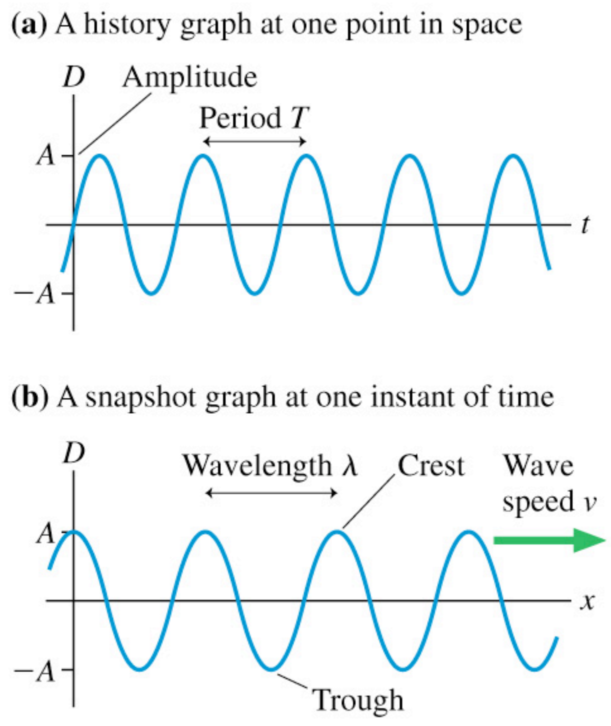
Lecture 8

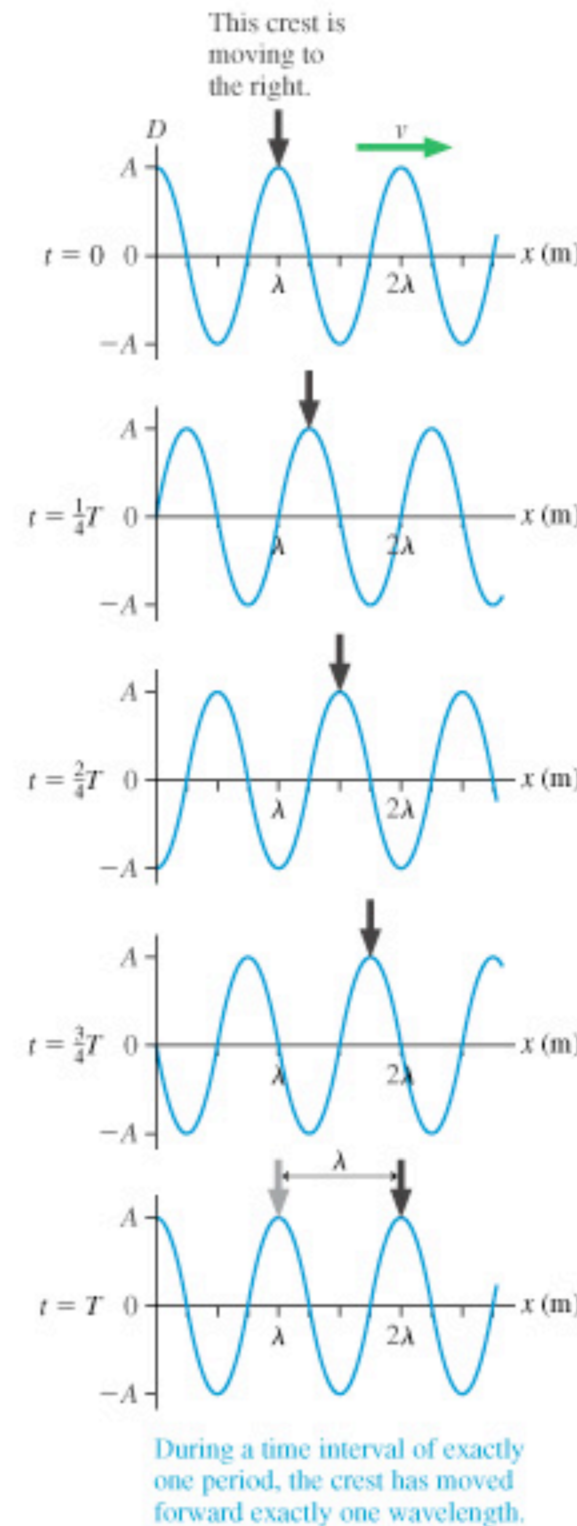
- Sinusoidal waves
- Wave speed on a string
- 2D/3D waves
- Sound and Light

