Lecture 21

- heat engines and refrigerators using ideal gas as working substance
- Brayton cycle
 Ideal gas Heat Engines

- closed cycle trajectory: clockwise for $W_{out} > 0$

$$W_{out} = W_{\text{expand}} - |W_{\text{compress}}| = \text{area inside closed curve}$$

### Ideal gas summary I

<table>
<thead>
<tr>
<th>Process</th>
<th>Gas law</th>
<th>Work $W_s$</th>
<th>Heat $Q$</th>
<th>Thermal energy $\Delta E_{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isochoric</td>
<td>$p_i/T_i = p_f/T_f$</td>
<td>0</td>
<td>$nC_V \Delta T$</td>
<td>$\Delta E_{th} = Q$</td>
</tr>
<tr>
<td>Isobaric</td>
<td>$V_i/T_i = V_f/T_f$</td>
<td>$p \Delta V$</td>
<td>$nC_p \Delta T$</td>
<td>$\Delta E_{th} = Q - W_s$</td>
</tr>
<tr>
<td>Isothermal</td>
<td>$p_iV_i = p_iV_f$</td>
<td>$nRT \ln \left( \frac{V_f}{V_i} \right)$</td>
<td>$Q = W_s$</td>
<td>$\Delta E_{th} = 0$</td>
</tr>
<tr>
<td>Adiabatic</td>
<td>$p_iV_i^{\gamma} = p_fV_f^{\gamma}$</td>
<td>$(p_iV_f - p_fV_i)/(1 - \gamma)$</td>
<td>0</td>
<td>$\Delta E_{th} = -W_s$</td>
</tr>
</tbody>
</table>
Ideal gas summary II

• $E_{th}$ depends only on T

Strategy for heat engine problems

• identify each process, draw pV diagram

• use ideal gas law to know n, p, V, T at one point

• use ideal gas law and equations for specific processes for p, V, T at beginning/end of each process

• calculate $Q$, $W_s$ and $\Delta E_{th}$ for each process

• $W_{out}$ by adding $W_s$'s: confirm by area within curve

• add positive values of Q to find $Q_H$

• check: $(\Delta E_{th})_{net} = 0$, $\eta < 1$, signs of $W_s$ and $Q$...

<table>
<thead>
<tr>
<th>TABLE 19.2 Properties of monatomic and diatomic gases</th>
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</thead>
<tbody>
<tr>
<td><strong>Monatomic</strong></td>
</tr>
<tr>
<td>$E_{th}$</td>
</tr>
<tr>
<td>$C_V$</td>
</tr>
<tr>
<td>$C_P$</td>
</tr>
<tr>
<td>$\gamma$</td>
</tr>
</tbody>
</table>
**Brayton cycle (heat engine)**

- adiabatic compression (1→2): raises T; isobaric expansion (2→3): raises T further, heat by fuel; adiabatic expansion (3→4): spins turbine, T still high; isobaric expansion (4→1): heat transferred to cooling fluid
  
  \[ T_H \geq T_3; \; T_C \leq T_1 \]

- **Thermal efficiency:** \( \eta = 1 - \frac{Q_C}{Q_H} \)

**Process 2 \rightarrow 3** (isobaric):

\[ Q_H = nC_P (T_3 - T_2) \]

**Process 4 \rightarrow 1** (isobaric):

\[ Q_C = |Q_{41}| = nC_P (T_4 - T_1) \]

\[ \Rightarrow \eta_B = 1 - \frac{T_4 - T_1}{T_3 - T_2} \]

Use \( pV = nRT \) and \( pV^\gamma = \text{constant (adiabatic)} \) to give \( p^{(1-\gamma)/\gamma}T = \text{constant} \)

\[ \Rightarrow T_1 = T_2 \left( \frac{p_2}{p_1} \right)^{\frac{1-\gamma}{\gamma}} = T_2 r_p^{\frac{1-\gamma}{\gamma}} \]

where \( r_p \equiv \frac{p_{\text{max}}}{p_{\text{min}}} \) and \( T_4 = T_3 r_p^{\frac{1-\gamma}{\gamma}} \)

\[ \Rightarrow \eta_B = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \] (increases with \( r_p \))
Brayton cycle (refrigerator)

- heat engine backward, ccw in pV: low-T heat exchanger is “refrigerator”
- sign of $W$ reversed, area inside curve is $W_{in}$: used to extract $Q_C$ from cold reservoir and exhaust $Q_H$ to hot...
- gas $T$ lower than $T_C$ (1 $\rightarrow$ 4), higher than $T_H$ (3 $\rightarrow$ 2) $\Rightarrow$ gas must reach $T_C$ by adiabatic expansion, $T_H$ by adiabatic compression
Comparison of Brayton cycle heat engine and refrigerator

- Brayton cycle refrigerator is not simply heat engine run backward, must change hot and cold reservoir: heat transferred into cold reservoir for heat engine \((T_C \leq T_1)\), from cold reservoir in refrigerator \((T_C \geq T_4)\); heat transferred from hot reservoir for heat engine \((T_H \geq T_3)\), into hot reservoir for refrigerator \((T_H \leq T_2)\)

- heat engine: heat transfer from hot to cold is spontaneous, extract useful work in this process via system...

- refrigerator: heat transfer from cold to hot not spontaneous, make it happen by doing work via system...