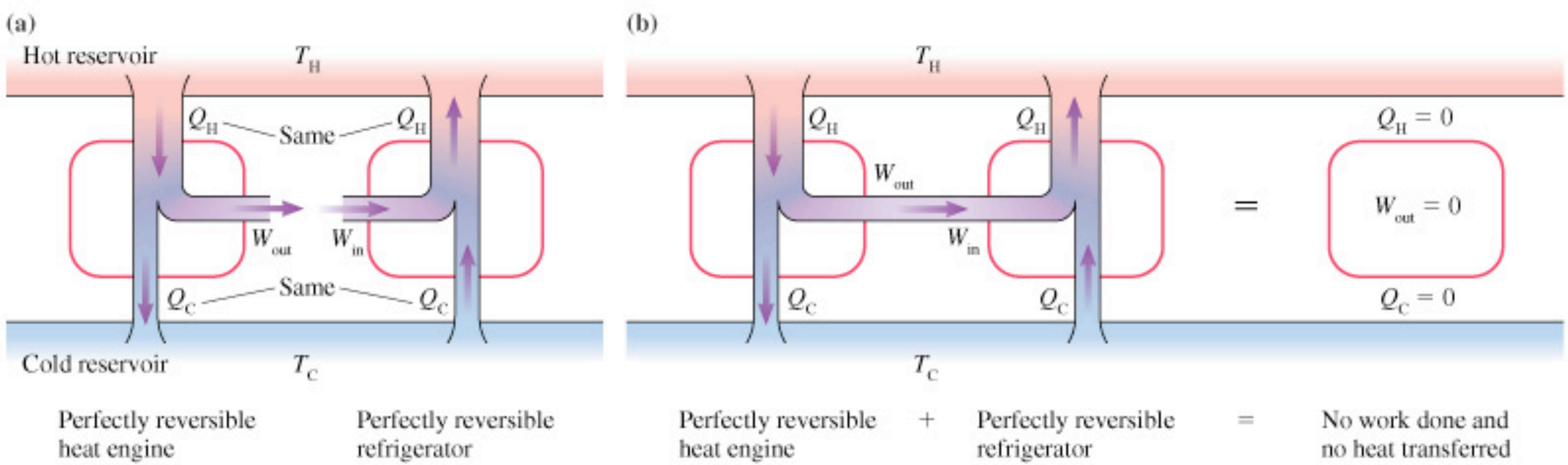
- What's most efficient heat engine/refrigerator operating between hot and cold reservoirs at temperatures T_C and T_H?

i.e., $\eta = 0.99$ allowed or is there an η_{max} (for given $T_{H,C}$) ?

- related: refrigerator is heat engine running "backwards"

- perfectly reversible engine: device can be operated between same two reservoirs, with same energy transfers (only direction reversed): cannot be Brayton-cycle engine (need to change temperatures of reservoirs)

