Lecture 1

Outline of PHYS 260: 6 topics

- Oscillations (chapter 14)
- Fluids (15)
- Waves (20, 21) (Optics in PHYS 270)
- Thermodynamics (16-19)

(applications of Newton's laws from PHYS 161: increasing complexity)

- Electrostatics (26-30)
- Electric Currents (31-32)

(beyond Newton's laws: continue with magnetism in PHYS 270)

Not much connection between topics: survey course

Outline for today

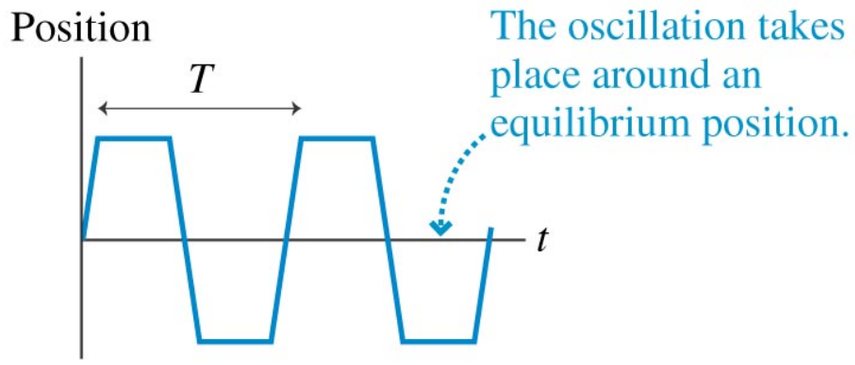
Chapter 14 (Oscillations)

- Kinematics of simple harmonic oscillations (mathematical description of motion): relation to uniform circular motion
- Dynamics: use conservation of energy and Newton's laws to relate kinematics to physical parameters (mass...)

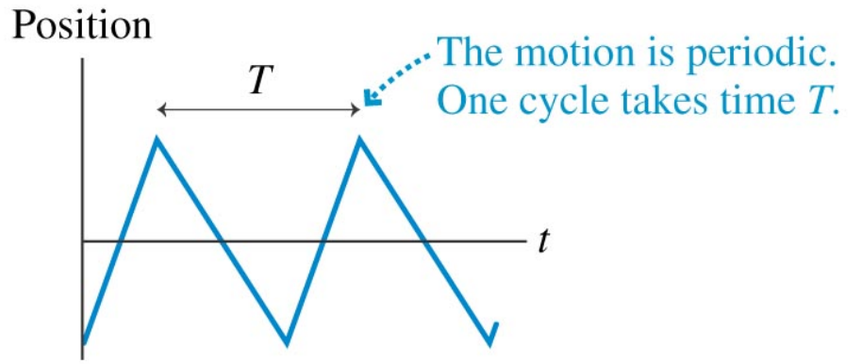
Review: uniform circular motion (4.5); restoring forces; elastic potential energy (10.4, 10.5); conservation of energy; energy diagrams (10.7)

Features of Oscillations

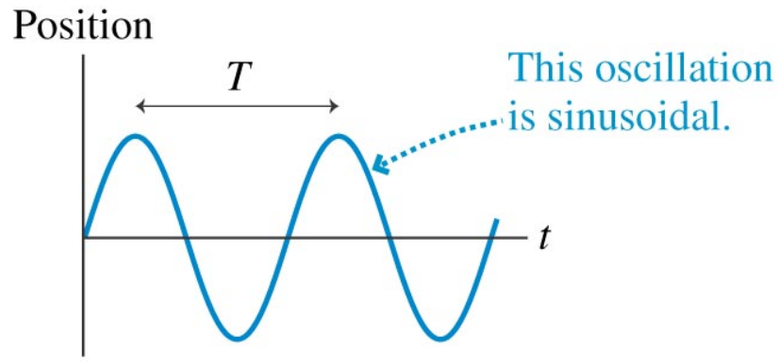
- back-and-forth motion about equilibrium position



