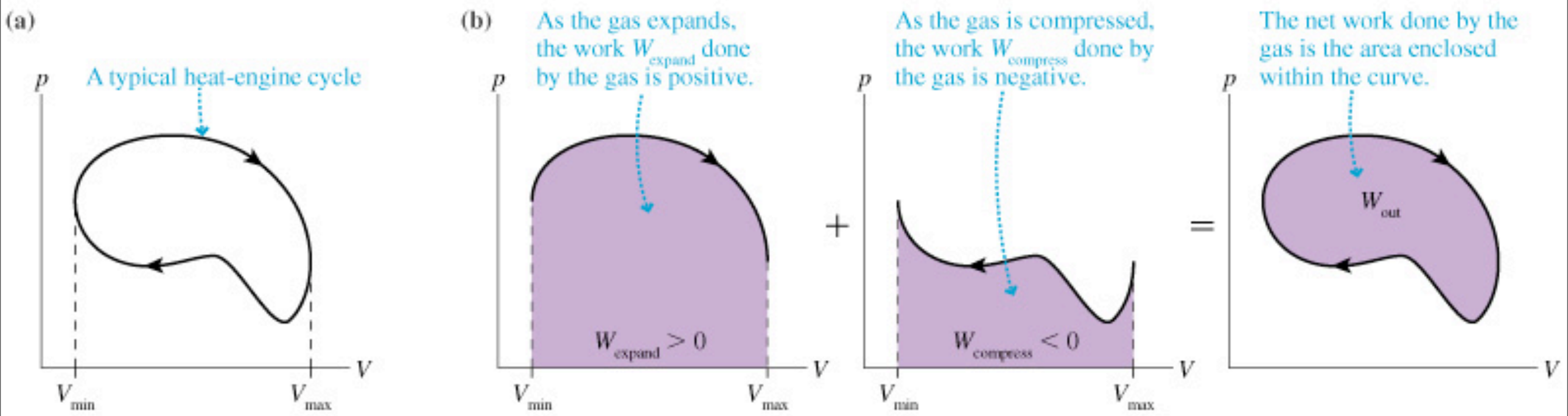
Lecture 21

- heat engines and refrigerators using ideal gas as working substance
- Brayton cycle

Ideal gas Heat Engines



- **closed cycle trajectory:** clockwise for $W_{out} > 0$

$$W_{out} = W_{expand} - |W_{compress}| = \text{area } \textit{inside} \text{ closed curve}$$

Ideal gas summary I

TABLE 19.1 Summary of ideal-gas processes

Process	Gas law	Work W_s	Heat Q	Thermal energy
Isochoric	$p_i/T_i = p_f/T_f$	0	$nC_V\Delta T$	$\Delta E_{th} = Q$
Isobaric	$V_i/T_i = V_f/T_f$	$p\Delta V$	$nC_P\Delta T$	$\Delta E_{th} = Q - W_s$
Isothermal	$p_iV_i = p_fV_f$	$nRT\ln(V_f/V_i)$ $pV\ln(V_f/V_i)$	$Q = W_s$	$\Delta E_{th} = 0$
Adiabatic	$p_iV_i^\gamma = p_fV_f^\gamma$	$(p_fV_f - p_iV_i)/(1-\gamma)$	0	$\Delta E_{th} = -W_s$

Ideal gas summary II

- E_{th} depends only on T

TABLE 19.2 Properties of monatomic and diatomic gases

	Monatomic	Diatomic
E_{th}	$\frac{3}{2}nRT$	$\frac{5}{2}nRT$
C_V	$\frac{3}{2}R$	$\frac{5}{2}R$
C_P	$\frac{5}{2}R$	$\frac{7}{2}R$
γ	$\frac{5}{3} = 1.67$	$\frac{7}{5} = 1.40$

