## Lecture 8

Sinusoidal waves

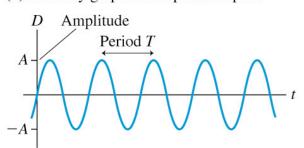
Wave speed on a string

• 2D/3D waves

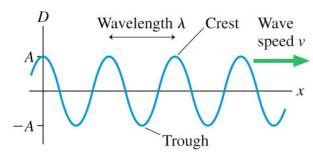
Sound and Light

#### (a) A history graph at one point in space

## Sinusoidal waves (graphical)



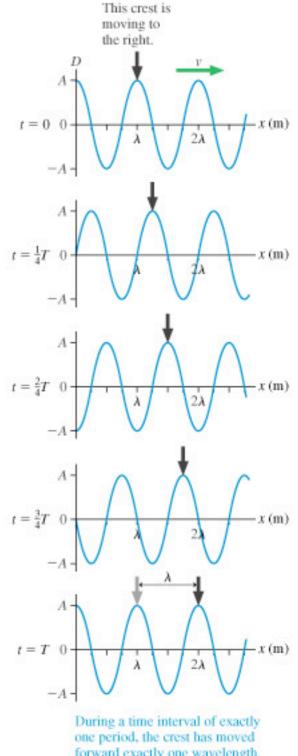
**(b)** A snapshot graph at one instant of time



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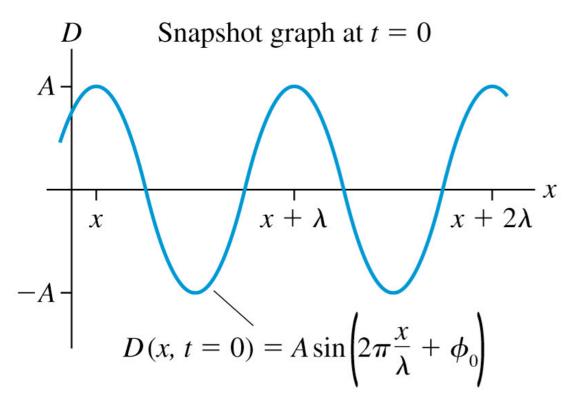
- generated by source in SHM
- snapshot and history graphs sinusoidal/periodic in space, time
- Wavelength ( $\lambda$ ): spatial analog of T, distance disturbance repeats
- In time T: (one oscillation for point) wave (crest) moves  $\lambda$

$$v = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T} = \lambda f$$



forward exactly one wavelength.

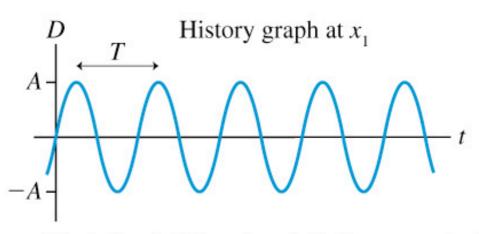
### Sinusoidal waves:mathematical



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### $D(x, t) = A\sin(kx - \omega t + \phi_0)$

(sinusoidal wave traveling in the positive *x*-direction)



If x is fixed,  $D(x_1, t) = A \sin(kx_1 - \omega t + \phi)$  gives a sinusoidal history graph at one point in space,  $x_1$ . It repeats every T s.

# Snapshot graph at $t_1$ A -A -ASnapshot graph at $t_1$ x

If t is fixed,  $D(x, t_1) = A \sin(kx - \omega t_1 + \phi)$  gives a sinusoidal snapshot graph at one instant of time,  $t_1$ . It repeats every  $\lambda$  m.

## • Set wave in motion by $x \rightarrow (x - vt)$

$$D(x,t) = A \sin \left[ 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) + \phi_0 \right]$$

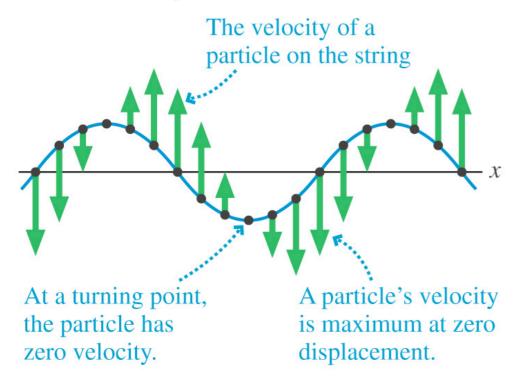
wave number, 
$$k = \frac{2\pi}{\lambda}$$
;  $\omega = \nu k$ 

 $\phi_0$  sets initial condition:

$$D(x=0, t=0) = A\sin\phi_0$$

## Waves on a string

The velocity of the wave —



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#### Newton's laws applied to string

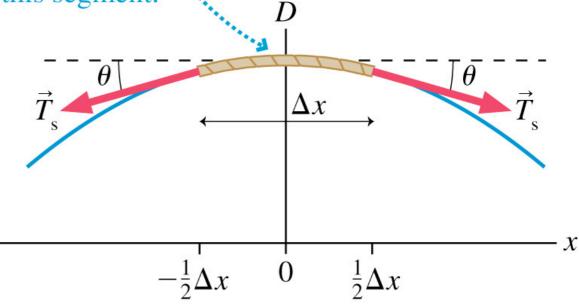
$$(F_{net})_y = ma_y = (\mu \Delta x) a_y$$
  
 $(F_{net})_y = 2T_s \sin \theta \approx -k^2 A T_s \Delta x$   
(evaluate slope of  $y = A \cos(kx)$ )

$$y(x,t) = A \sin(kx - \omega t + \phi_0)$$

$$v_y = -\omega A \cos(kx - \omega t + \phi_0)$$

$$a_y y = -\omega^2 A \sin(kx - \omega t + \phi_0)$$

A small segment of the string at the crest of the wave. Because of the curvature of the string, the tension forces exert a net downward force on this segment.