- period (T): time for 1 cycle



- frequency (f = 1/ T): number of cycles per second



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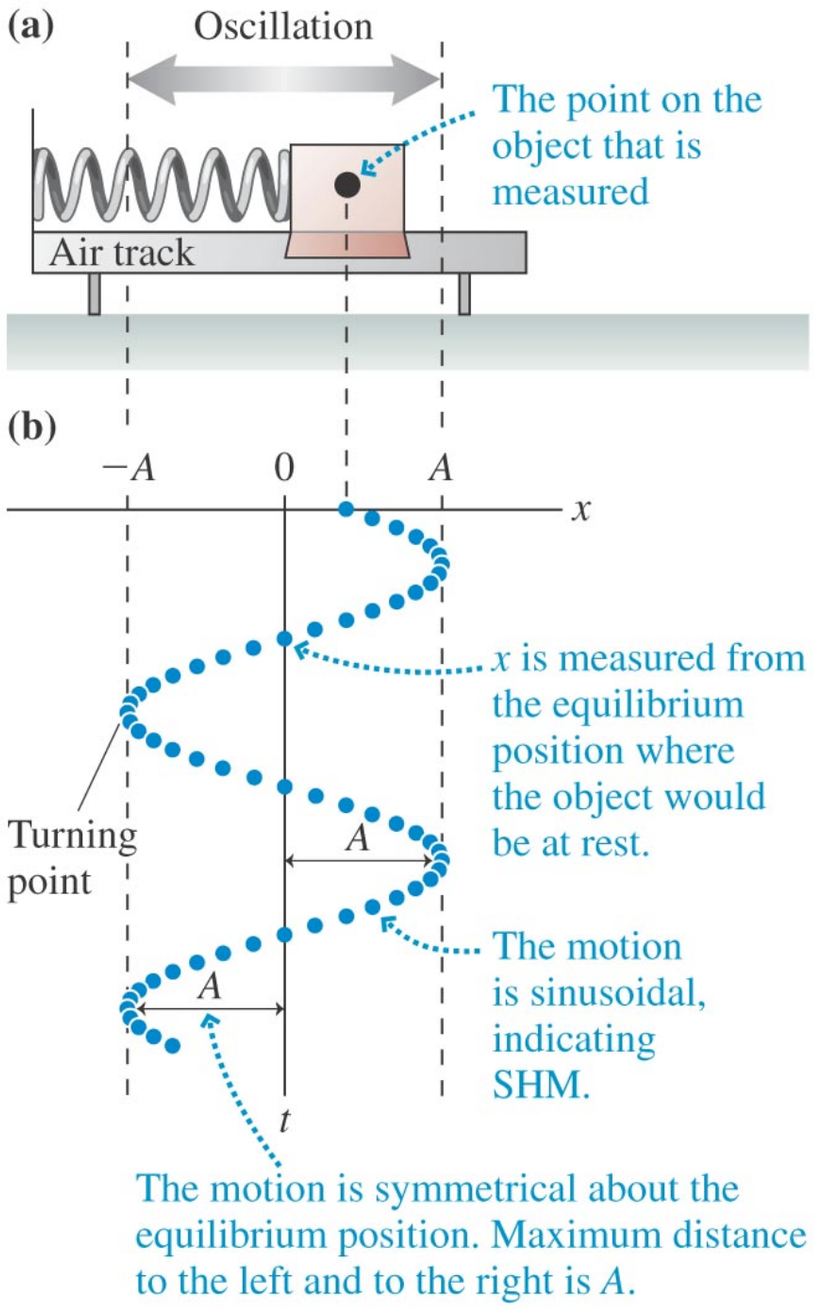
- units: 1 hertz (Hz) = 1 cycle/second = 1/ s
- 1 k Hz = 1000 Hz
  T = 1 / (1000 / s) = 1 ms