Sinusoidal waves (graphical)

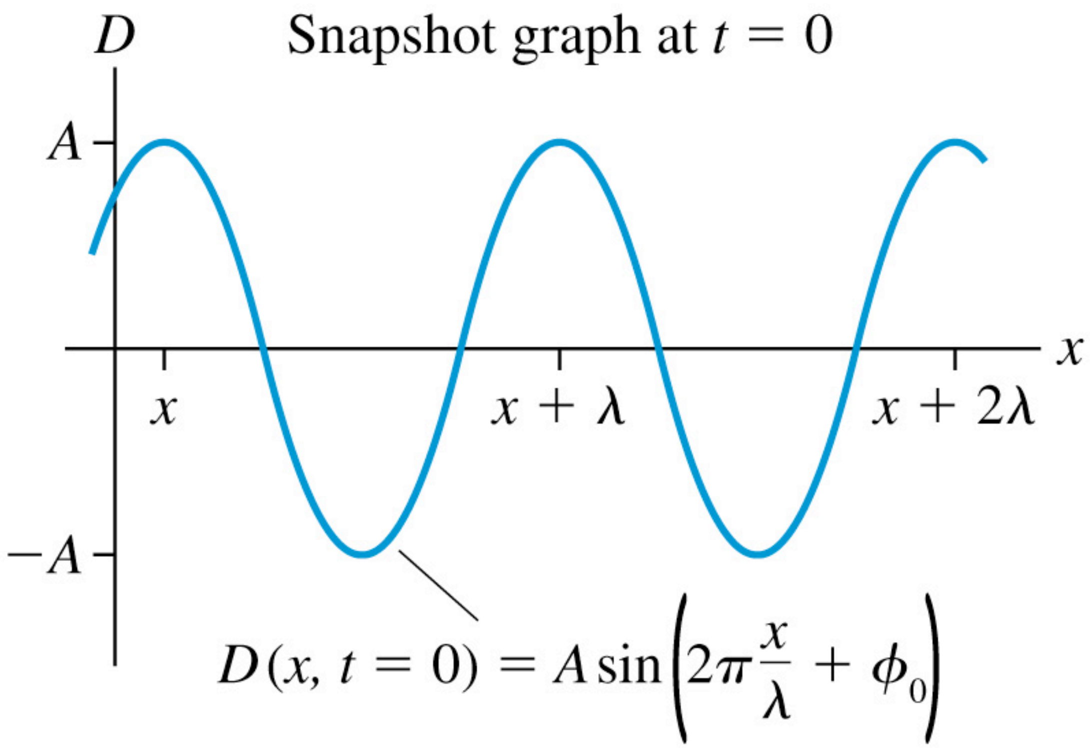


- generated by source in SHM
- snapshot and history graphs sinusoidal/periodic in space, time
- <u>Wavelength</u> (λ): spatial analog of T, distance disturbance repeats
- In time T: (one oscillation for point) wave (crest) moves λ

