Lecture 33

• electric potential energy: uniform field; point charges; dipole

• electric potential
Electric Potential Energy in uniform field

The gravitational field does work on the particle. We can express the work as a change in gravitational potential energy.

The electric field does work on the particle. We can express the change in electric potential energy as a change in the electric potential energy.

Analogy with gravitational field:

\[ W_{grav} = w \Delta r \cos 0^\circ = mg (y_i - y_f) \]
\[ \Delta U_{grav} = -W_{grav} (i \rightarrow f) \Rightarrow U_{grav} = U_0 + mgy \]

Work done on charge by E

\[ W_{elec} = F \Delta r \cos 0^\circ = qE (s_i - s_f) \ldots \Rightarrow \]
\[ U_{elec} = U_0 + qEs \]
(potential energy of charge q in a uniform electric field)
Potential energy of Point Charges

- **Analogy with mass-spring system:**

\[
W_{sp} = \int_{x_i}^{x_f} F \, dx = -k \int_{x_i}^{x_f} x \, dx \ldots \Rightarrow U_{sp} = \frac{1}{2} k x^2
\]

- **Work done by** \( F_{1 \text{ on } 2} \) \( \text{on} \) \( q_2 \)

\[
W_{elec} = \int_{x_i}^{x_f} F_{1 \text{ on } 2} \, dx = K q_1 q_2 \int_{x_i}^{x_f} \frac{1}{x^2} = K q_1 q_2 \left( -\frac{1}{x_f} + \frac{1}{x_i} \right) \ldots \Rightarrow
\]

\[
U_{elec} = \frac{K q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r} \quad \text{(two point charges)} \quad \text{(positive for like charges...)}
\]
Potential energy of point charges II

- Charges released from rest will move in the direction of decreasing potential energy (like charges move apart...)
- Two like charges shot towards each other reach minimum separation
- Two opposite charges shot apart... slow down... reach maximum...
Potential energy of point charges III

- Electric force is conservative: work done independent of path (potential energy can be defined): divide path into...

- $U_{elec} \rightarrow 0 \text{ as } r \rightarrow \infty$: think of zero as “no interaction”

- negative energies: less energy than for infinitely far apart, at rest

- $E_1 < 0 \text{ (bound system)}; E_2 > 0 \text{ (can escape, barely for } E = 0)$
Potential Energy of Multiple Charges/Dipole

- each pair counted once:

\[ U_{elec} = \sum_{i<j} \frac{K q_i q_j}{r_{ij}} \]

(a)

\[ dW_+ = \vec{F}_+ \cdot d\vec{s} = F_+ ds \cos \theta \]

with \( ds = r d\phi = \left( \frac{1}{2} d \right) d\phi \) (pivot at center of dipole)...  

\[ W_{elec} = -pE \int_{\phi_i}^{\phi_f} \sin \phi d\phi = pE (\cos \phi_f - \cos \phi_i) \]

\[ U_{dipole} = -pE \cos \phi = -\vec{p} \cdot \vec{E} \]

- work done as dipole rotates:

(b)  

Turning points for oscillation with energy \( E_{mech} \)  

Stable equilibrium at \( \theta = 0^\circ \)  

Unstable equilibrium at \( \theta = \pm 180^\circ \)
Electric Potential

- Analogy with $E$

  force on $q = [\text{charge } q] \times [\text{alteration of space by source}]

  potential energy of $q + \text{sources} = [\text{charge } q] \times [\text{potential for interaction of sources}]

- $q$ does “separate out”: $V \equiv \frac{U_{q+\text{sources}}}{q}$

  $U_{q+\text{sources}} = qV$

- Units: 1 volt (V) = 1 J/C

- $V$ depends only on sources: ability to have interaction if charge $q$ “shows up”

- source charges exert influence via $V$: “ignore” sources once $V$ is known
Using Electric Potential

- If \( V \) is changing, particle slows down...
- potential difference (voltage): \( \Delta V = V_f - V_i \)

A proton with speed \( 2 \times 10^{-5} \) m/s passes thru’ a region with potential difference of 100 V. What is its final speed?

\[
K_f - K_i + q\Delta V = 0 \quad \text{gives} \quad \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - q\Delta V \rightarrow \\
v_f = \sqrt{v_i^2 - \frac{2q}{m}\Delta V} = 1.44 \times 10^{-5} \text{ m/s}
\]