Special case: Simple Harmonic Motion



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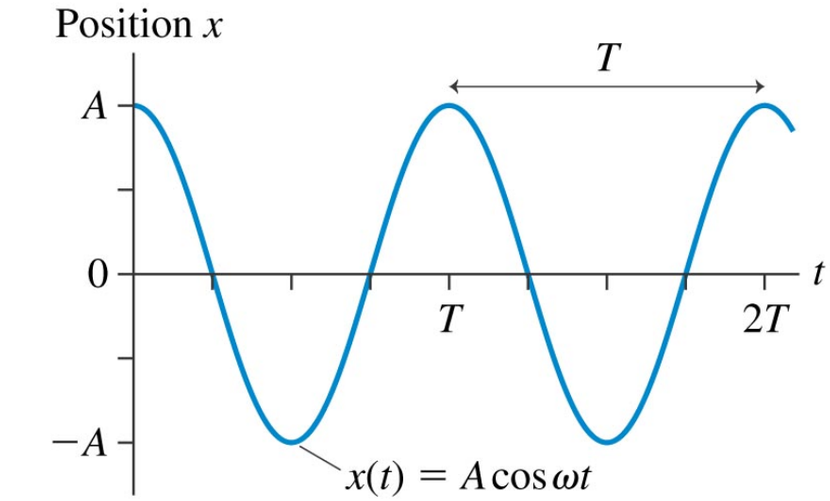
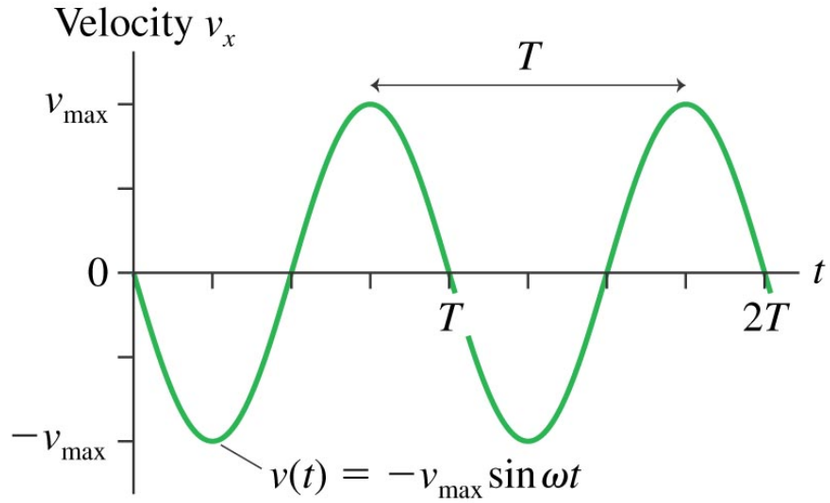
- amplitude (A): max. displacement from equilibrium position (x=0)
- velocity (v) = dx/dt
- v=0 at x = +A, -A
- v = v_{max} at x = 0





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3 Questions

- v_{max} related to A?

- T (or f) related to physical parameters (mass, spring constant)

