Summaries of chapters for final exam (can<u>not</u> be brought to the exam)

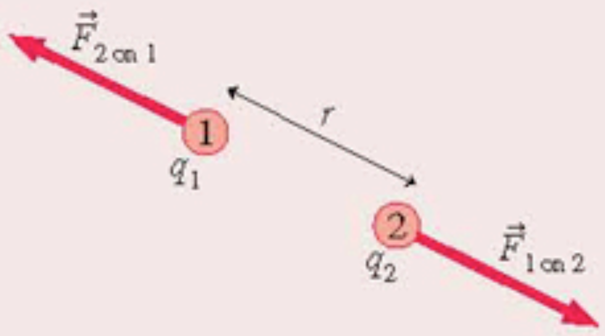
Chapter 25 (Electric Charges and Forces)

GENERAL PRINCIPLES

Coulomb's Law

The forces between two charged particles q_1 and q_2 separated by distance r are

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{K|q_1||q_2|}{r^2}$$



These forces are an action/reaction pair directed along the line joining the particles.

- The forces are repulsive for two like charges, attractive for two opposite charges.
- The net force on a charge is the sum of the forces from all other charges.
- The unit of charge is the coulomb (C).

Chapter 25 (continued)

IMPORTANT CONCEPTS

The Charge Model

There are **two kinds of charge,** called *positive* and *negative*.

- Fundamental charges are protons and electrons, with charge $\pm e$ where $e = 1.60 \times 10^{-19}$ C.
- Objects are charged by adding or removing electrons.
- The amount of charge is $q = (N_p - N_e)e$.
- An object with an equal number of protons and electrons is neutral, meaning no *net* charge.

Charged objects exert **electric forces** on each other.

- Like charges repel, opposite charges attract.
- The force increases as the charge increases.
- The force decreases as the distance increases.

There are **two types of material,** insulators and conductors.

- Charge remains fixed in or on an insulator.
- Charge moves easily through or along conductors.
- Charge is transferred by contact between objects.

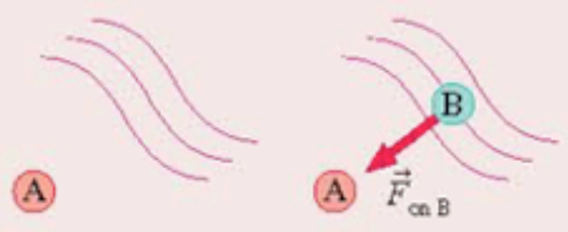
Charged objects attract neutral objects.

- Charge polarizes metal by shifting the electron sea.
- Charge polarizes atoms, creating electric dipoles.
- The polarization force is always an attractive force.

The Field Model

Charges interact with each other via the electric field $\vec{E}$.

- Charge A alters the space around it by creating an electric field.

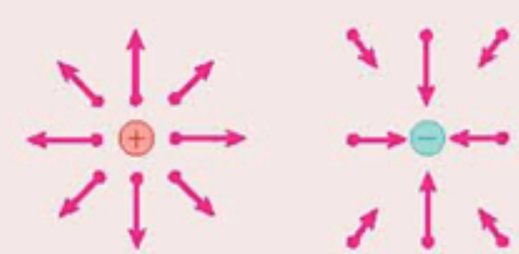


- The field is the agent that exerts a force. The force on charge q_B is $\vec{F}_{\text{on B}} = q_B\vec{E}$.

An electric field is identified and measured in terms of the force on a **probe charge** q.

$$\vec{E} = \vec{F}_{\text{on } q}/q$$

- The electric field exists at all points in space.
- An electric field vector shows the field only at one point, the point at the tail of the vector.



The electric field of a **point charge** is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Chapter 26 (The Electric Field)

GENERAL PRINCIPLES

Sources of $\vec{E}$

Electric fields are created by charges.

Two major tools for calculating $\vec{E}$ are

- The field of a point charge

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

- The principle of superposition

Multiple point charges

Use superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ \ldots$

Continuous distribution of charge

- Divide the charge into point-like ΔQ
- Find the field of each ΔQ
- Find $\vec{E}$ by summing the fields of all ΔQ

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a **charge density** (λ or η) and an integration coordinate.

