Lecture 8

- Sinusoidal waves
- Wave speed on a string
- 2D/3D waves
- Sound and Light
Sinusoidal waves (graphical)

- generated by source in SHM
- snapshot and history graphs sinusoidal/periodic in space, time
- Wavelength ($\lambda$): spatial analog of $T$, distance disturbance repeats
- In time $T$: (one oscillation for point) wave (crest) moves $\lambda$

\[ v = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T} = \lambda f \]
### Sinusoidal waves: mathematical

**Set wave in motion by** \( x \rightarrow (x - vt) \)

\[
D(x, t) = A \sin \left[ 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) + \phi_0 \right]
\]

**Wave number**, \( k = \frac{2\pi}{\lambda}; \omega = \nu k \)

\( \phi_0 \) sets initial condition:

\[
D(x = 0, t = 0) = A \sin \phi_0
\]

**D(x, t) = A \sin (kx - \omega t + \phi_0)**

(sinusoidal wave traveling in the positive \( x \)-direction)

If \( x \) is fixed, \( D(x_1, t) = A \sin(kx_1 - \omega t + \phi) \) gives a sinusoidal history graph at one point in space, \( x_1 \). It repeats every \( T \) s.

If \( t \) is fixed, \( D(x, t_1) = A \sin(kx - \omega t_1 + \phi) \) gives a sinusoidal snapshot graph at one instant of time, \( t_1 \). It repeats every \( \lambda \) m.
Waves on a string

The velocity of the wave

The velocity of a particle on the string

At a turning point, the particle has zero velocity.

A particle’s velocity is maximum at zero displacement.

Newton’s laws applied to string

\[ y(x, t) = A \sin(kx - \omega t + \phi_0) \]
\[ v_y = -\omega A \cos(kx - \omega t + \phi_0) \]
\[ a_y = -\omega^2 A \sin(kx - \omega t + \phi_0) \]

\[ (F_{net})_y = m a_y = (\mu \Delta x) a_y \]
\[ (F_{net})_y = 2T_s \sin \theta \approx -k^2 AT_s \Delta x \]

(evaluate slope of \( y = A \cos(kx) \))

\[ \Rightarrow v = \sqrt{\frac{T_s}{\mu}} \quad \text{(independent of A/shape)} \]
2D/3D waves

(a) Wave fronts are the crests of the wave. They are spaced one wavelength apart.

![Diagram of 2D circular waves]

The circular wave fronts move outward from the source at speed v.

(b) Very far away from the source, small sections of the wave fronts appear to be straight lines.

![Diagram of 3D spherical waves]

Very far from the source, small segments of spherical wave fronts appear to be planes. The wave is cresting at every point in these planes.

- 2D circular waves: *wavefronts* (lines locating crests), small section appear as straight lines far away

- 3D spherical waves...appear as planes far away, described by D(x, t) (same at every point in yz plane)
2D/3D waves

\[ D(r, t) = A(r) \sin (kr - \omega t + \phi_0) \]
with \( A(r) \) decreasing with \( r \)

Phase and phase difference

phase, \( \phi = kx - \omega t + \phi_0 \)

\[ D(x, t) = A \sin \phi \]

- wavefronts are surfaces of same displacement constant phase

phase difference, \( \Delta \phi = 2\pi \frac{\Delta x}{\lambda} \)
\( \Delta \phi = 2\pi \) between adjacent wavefronts (separated by \( \lambda \))
Sound waves

\[ v_{\text{sound}} \text{ in air at } 20^\circ = 343 \text{ m/s} \]
(larger in liquid/solid)

- human ears: 20 Hz to 20 k Hz

- ultrasound: > 20 k Hz
Electromagnetic (EM) waves

- Oscillations of EM field, can travel in vacuum
e.g. light from stars
  \[ v_{\text{light}} = c = 3 \times 10^8 \text{ m/s in vacuum} \]
  \( \gg v_{\text{sound}} \)

- Visible spectrum: 400 nm (violet/blue) to 700 nm (orange/red)

  \[ \ll \lambda_{\text{sound}} \Rightarrow f_{\text{light}} \gg f_{\text{sound}} \]

- EM spectrum: visible + higher frequencies (UV/X rays) + lower frequencies (IR/micro/radio waves)

- Index of refraction (light slowed down):
  \[ n = \frac{\text{speed of light in vacuum}}{\text{speed of light in material}} = \frac{c}{v} \]

Frequency does not change (e.g., sound wave hitting water):
  \[ f_{\text{vac.}} \left( = \frac{c}{\lambda_{\text{vac.}}} \right) = f_{\text{mat.}} \left( = \frac{v_{\text{mat.}}}{\lambda_{\text{mat.}}} \right) \]

\[ \Rightarrow \lambda_{\text{mat.}} < \lambda_{\text{vac.}} \]