- use heat engine to drive refrigerator: no net heat transfer

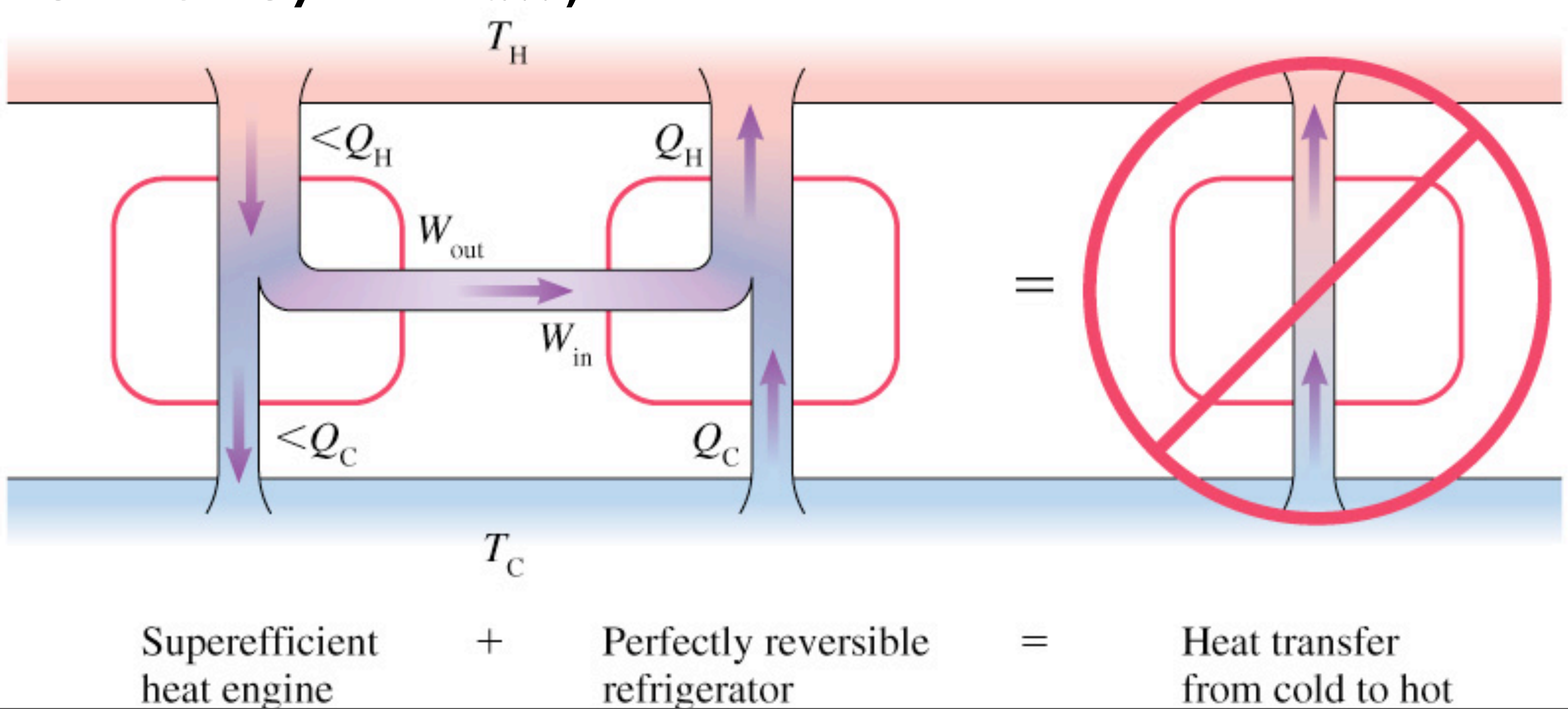


Limits of efficiency I

- Proof by Contradiction (II): suppose heat engine with more efficiency than perfectly reversible → for same W_{out}, new heat engine exhausts/needs less heat to/from cold/hot reservoir:

$$\eta = \frac{W_{out}}{Q_H} \text{ and } W_{out} = Q_H - Q_C$$

- use it to operate perfectly reversible refrigerator: engine extracts less heat from hot reservoir than refrigerator exhausts... heat transferred from cold to hot without outside assistance (forbidden by 2nd law)