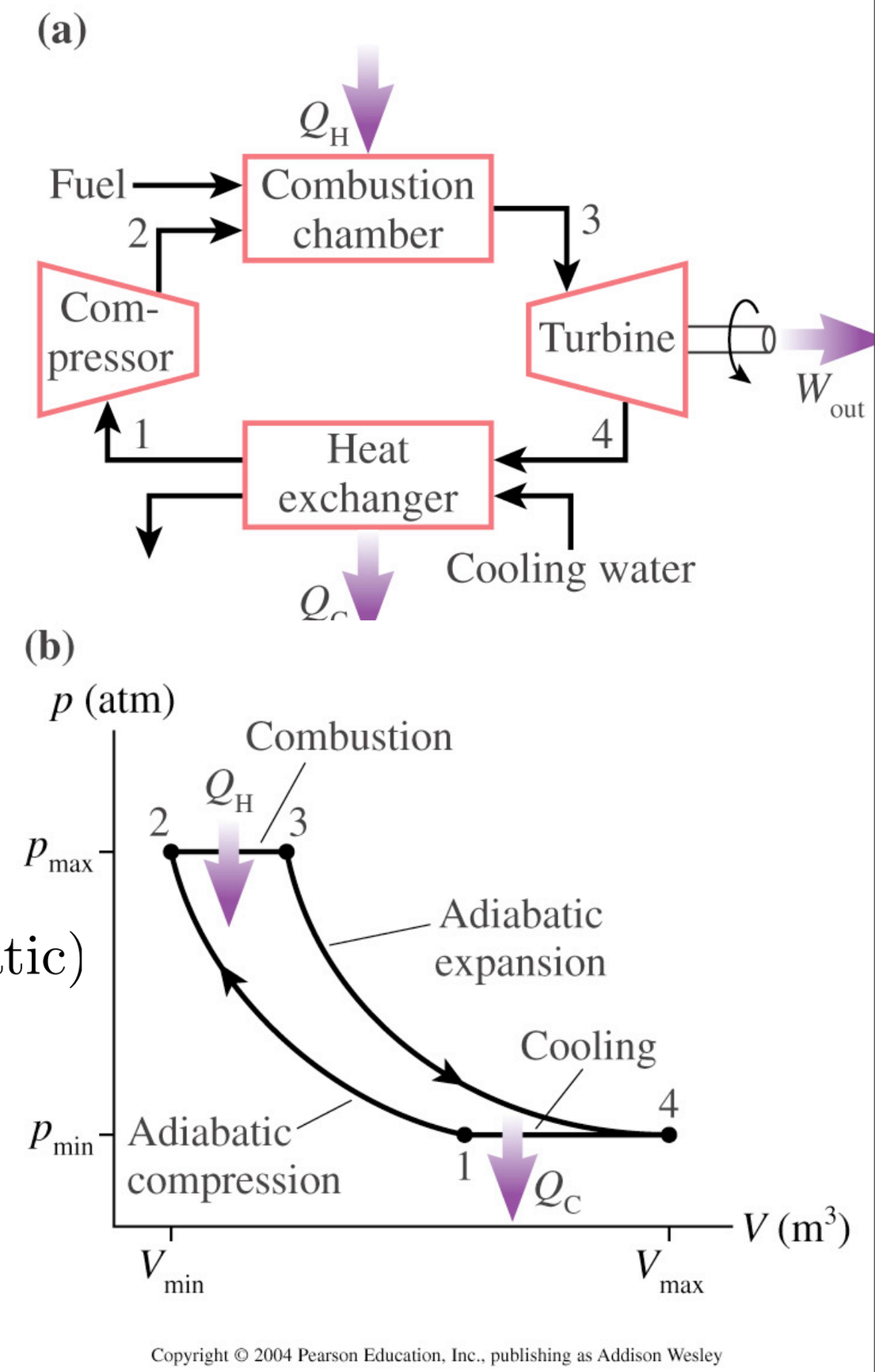
Strategy for heat engine problems

- identify each process, draw pV diagram
- use ideal gas law to know n, p, V, T at one point
- use ideal gas law and equations for specific processes for p, V, T at beginning/end of each process
- calculate Q, W_s and ΔE_{th} for each process
- W_{out} by adding W_s`s: confirm by area within curve
- add positive values of Q to find Q_H
- check: $(\Delta E_{th})_{net} = 0$, $\eta < 1$, signs of W_s and Q...