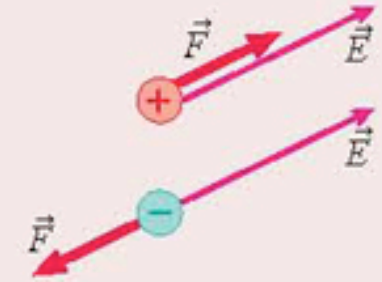
Consequences of $\vec{E}$

The electric field exerts a force on a charged particle.

$$\vec{F} = q\vec{E}$$

The force causes acceleration

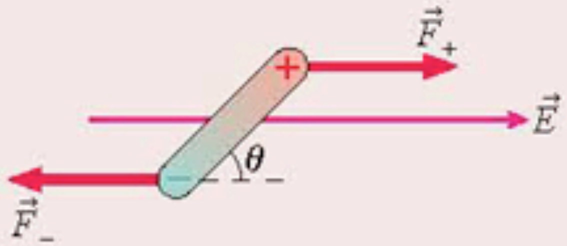
$$\vec{a} = (q/m)\vec{E}$$



Trajectories of charged particles are calculated with kinematics.

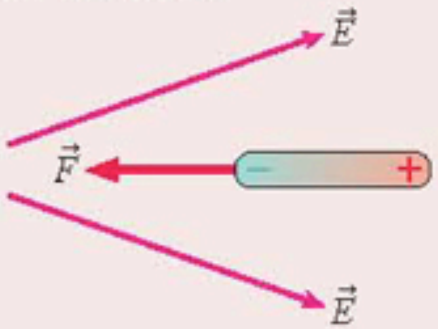
The electric field exerts a torque on a dipole.

$$\tau = pE\sin\theta$$



The torque tends to align the dipoles with the field.

In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.

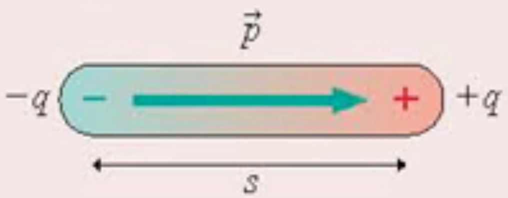


Chapter 26 (continued)

APPLICATIONS

The following fields are important models of the electric field:

Electric dipole



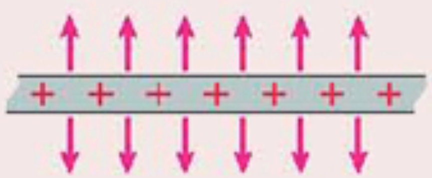
The electric dipole moment is

$\vec{p} = (qs, \text{from negative to positive})$

Field on axis $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{2\vec{p}}{r^3}$

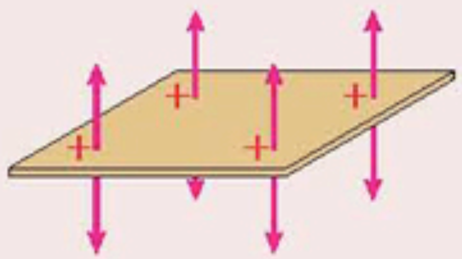
Field in bisecting plane $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{\vec{p}}{r^3}$

Infinite line of charge with linear charge density λ



$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}, \text{perpendicular to line}\right)$$

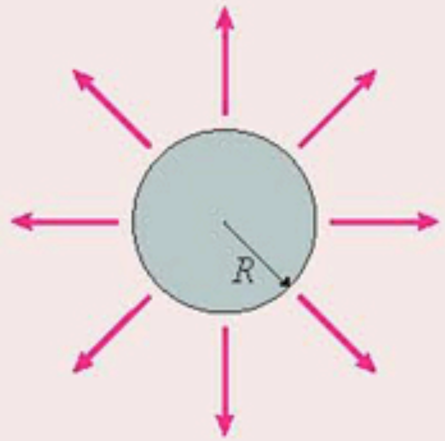
Infinite plane of charge with surface charge density η



$$\vec{E} = \left(\frac{\eta}{2\epsilon_0}, \text{perpendicular to plane}\right)$$