$$v = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T} = \lambda f$$

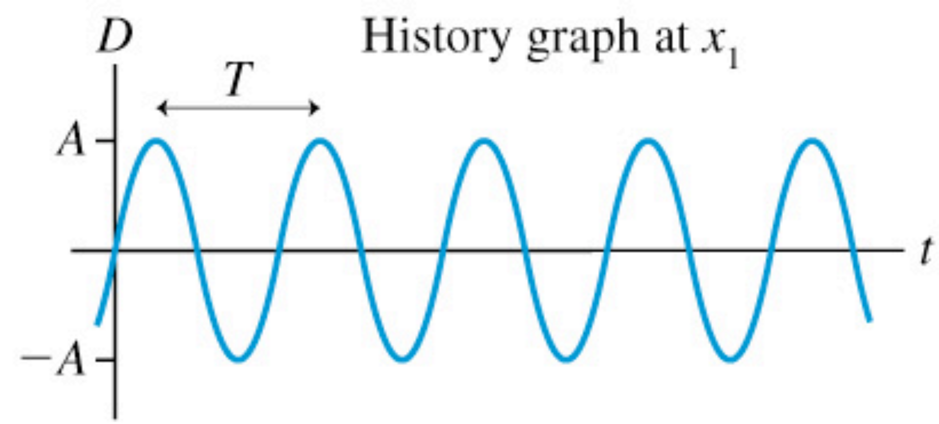


Sinusoidal waves:mathematical

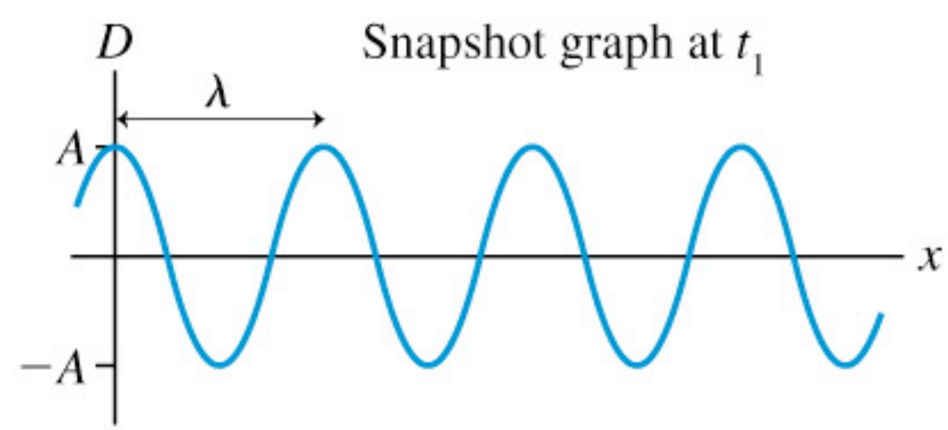


$D(x, t) = A\sin(kx - \omega t + \phi_0)$

(sinusoidal wave traveling in the positive x-direction)



If x is fixed, $D(x_1, t) = A\sin(kx_1 - \omega t + \phi)$ gives a sinusoidal history graph at one point in space, x_1. It repeats every T s.



If t is fixed, $D(x, t_1) = A\sin(kx - \omega t_1 + \phi)$ gives a sinusoidal snapshot graph at one instant of time, t_1. It repeats every λ m.

- Set wave in motion by $x \rightarrow (x - vt)$

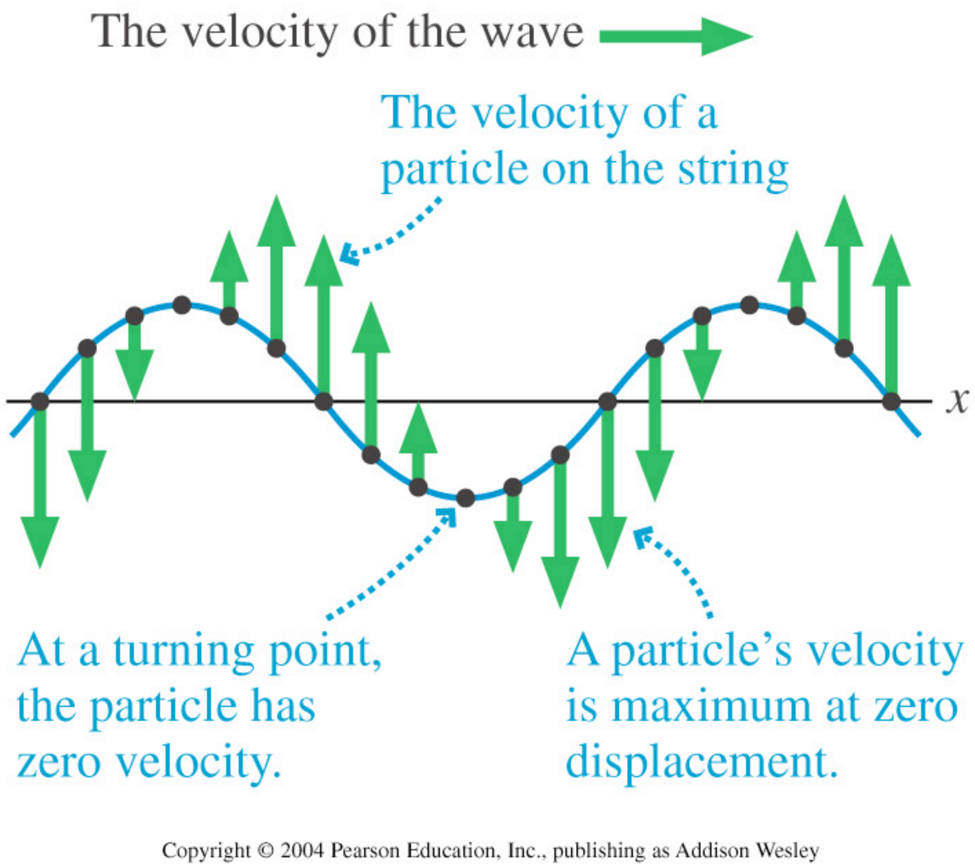
$$D(x,t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \phi_0\right]$$