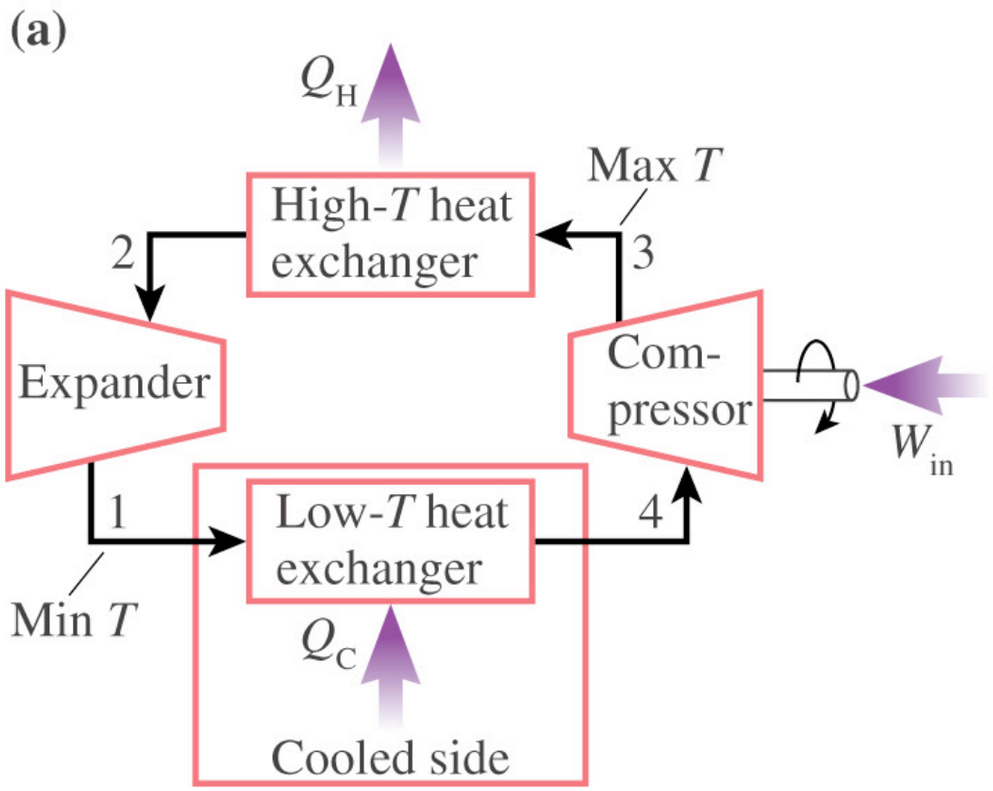
Brayton cycle (heat engine)

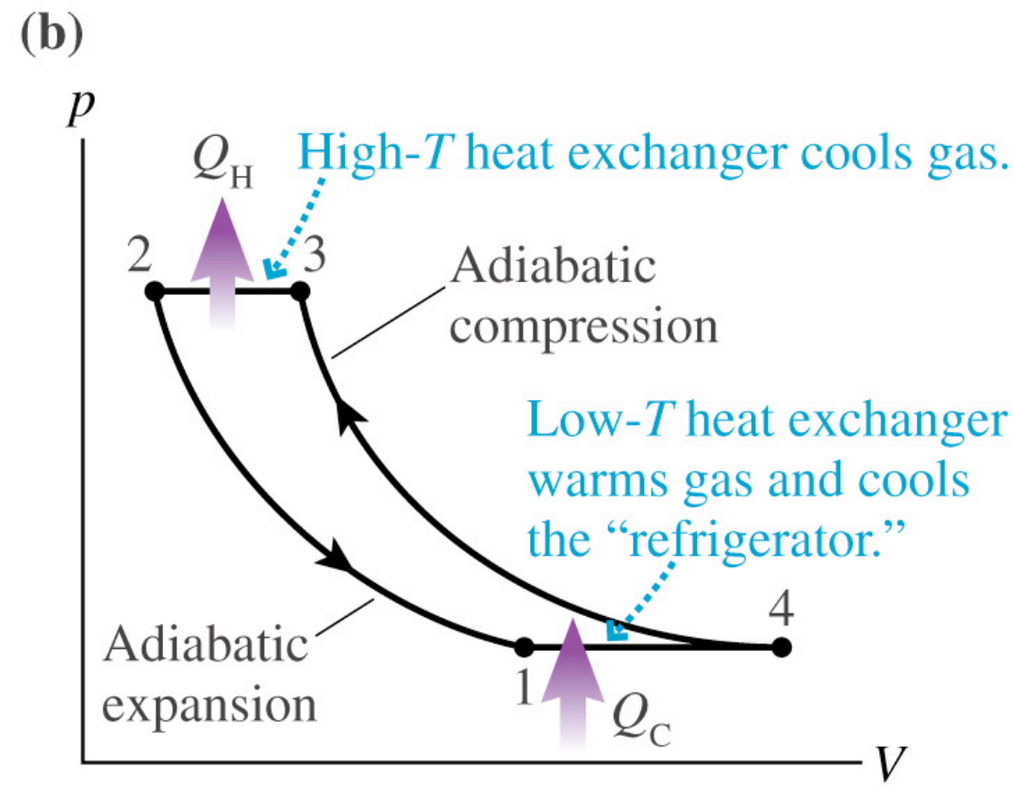
- adiabatic compression (1→2): raises T; isobaric expansion (2→3): raises T further, heat by fuel; adiabatic expansion (3→4): spins turbine, T still high; isobaric expansion (4→1): heat transferred to cooling fluid
  $T_H \geq T_3$; $T_C \leq T_1$
- Thermal efficiency: $\eta = 1 - \frac{Q_C}{Q_H}$