Sphere of charge

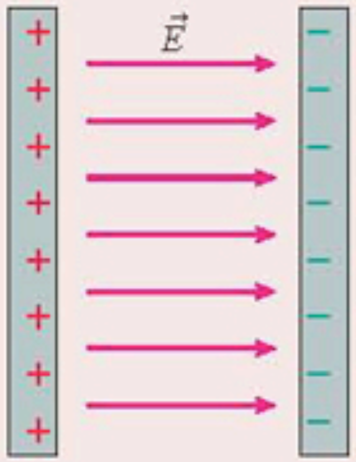
Same as a point charge Q for $r > R$



Parallel-plate capacitor

The electric field inside an ideal capacitor is a **uniform electric field**

$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{from positive to negative}\right)$$

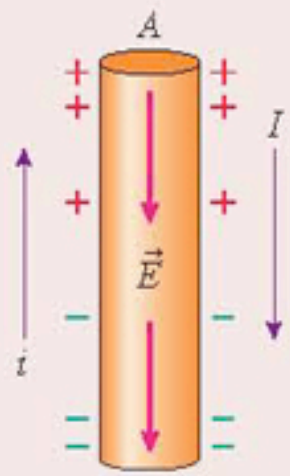


A real capacitor has a weak **fringe field** around it.

Chapter 28 (Current and Conductivity)

GENERAL PRINCIPLES

Current is a nonequilibrium motion of charges sustained by an electric field. Nonuniform surface charge density creates an electric field in a wire. The electric field pushes the electron current i in a direction opposite to $\vec{E}$. The conventional current I is in the direction in which positive charge *seems* to move.



Electron current

i = rate of electron flow

$N_e = i\Delta t$

Conventional current

I = rate of charge flow $= ei$

$Q = I\Delta t$

Current density

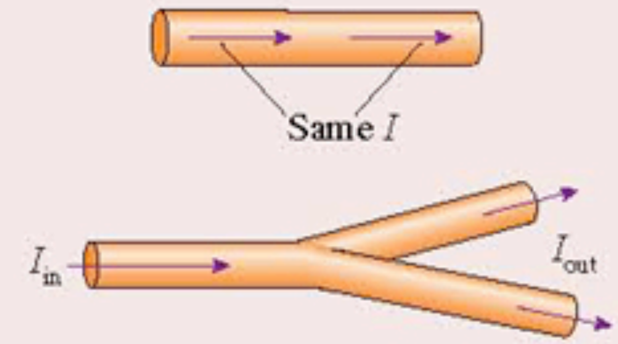
$J = I/A$

Conservation of Current

The current is the same at any two points in a wire.

At a junction,

$$\sum I_{in} = \sum I_{out}$$



Batteries

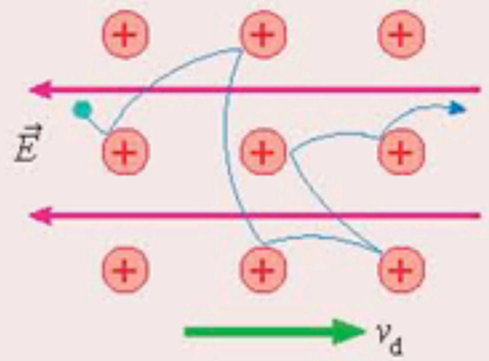
The role of a battery is to maintain a charge separation and thus sustain a current. Chemical reactions power the "charge escalator" that moves electrons from the positive terminal to the negative terminal.

Chapter 28 (continued)

IMPORTANT CONCEPTS

Sea of electrons

Conduction electrons move freely around the positive ions that form the atomic lattice.



Conduction

An electric field causes a slow drift at speed v_d to be superimposed on the rapid but random thermal motions of the electrons.

Collisions of electrons with the ions transfer energy to the atoms. This makes the wire warm and lightbulbs glow. More collisions mean a higher resistivity ρ and a lower conductivity σ.

The **drift speed** is $v_d = \frac{e\tau}{m}E$

where τ is the mean time between collisions. v_d is related to the electron current by

$$i = nAv_d$$

where n is the electron density.

An electric field E in a conductor causes a current density $J = nev_d = \sigma E$ where the **conductivity** is

$$\sigma = \frac{ne^2\tau}{m}$$

The **resistivity** is $\rho = 1/\sigma$.

Chapter 29 (The Electric Potential)

GENERAL PRINCIPLES

Sources of *V*

The electric potential, like the electric field, is created by charges.