- derive motion from Newton's laws

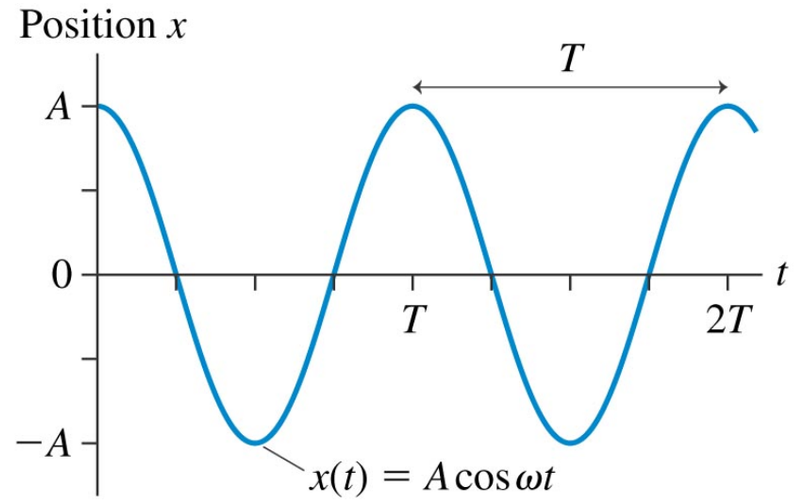
1. Mathematical description

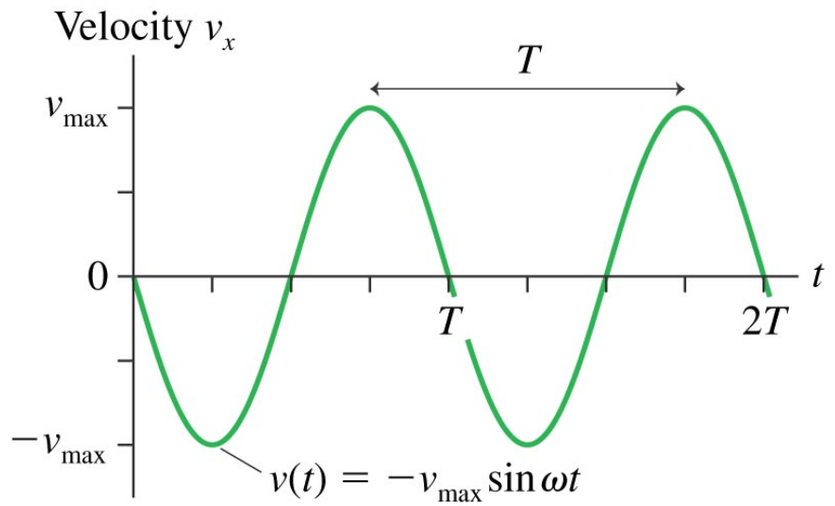
- focus on spring-mass (but general)
- empirical data (theory in next lecture):

$$\begin{aligned} x(t) &= A\cos\left(\frac{2\pi t}{T}\right) \\ &= A\cos\left(2\pi f t\right) \\ &= A\cos\omega t \end{aligned}$$

$$\omega \text{ (angular frequency)} = 2\pi f = 2\pi/T$$

(in radions/second, **not cycles/second**)





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- from graph: $v_x(t) = -v_{max}\sin\left(\frac{2\pi t}{T}\right)$