<u>wave number,</u> $k = \frac{2\pi}{\lambda}; \omega = \nu k$

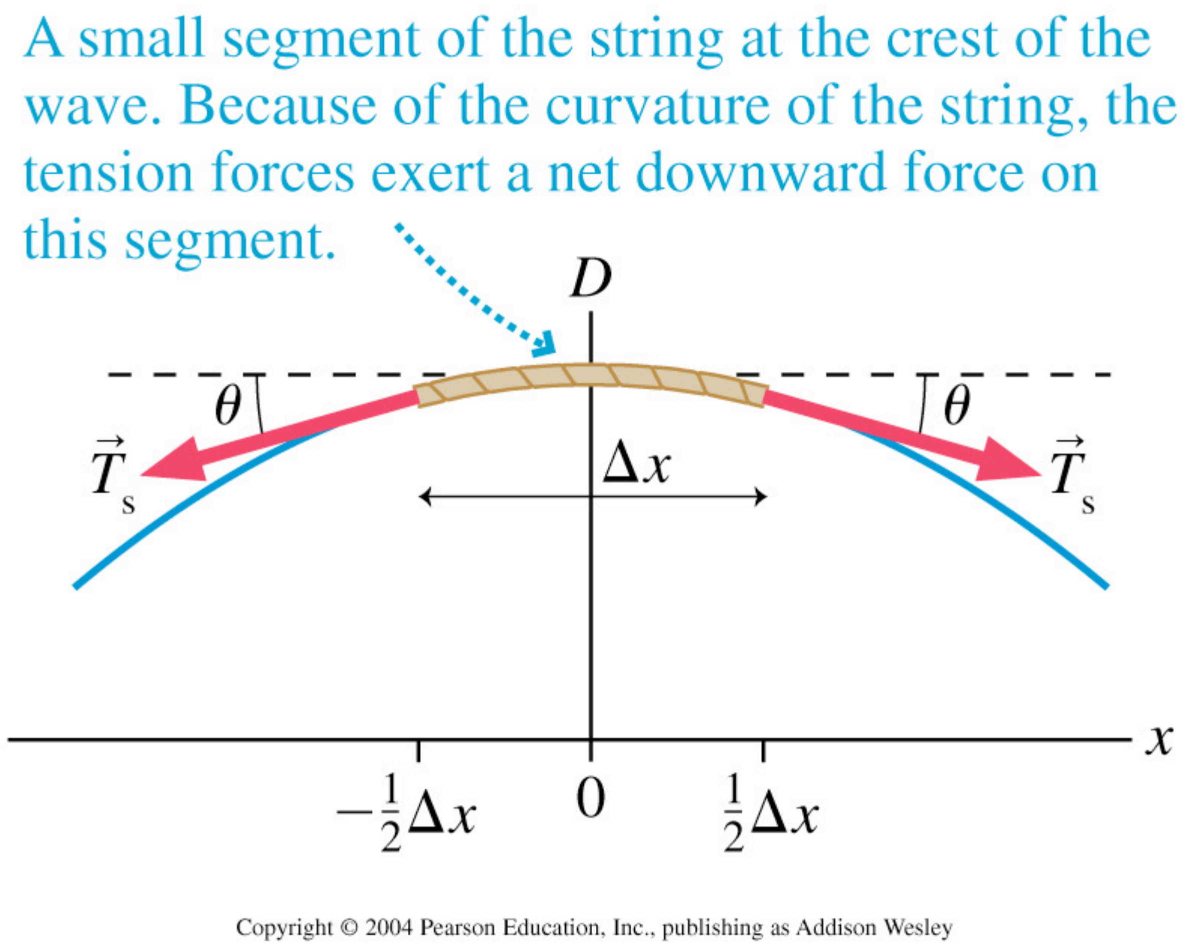
ϕ_0 sets initial condition:

$$D(x = 0, t = 0) = A\sin\phi_0$$

Waves on a string



$$y(x,t) = A\sin\left(kx - \omega t + \phi_0\right)$$
$$v_y = -\omega A\cos\left(kx - \omega t + \phi_0\right)$$
$$a_y y = -\omega^2 A\sin\left(kx - \omega t + \phi_0\right)$$