Two major tools for calculating *V* are

- The potential of a point charge $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$
- The principle of superposition

Multiple point charges

Use superposition: $V = V_1 + V_2 + V_3 + \dots$

Continuous distribution of charge

- Divide the charge into point-like ΔQ.
- Find the potential of each ΔQ.
- Find *V* by summing the potentials of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a charge density and an integration coordinate. Calculating *V* is usually easier than calculating $\vec{E}$ because the potential is a scalar.

Consequences of *V*

A charged particle has potential energy

$$U = qV$$

at a point where source charges have created an electric potential *V*.

The electric force is a conservative force, so the mechanical energy is conserved for a charged particle in an electric potential:

$$K_f + U_f = K_i + U_i$$

The potential energy of **two point charges** separated by distance *r* is

$$U_{q_1+q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

The **zero point** of potential and potential energy is chosen to be convenient. For point charges, we let $U = 0$ when $r \to \infty$.

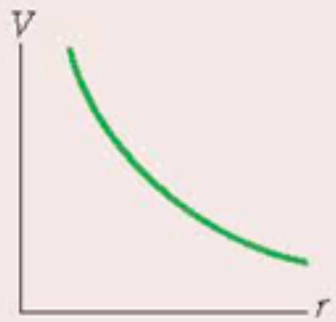
The potential energy in an electric field of an **electric dipole** with dipole moment $\vec{p}$ is

$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

Chapter 29 (continued)

APPLICATIONS

Graphical representations of the potential:



Potential graph

Equipotential surfaces

Contour map

Elevation graph

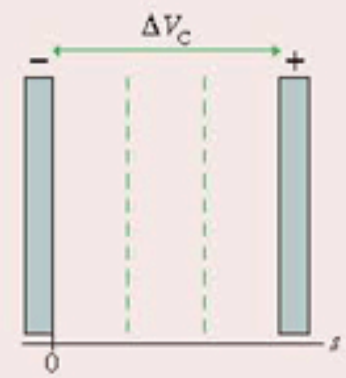
Sphere of charge Q

Same as a point charge if $r \geq R$.

Parallel-plate capacitor

$V = Es$, where s is measured from the negative plate. The electric field inside is

$$E = \Delta V_C / d$$



Units

Electric potential: 1 V = 1 J/C

Electric field: 1 V/m = 1 N/C

Chapter 30 (Potential and Field)

GENERAL PRINCIPLES

Connecting V and $\vec{E}$

The electric potential and the electric field are two different perspectives of how source charges alter the space around them. V and $\vec{E}$ are related by

$$\Delta V = V(s_f) - V(s_i) = -\int_{s_i}^{s_f} E_s\,ds$$

where s is measured from point i to point f and E_s is the component of $\vec{E}$ parallel to the line of integration.

Graphically

ΔV = the negative of the area under the E_s graph

and

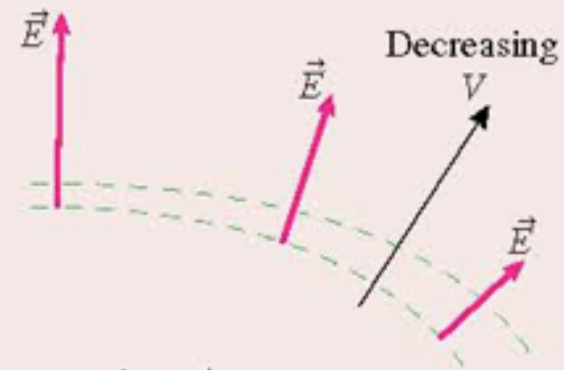
$$E_s = -\frac{dV}{ds}$$

= the negative of the slope of the potential graph.

The Geometry of Potential and Field

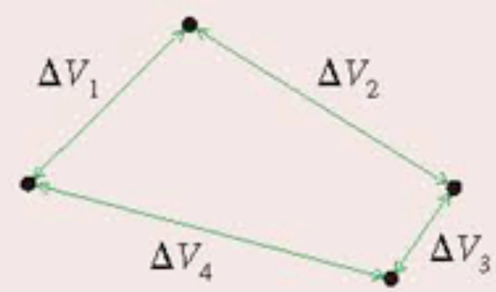
The electric field

- Is perpendicular to the equipotential surfaces.
- Points "downhill" in the direction of decreasing V.
- Is inversely proportional to the spacing Δs between the equipotential surfaces.