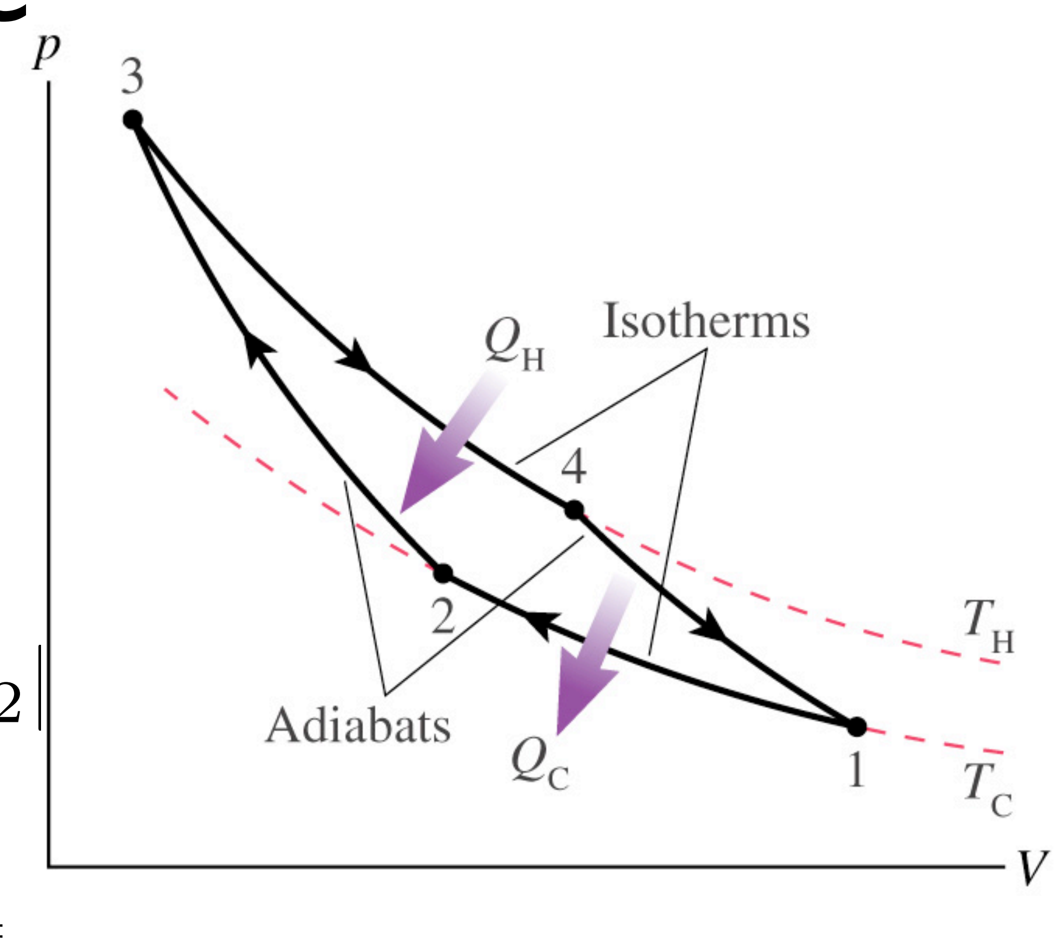
Limits of efficiency II

- 2nd law, informal statements # 5, 6: no heat engine more efficient than perfectly reversible engine operating between two reservoirs...no refrigerator has larger coefficient of performance

Conditions for reversible engine: Carnot

- so far, exists; next, design it and calculate efficiency (η_{max})
- exchange of energy in mechanical interactions (pushes on piston) reversible if (i) $W_{out} = W_{in}$ and (ii) system returns to initial T...only if motion is frictionless
- heat transfer thru' an finite temperature difference is irreversible
- reversible if heat transferred infinitely slowly (infinitesimal temperature difference) in isothermal process
- must use (i) frictionless, no heat transfer ($Q = 0$) and (ii) heat transfer in isothermal processes ($\Delta E_{th} = 0$):
Carnot engine (maximum η and K)

Carnot cycle



- enough to determine efficiency of Carnot engine using <u>ideal gas</u>
- ideal-gas cycle: 2 isothermal ($\Delta E_{th} = 0$) and 2 adiabatic processes ($Q = 0$)
- slow isothermal compression (1→2): $|Q_{12}|$ removed; adiabatic compression (2→3) till T_H; isothermal expansion (3→4): Q_{34} transferred; adiabatic expansion (4→1) to T_C
- work during 4 processes; heat transferred during 2 isothermal...
- Find 2 Q's for thermal efficiency: $\eta = 1 - \frac{Q_C}{Q_H}$

$$Q_{12} = -nRT_C \ln \frac{V_1}{V_2};\ Q_C = |Q_{12}|$$
$$Q_{34} = nRT_H \ln \frac{V_4}{V_3}$$
$$\Rightarrow \eta_{Carnot} = 1 - \frac{T_C}{T_H} \frac{\ln(V_1/V_2)}{\ln(V_4/V_3)}$$

Maximum (Carnot) efficiency

Using $TV^{\gamma-1} = \text{constant}$ for adiabatic, $\frac{V_1}{V_2} = \frac{V_4}{V_3} \Rightarrow$

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H} \qquad \text{(Carnot thermal efficiency)}$$

- Similarly, for refrigerator

$$K_{\text{Carnot}} = \frac{T_C}{T_H - T_C} \qquad \text{(Carnot coefficient of performance)}$$

- Earlier: $\eta = 1$ not allowed by 2nd law, but 0.99 is...
- Next, can't be more efficient than perfectly reversible
- Now, result for Carnot thermal efficiency
- 2nd law informal statements #7, 8: no heat engine/refrigerator can exceed $\eta_{Carnot} = 1 - \frac{T_C}{T_H}$ and $K_{Carnot} = \frac{T_C}{T_H - T_C}$
- high efficiency requires $T_H \gg T_C$, difficult in practice...
- $\eta \not> 1$ expected from energy conservation vs. limits from 2nd law

Example

- A Carnot engine operating between energy reservoirs at temperatures 300 K and 500 K produces a power output of 1000 W. What are (a) the thermal efficiency of this engine, (b) the rate of heat input, in W, and (c) the rate of heat output, in W?