- using calculus:

$$\begin{aligned} v_x(t) &= \frac{dx}{dt} \\ &= -\omega A\sin\left(\frac{2\pi t}{T}\right) \end{aligned}$$

$$\Rightarrow \quad v_{\max} = \frac{2\pi A}{T} = 2\pi f A = \omega A$$

- stretch spring more ⇒ mass moves faster

Example

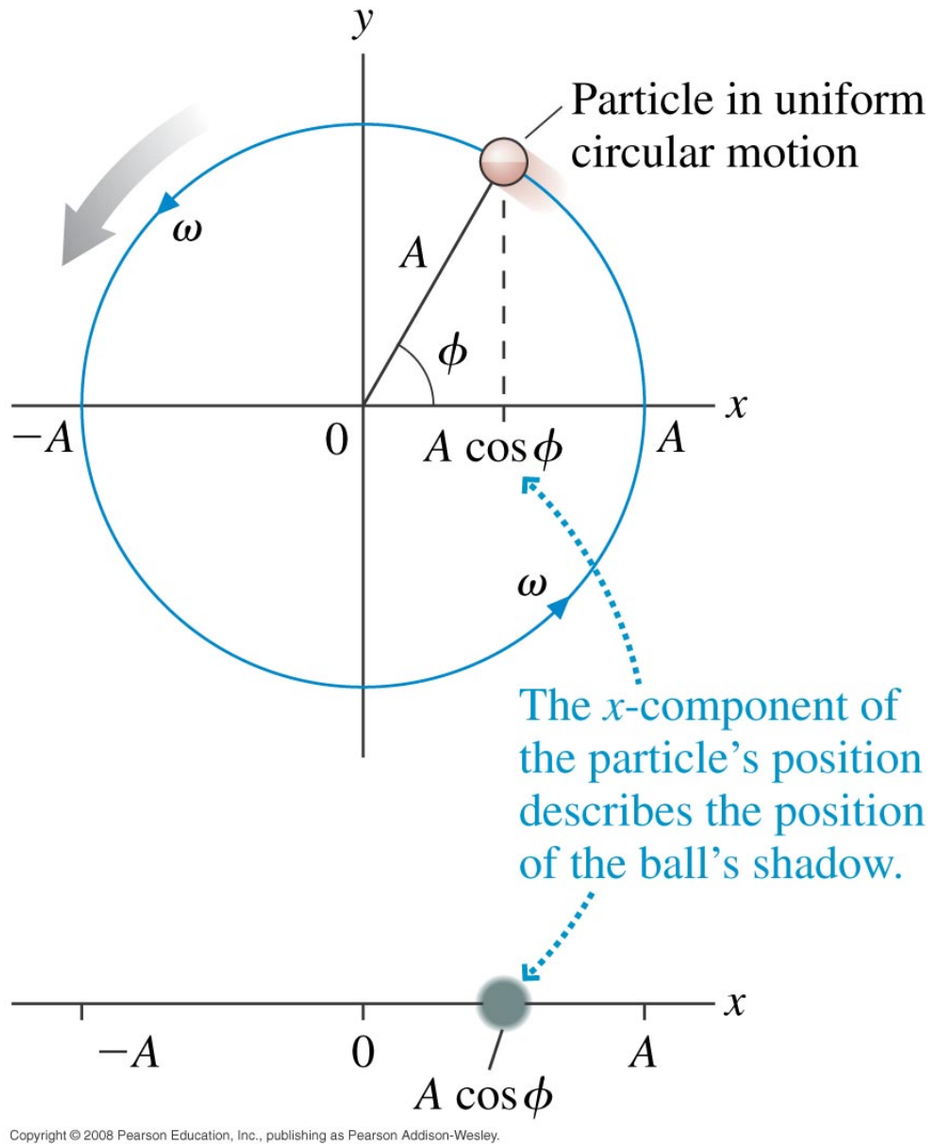
- An object undergoing SHM has a maximum displacement of 4.7 m at t = 0 s. If the angular frequency of oscillation is 1.6 rad/s, what is the object's (a) displacement and (b) speed when t = 3.5 s?