Conservation of Energy

The sum of all potential differences around a closed path is zero. $\sum(\Delta V)_i = 0$.



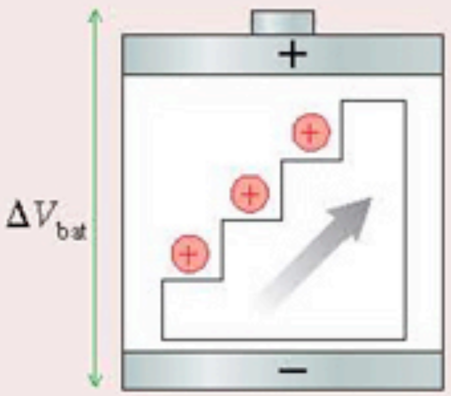
Chapter 30 (continued)

IMPORTANT CONCEPTS

A **battery** is a source of potential.
The charge escalator in a battery uses chemical reactions to move charges from the negative terminal to the positive terminal.

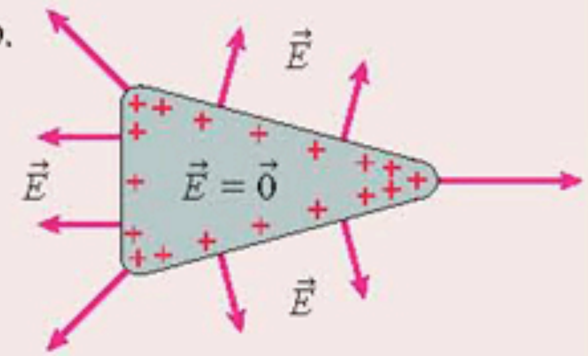
$$\Delta V_{\text{bat}} = \mathcal{E}$$

where the emf $\mathcal{E}$ is the work per charge done by the charge escalator.



For a conductor in electrostatic equilibrium

- The interior electric field is zero.
- The exterior electric field is perpendicular to the surface.
- The surface is an equipotential.
- The interior is at the same potential as the surface.



APPLICATIONS

Resistors

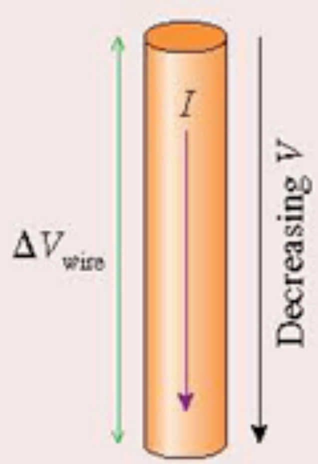
A potential difference ΔV_{wire} between the ends of a wire creates an electric field inside the wire

$$E_{\text{wire}} = \frac{\Delta V_{\text{wire}}}{L}$$

The electric field causes a current

$$I = \frac{\Delta V_{\text{wire}}}{R}$$

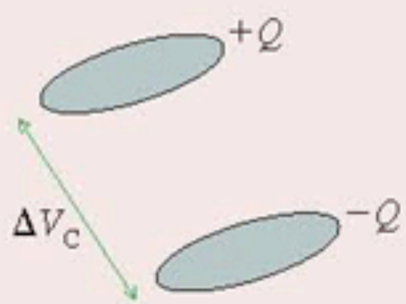
where $R = \dfrac{\rho L}{A}$ is the wire's **resistance.**



Capacitors

The **capacitance** of two conductors charged to $\pm Q$ is

$$C = \frac{Q}{\Delta V_C}$$

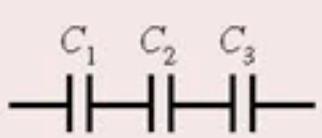


The energy stored in a capacitor is

$$U_C = \tfrac{1}{2} C (\Delta V_C)^2$$

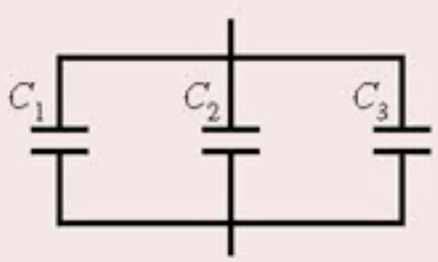
Series capacitors

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \right)^{-1}$$



Parallel capacitors

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$$



Chapter 31(Fundamentals of Circuits)

GENERAL STRATEGY

MODEL Assume that wires and, where appropriate, batteries are ideal.