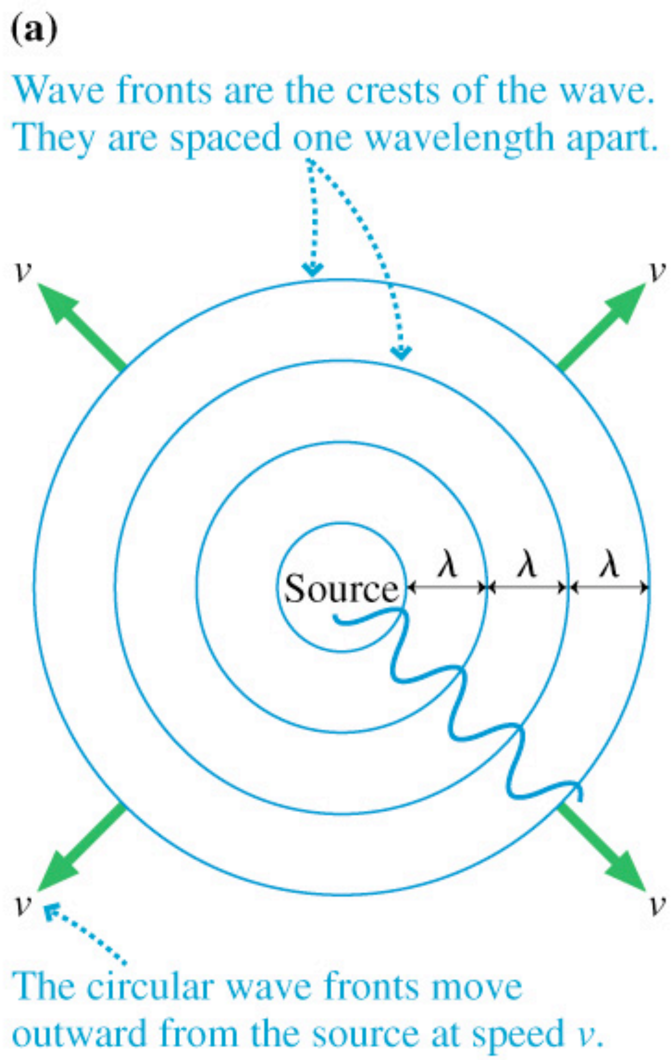


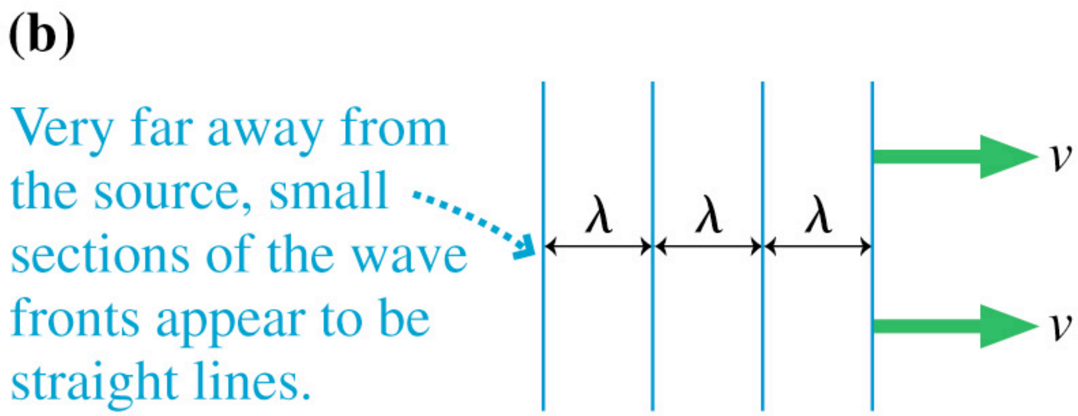
- Newton's laws applied to string

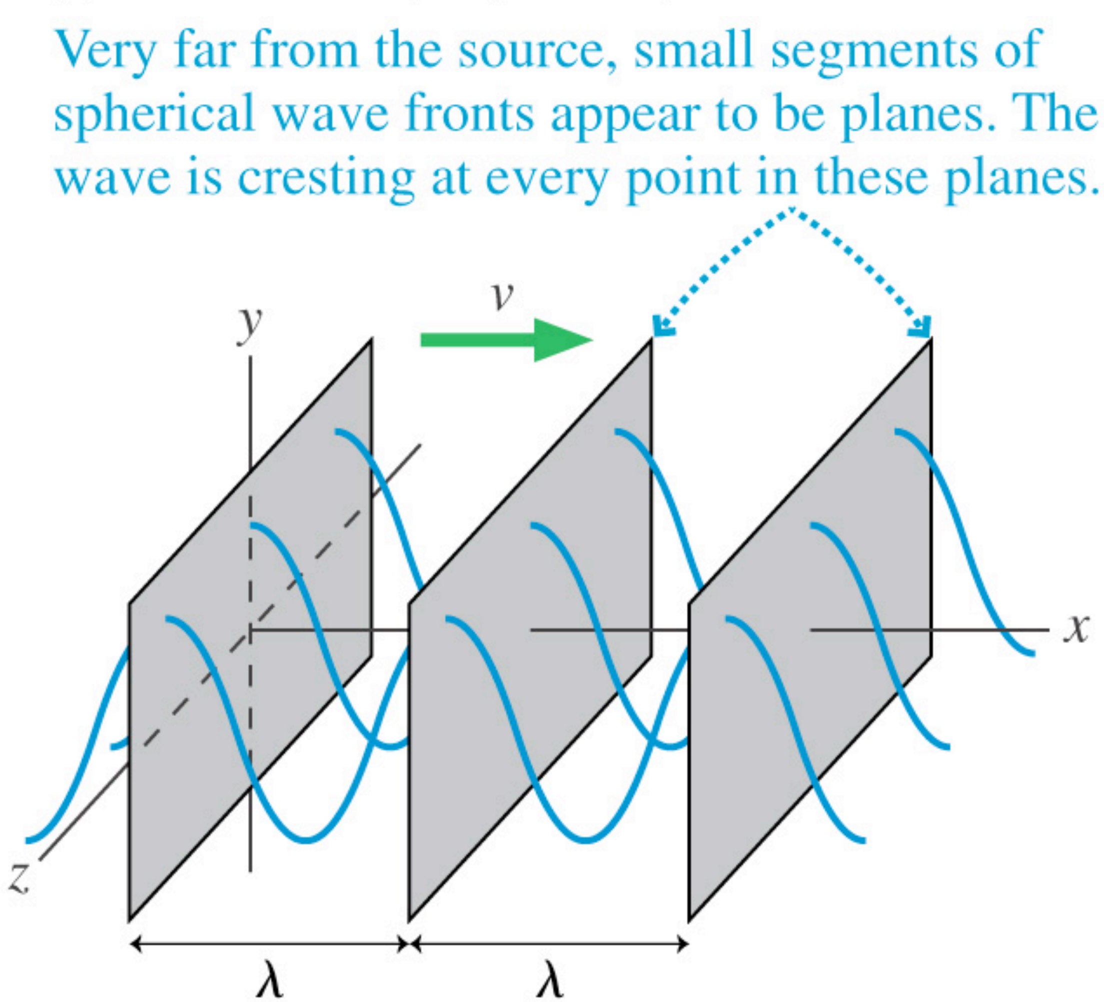
$$(F_{net})_y = ma_y = (\mu\Delta x)\, a_y$$
$$(F_{net})_y = 2T_s\sin\theta \approx -k^2 A T_s \Delta x$$
$$\text{(evaluate slope of } y = A\cos(kx))$$

$$\Rightarrow v = \sqrt{\frac{T_s}{\mu}}$$ (independent of A/shape)

2D/3D waves







- 2D circular waves: wavefronts (lines locating crests), small section appear as straight lines far away
- 3D spherical waves...appear as planes far away, described by D(x, t) (same at every point in yz plane)