Relation to Circular motion (I)

- general initial condition:

$$x_0 \neq A \text{ at } t = 0$$

- SHM: projection of uniform circular motion onto 1 dimension



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- $x = A\cos\phi = A\cos\omega t$

$(\phi = \omega t$: uniform circular motion with $\phi = 0$ at $t = 0)$

Relation to Circular motion (II)

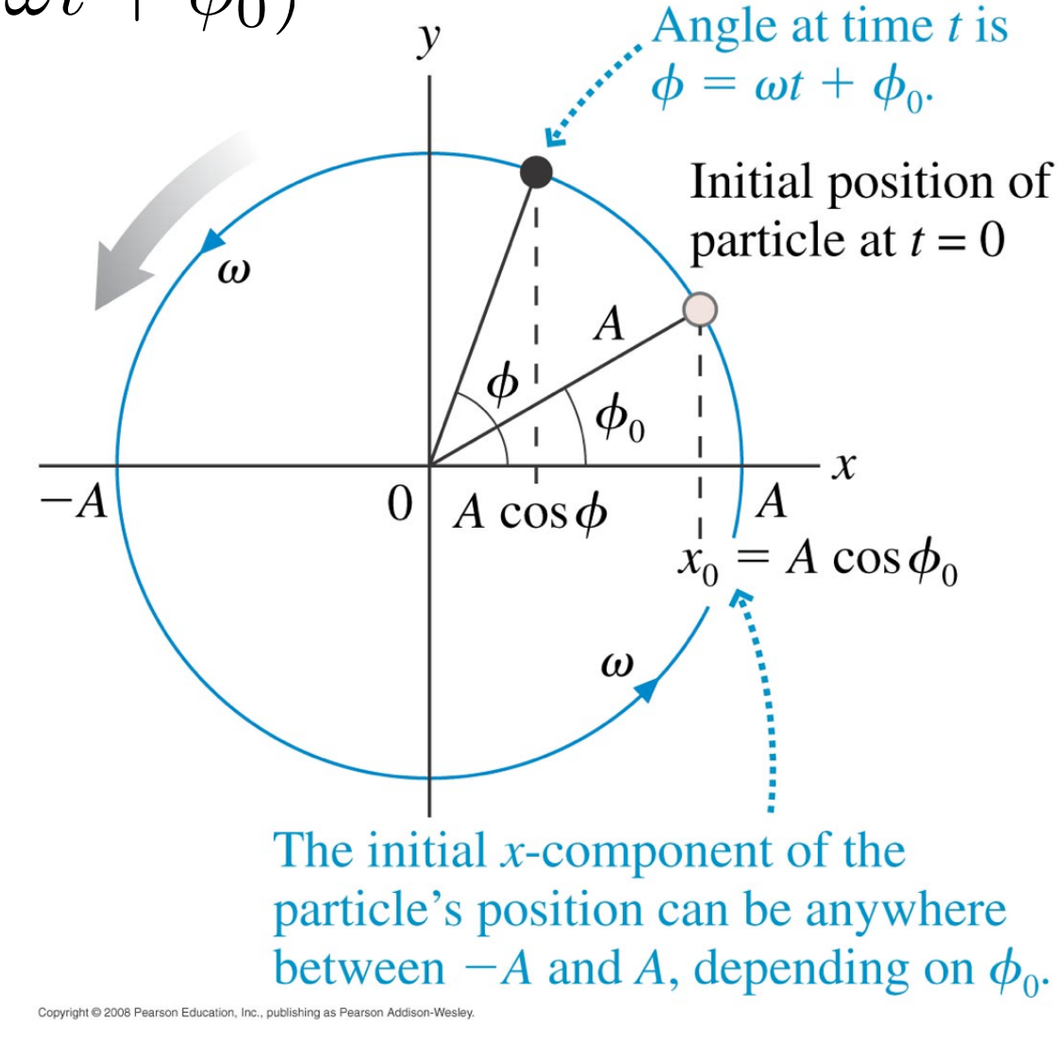
- In general:

$$\phi(t=0) \equiv \phi_0 \neq 0 \rightarrow$$
$$x(t) = A\cos\phi(t) = A\cos(\omega t + \phi_0);$$
$$v_x(t) = -\omega A\sin(\omega t + \phi_0) = v_{max}\sin(\omega t + \phi_0)$$

- $\phi(t)$ phase of oscillation (or angle of circular motion)
- ϕ_0 phase constant (sets initial condition: starting point on circle):

$$x_0 = A\cos\phi_0;$$
$$v_{0\,x} = -\omega A\sin\phi_0$$

vs. $\cos\phi_0$ in x_0



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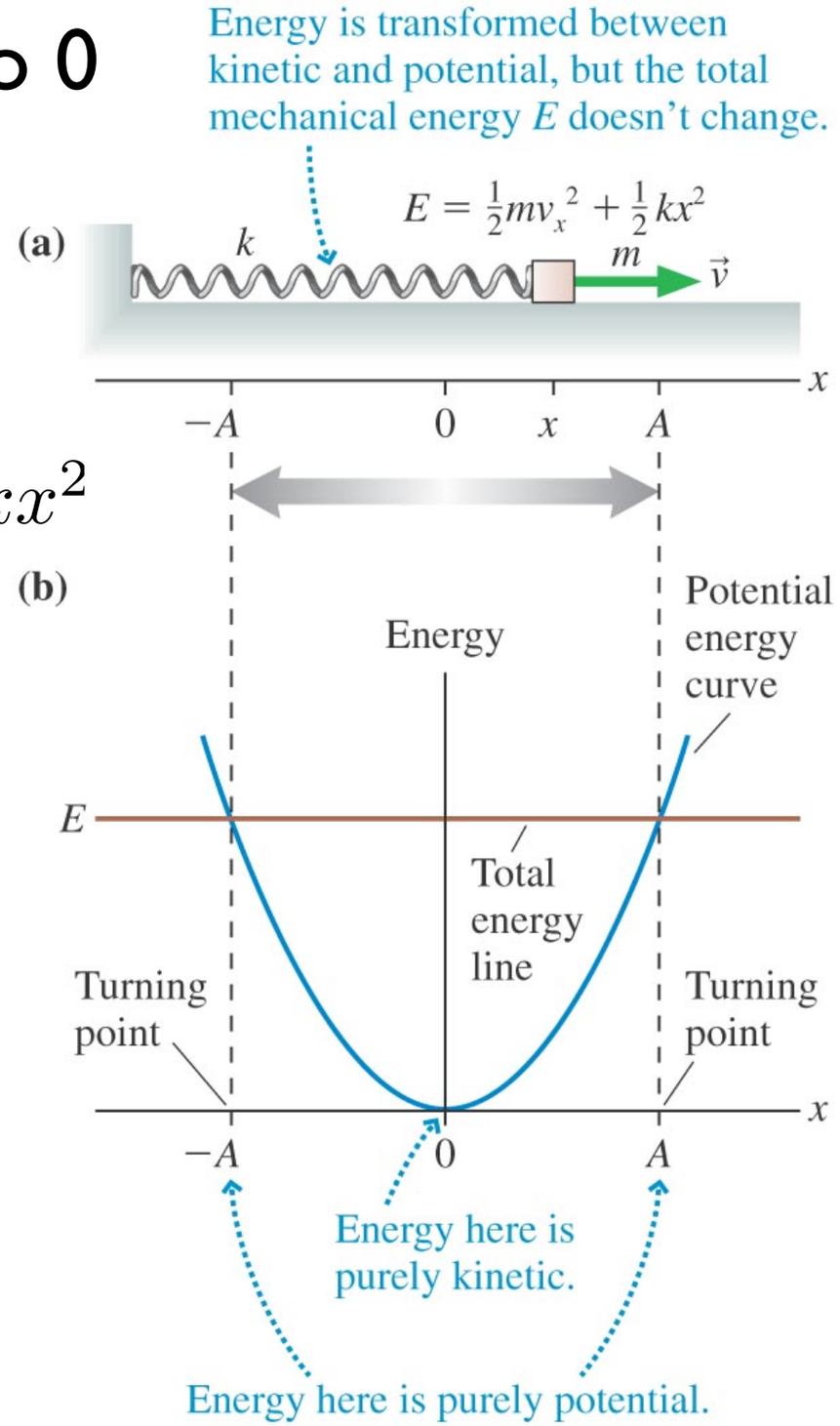
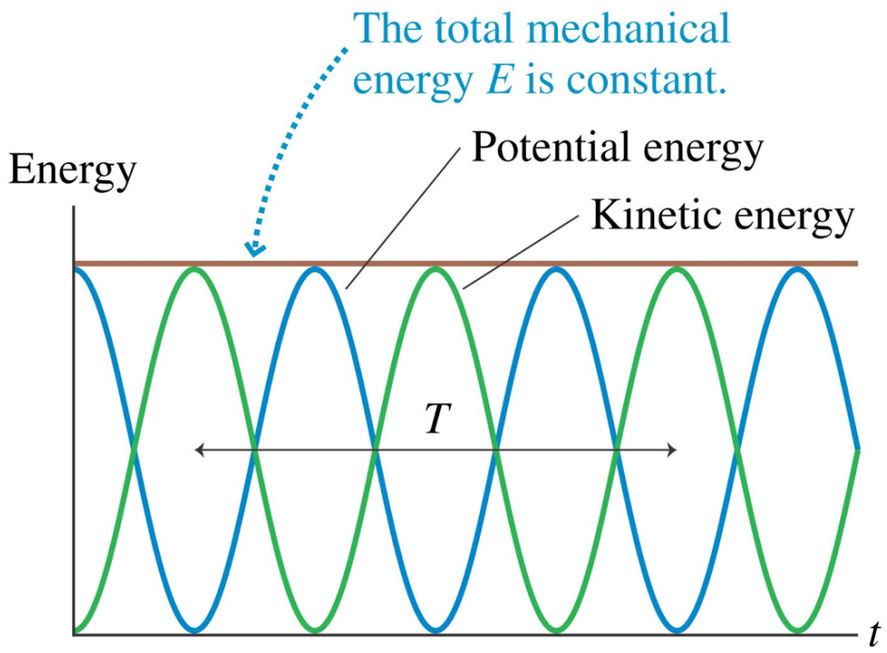
2. Use conservation of energy to...

- ...relate A, ω to m, k (spring constant)
- assume no friction (energy conserved)
- potential energy: $U = \frac{1}{2}k\,(\Delta x)^2$ $(\Delta x = x - x_c)$ set to 0
- mechanical energy: $E = K + U = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$
- at turning point: $E = U = \frac{1}{2}kA^2$
- at equilibrium: $E = \frac{1}{2}mv_{max}^2$



$$\omega = \sqrt{\frac{k}{m}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

(independent of A)



$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m(v_{\max})^2 \quad \text{(conservation of energy)}$$

(period of oscillation is half)