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$$\Rightarrow v = \sqrt{\frac{T_s}{\mu}}$$
 (

(independent of A/shape)

#### 2D/3D waves

Wave fronts are the crests of the wave. They are spaced one wavelength apart.

Source

N

The circular wave fronts move

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outward from the source at speed  $\nu$ .

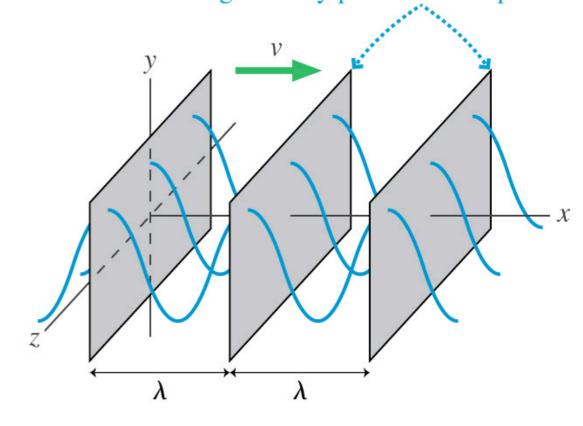
- 2D circular waves: wavefronts (lines locating crests), small section appear as straight lines far away
- 3D spherical waves...appear as planes far away, described by D(x, t) (same at every point in yz plane)

**(b)** 

Very far away from the source, small  $\lambda$   $\lambda$   $\lambda$  sections of the wave fronts appear to be straight lines.

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Very far from the source, small segments of spherical wave fronts appear to be planes. The wave is cresting at every point in these planes.



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## 2D/3D waves

$$D(r,t) = A(r)\sin(kr - \omega t + \phi_0)$$
  
with  $A(r)$  decreasing with  $r$ 

## Phase and phase difference

phase, 
$$\phi = kx - \omega t + \phi_0$$
  

$$D(x,t) = A\sin\phi$$

 wavefronts are surfaces of same displacement constant phase

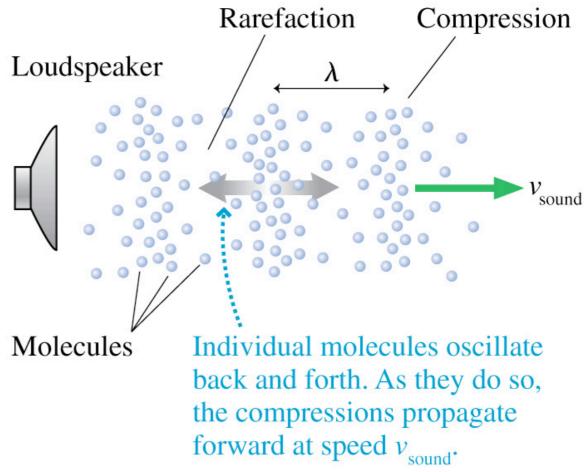
phase difference, 
$$\Delta \phi = 2\pi \frac{\Delta x}{\lambda}$$
  
 $\Delta \phi = 2\pi$  between adjacent wavefronts  
(separated by  $\lambda$ )

## Sound waves

 $v_{sound}$  in air at  $20^{\circ} = 343 \ m/s$  (larger in liquid/solid)

human ears: 20 Hz to20 k Hz

ultrasound: > 20 k Hz



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## Electromagnetic (EM) waves

 oscillations of EM field, can travel in vacuum e.g. light from stars

$$v_{light} = c = 3 \times 10^8 \ m/s \text{ in vacuum}$$
  
( $\gg v_{sound}$ )

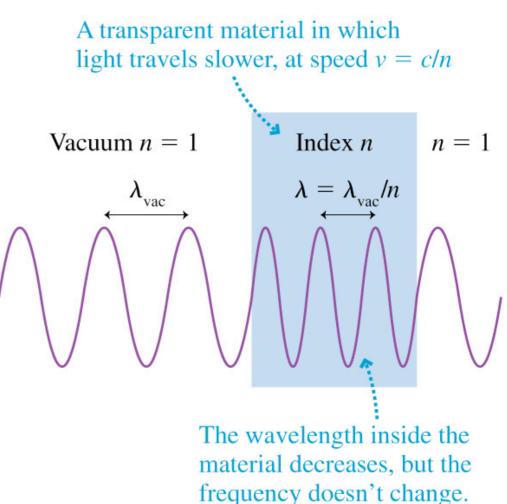
 visible spectrum: 400 nm (violet/blue to 700 nm (orange/red)

$$\ll \lambda_{sound} \Rightarrow f_{light} \gg f_{sound}$$

- EM spectrum: visible + higher frequencie
   (UV/X rays) + lower frequencies
   (IR/micro/radio waves)
- index of refraction (light slowed down):

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in material}} = \frac{c}{v}$$

frequency does <u>not</u> change (e.g.,  $f_{vac.}\left(=\frac{c}{\lambda_{vac.}}\right) = f_{mat.}\left(=\frac{v_{mat.}}{\lambda_{mat.}}\right)$  sound wave hitting water):  $\Rightarrow \lambda_{mat.} < \lambda_{vac.}$ 



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