2D/3D waves

$$D(r,t) = A(r)\sin(kr - \omega t + \phi_0)$$
with $A(r)$ decreasing with r

Phase and phase difference

$$\text{phase, } \phi = kx - \omega t + \phi_0$$
$$D(x,t) = A\sin\phi$$

- wavefronts are surfaces of same displacement constant phase

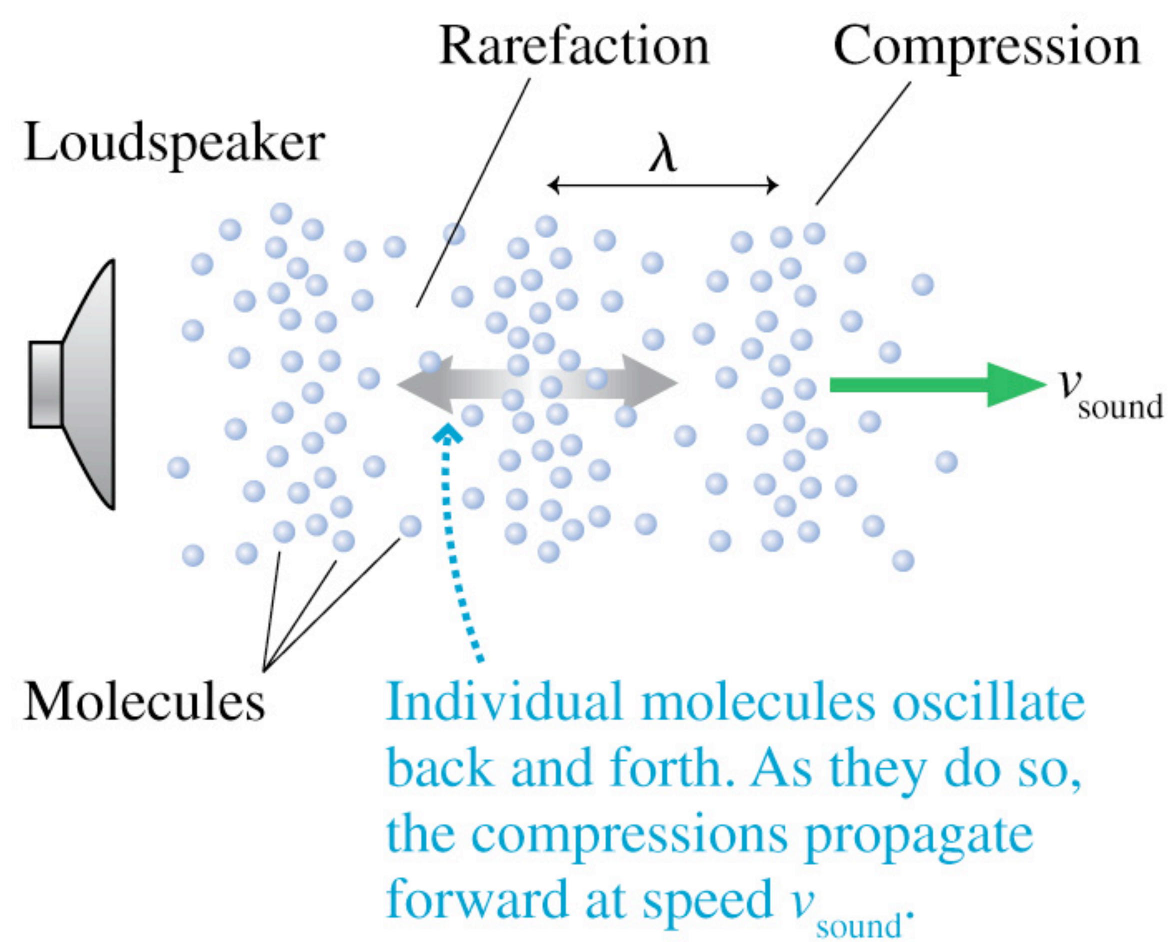
$$\text{phase difference, } \Delta\phi = 2\pi\frac{\Delta x}{\lambda}$$
$\Delta\phi = 2\pi$ between adjacent wavefronts (separated by λ)

Sound waves

$$v_{sound} \text{ in air at } 20^\circ = 343\ m/s$$
(larger in liquid/solid)