VISUALIZE Draw a circuit diagram. Label known and unknown quantities.

SOLVE The solution is based on Kirchhoff's laws.

- Reduce the circuit to the smallest possible number of equivalent resistors.
- Find the current and the potential difference.
- "Rebuild" the circuit to find I and ΔV for each resistor.

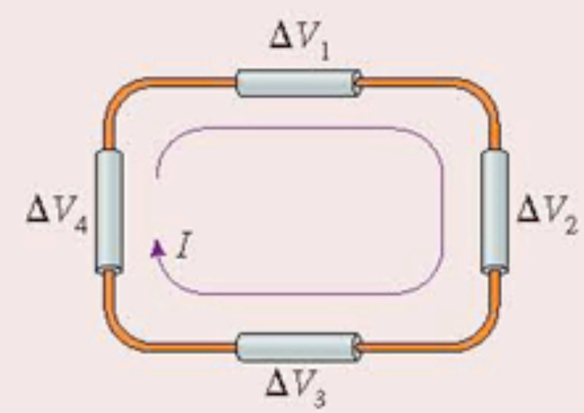
ASSESS Verify that

- The sum of potential differences across series resistors matches ΔV for the equivalent resistor.
- The sum of the currents through parallel resistors matches I for the equivalent resistor.

Kirchhoff's loop law

For a closed loop:

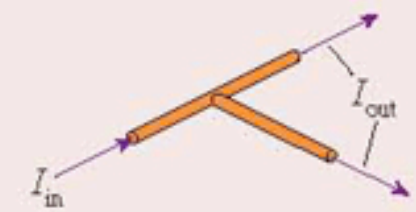
- Assign a direction to the current I.
- $\sum_i (\Delta V)_i = 0$



Kirchhoff's junction law

For a junction:

- $\sum I_{in} = \sum I_{out}$



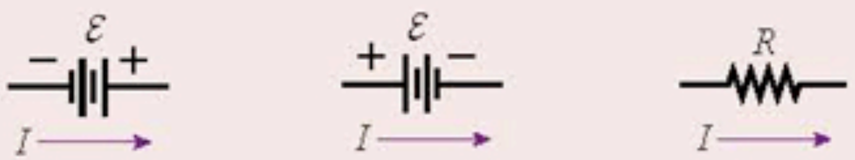
IMPORTANT CONCEPTS

Ohm's law

A potential difference ΔV between the ends of a conductor with resistance R creates a current

$$I = \frac{\Delta V}{R}$$

Signs of ΔV



$$\Delta V_{\text{bat}} = +\mathcal{E} \qquad \Delta V_{\text{bat}} = -\mathcal{E} \qquad \Delta V_{\text{R}} = -IR$$

The energy used by a circuit is supplied by the emf $\mathcal{E}$ of the battery through the energy transformations

$$E_{\text{chem}} \rightarrow U \rightarrow K \rightarrow E_{\text{th}}$$

The battery *supplies* energy at the rate

$$P_{\text{bat}} = I\mathcal{E}$$

The resistors *dissipate* energy at the rate

$$P_{\text{R}} = I\Delta V_{\text{R}} = I^2R = \frac{(\Delta V_{\text{R}})^2}{R}$$

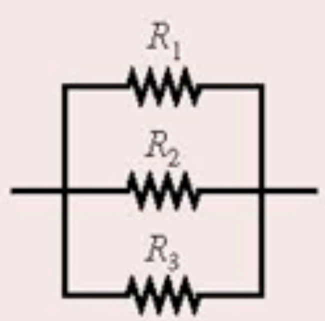
APPLICATIONS

Series resistors

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \ \ldots$$

Parallel resistors

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ \ldots\right)^{-1}$$



RC circuits

The discharge of a capacitor through a resistor satisfies:

$$Q = Q_0 e^{-t/\tau}$$

$$I = -\frac{dQ}{dt} = \frac{Q_0}{\tau} e^{-t/\tau} = I_0 e^{-t/\tau}$$

where $\tau = RC$ is the **time constant.**