Process $2 \rightarrow 3$ (isobaric):
$Q_H = nC_P\,(T_3 - T_2)$
Process $4 \rightarrow 1$ (isobaric):
$Q_C = |Q_{41}| = nC_P\,(T_4 - T_1)$
$\Rightarrow \eta_B = 1 - \frac{T_4 - T_1}{T_3 - T_2}$
Use $pV = nRT$ and $pV^\gamma =$ constant (adiabatic) to give $p^{(1-\gamma)/\gamma}T =$ constant
$\Rightarrow T_1 = T_2 \left(\frac{p_2}{p_1}\right)^{(1-\gamma)/\gamma} = T_2 r_p^{(1-\gamma)/\gamma}$,
where $r_p \equiv \frac{p_{max}}{p_{min}}$ and $T_4 = T_3 r_p^{(1-\gamma)/\gamma}$
$\Rightarrow \eta_B = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}}$ (increases with r_p)

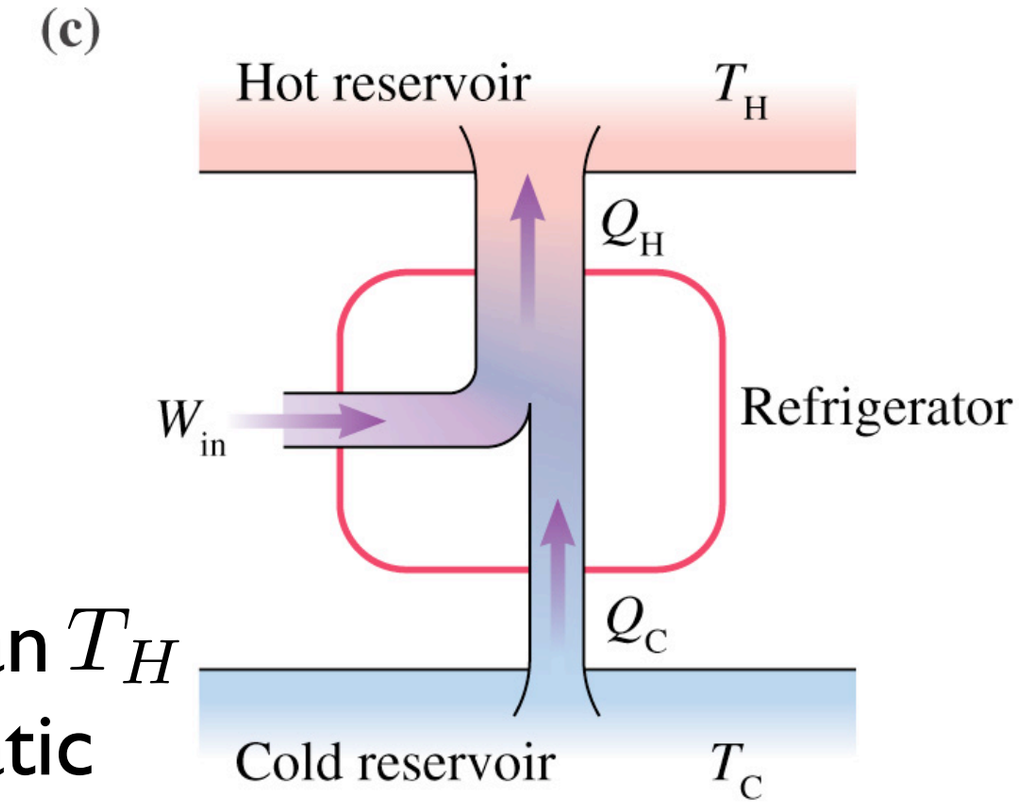


Brayton cycle (refrigerator)





- heat engine backward, ccw in pV: low-T heat exchanger is "refrigerator"
- sign of W reversed, area inside curve is W_{in}: used to extract Q_C from cold reservoir and exhaust Q_H to hot...
- gas T lower than T_C (1 → 4), higher than T_H (3 → 2) ⇒ gas must reach T_C by adiabatic expansion, T_H by adiabatic compression



Comparison of Brayton cycle heat engine and refrigerator

- Brayton cycle refrigerator is <u>not</u> simply heat engine run backward, must <u>change hot and cold reservoir</u>: heat transferred <u>into</u> <u>cold</u> reservoir for <u>heat engine</u> $(T_C \leq T_1)$, from cold reservoir in refrigerator $(T_C \geq T_4)$; heat transferred <u>from</u> <u>hot</u> reservoir for <u>heat engine</u> $(T_H \geq T_3)$, into hot reservoir for refrigerator $(T_H \leq T_2)$

- <u>heat engine</u>: heat transfer from hot to cold is spontaneous, extract useful work in this process via system...

- <u>refrigerator</u>: heat transfer from cold to hot not spontaneous, make it happen by doing work via system...