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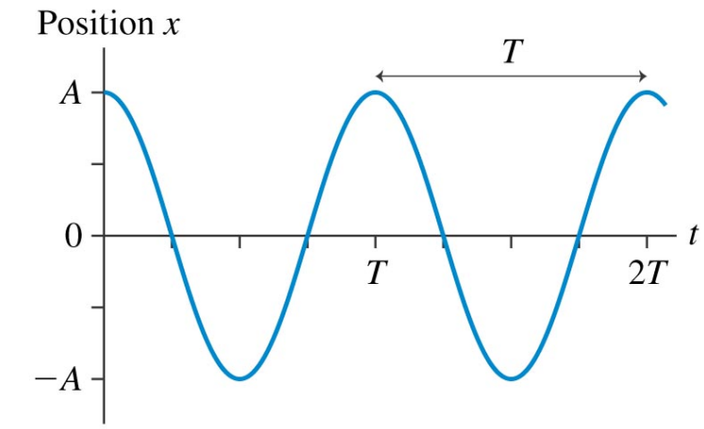
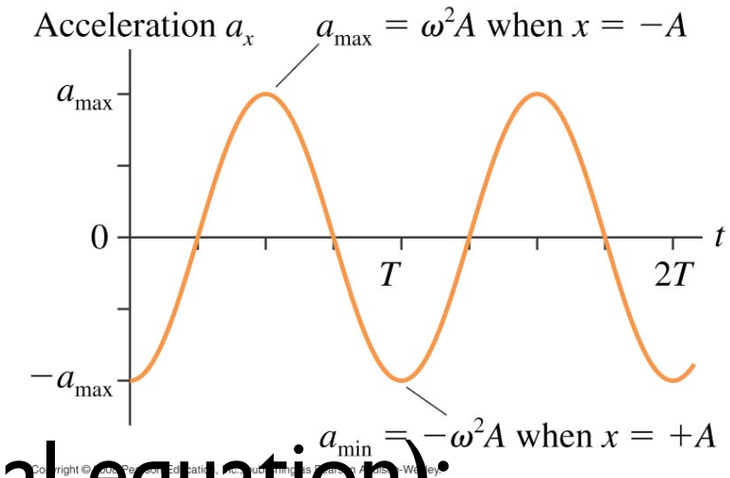
3. Newton's laws

$$\begin{aligned} a_x &= \frac{dv_x}{dt} \\ &= -\omega^2 A\cos\omega t \\ &= -\omega^2 x(t) \end{aligned}$$





- no friction/gravity

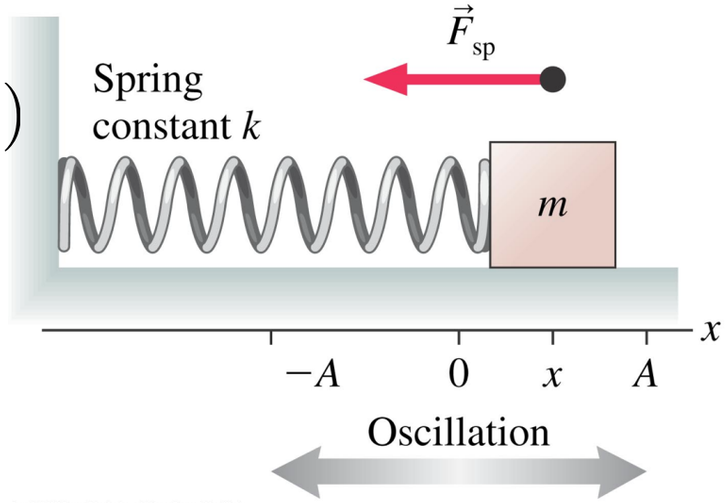
$$\begin{aligned} (F_{net})_x &= (F_{spring})_x \\ &= -k\Delta x = -kx \\ &= ma_x \end{aligned}$$

- acceleration not constant (2nd order differential equation):

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

unspecified constants

- unique solution ➡ guess: $x(t) = A\cos(\omega t + \phi_0)$



- verify: $\frac{dx}{dt} = -\omega A\sin\ldots \rightarrow \frac{dx^2}{dt^2} = -\omega^2 A\cos\ldots$
- satisfied if $\omega^2 = k/m$ (same as energy...)
- assumption $x(t) = A\cos(\omega t + \phi_0)$ justified by Newton's + Hooke's laws (theory agrees with experiment!)