- human ears: 20 Hz to 20 k Hz

- ultrasound: > 20 k Hz



Electromagnetic (EM) waves

- oscillations of EM field, can travel <u>in vacuum</u> e.g. light from stars

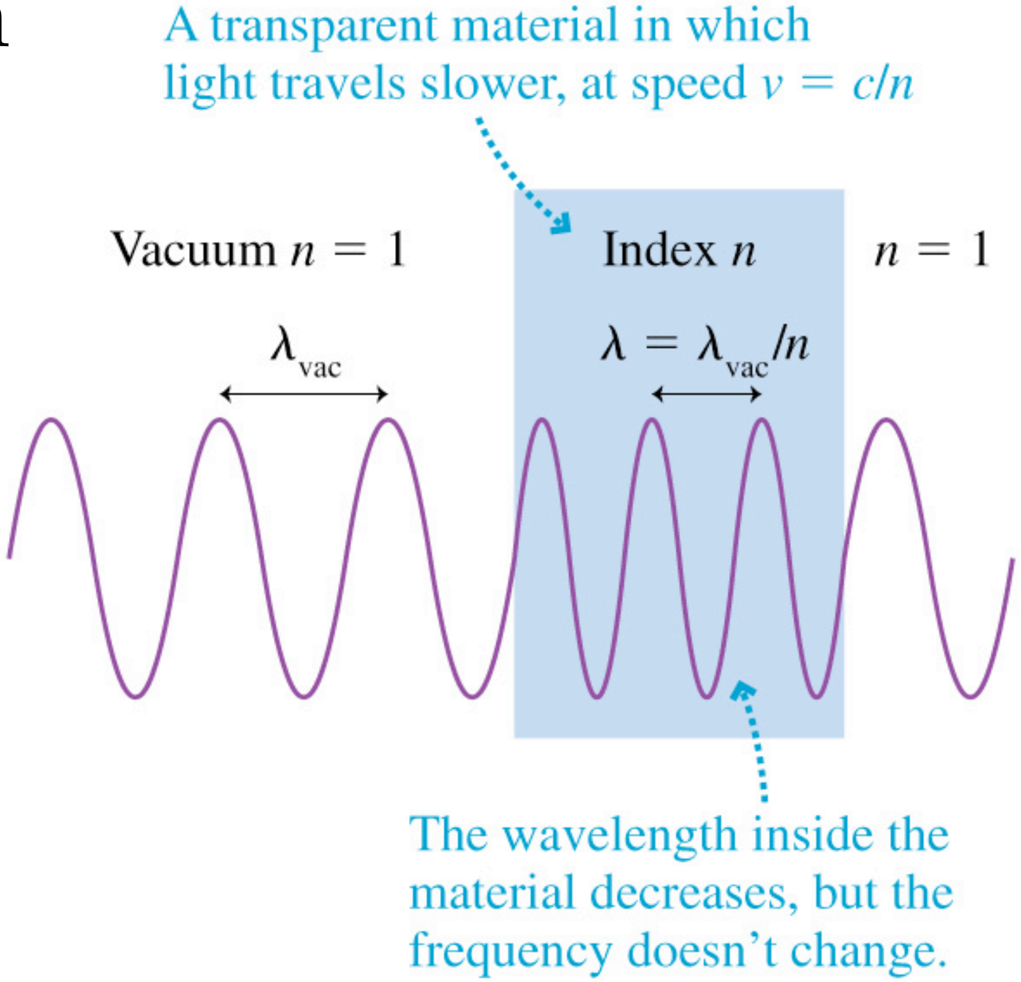
$$v_{light} = c = 3 \times 10^8 \; m/s \text{ in vacuum}$$

$(\gg v_{sound})$

- <u>visible</u> spectrum: 400 nm (violet/blue to 700 nm (orange/red)

$$\ll \lambda_{sound} \Rightarrow f_{light} \gg f_{sound}$$

- EM spectrum: visible + higher frequencie (UV/X rays) + lower frequencies (IR/micro/radio waves)



- <u>index of refraction</u> (light slowed down):

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in material}} = \frac{c}{v}$$

<u>frequency</u> does <u>not</u> change (e.g., sound wave hitting water):

$$f_{vac.} \left(= \frac{c}{\lambda_{vac.}} \right) = f_{mat.} \left(= \frac{v_{mat.}}{\lambda_{mat.}} \right)$$

$$\Rightarrow \lambda_{mat.} < \lambda_{vac.}$$