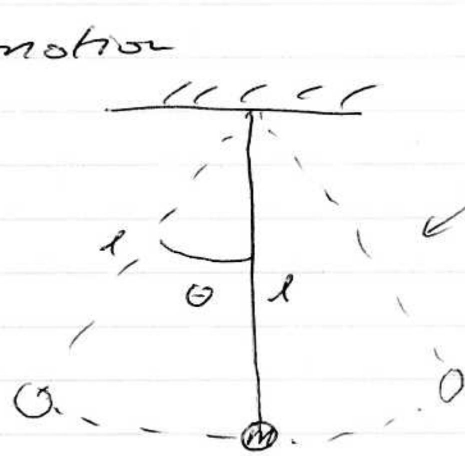
Pendular motion



assume string is massless

This really a rotation & torque problem.

The mass swing through an angle θ a constant distance l from the pivot point

The relevant equation of motion can be found through

$$\tau = \overset{\dot{\Omega}}{\cancel{I\dot{\omega}}} = I\frac{d^2\theta}{dt^2}$$

$\dot{\Omega}$ is angular acceleration

Since this is an oscillation we will posit a proportional restoring torque

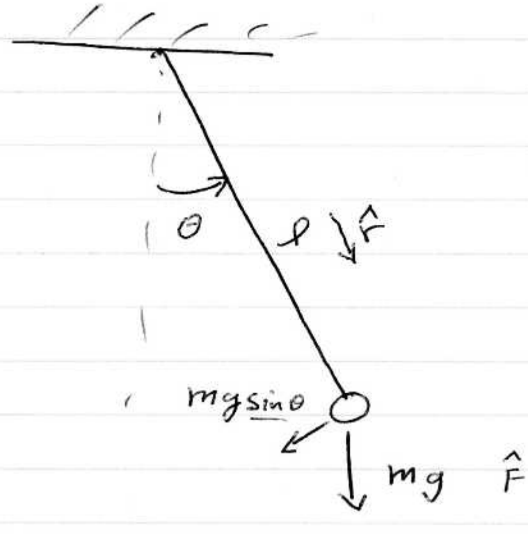
$$\tau_{restoring} = -\kappa_{eff}\,\theta$$

for small angle

in this case

$$\omega = \sqrt{\frac{\kappa_{eff}}{I}}$$

then solutions would be the same as we had for the linear Harmonic oscillator



positive θ rotation $\hat{\theta}$ out of page

$$I \frac{d^2\theta}{dt^2}\hat{\theta} = \vec{\tau} = \underbrace{\vec{r} \times \vec{F}}_{-l\,mg\sin\theta\,\hat{\theta}}$$

($\hat{\theta}$ into page)

$$\frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\sin\theta$$

Now this will conform to our requirement for SHM if we can replace $\sin\theta$ by θ

Small angles $\quad \sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$

small θ $\quad \sin\theta \simeq \theta$

Note $\quad \cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots$

$$\frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta$$

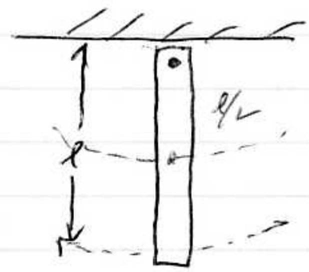
$$I = ml^2$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

$$\boxed{\Rightarrow \quad \theta(t) = \theta_0 \cos(\omega t + \phi) \qquad \omega = \sqrt{\frac{g}{l}} \quad \text{No dependence on } m!}$$

In the cases we have considered the string or wire was considered massless so all mass at the end of string

What if we have a swinging plank



For Uniform plank CM in center

now what?

We have to determine the correct

I and apply to CM.

Think about the case when the pivot point may be somewhere else.

Energy Considerations in SHM

~~Back in Ch 10 of the text the energy of~~

It is clear that stretching or compressing a spring requires work on the spring. This was discussed back in Ch 10. By the same token when the spring relaxes back to its equilibrium length it does work on the mass. Thus it is possible to store Energy in a spring

$$W = \int_{x_1}^{x_2} F \cdot dx$$

where $\vec{x} \parallel \vec{F}$

Let's calculate for ~~Hook~~ our restoring force
To stretch the spring, we must exert a force $k\Delta x$

Formally we can write

$$W = \int_{x_1}^{x_2} +kx\, dx = \frac{1}{2}k x_2 - \frac{1}{2}k x_1$$

This is the work we do on the spring

3/4

If we let the spring relax while holding one end. We then do negative work. (Newton 3rd law)

W done by Spring

$$\underbrace{W_{el}}\ = \frac{1}{2} k_1 x_1^2 - \frac{1}{2} k x_i^2$$

This is the elastic work (no heat involved)

As was done w/ gravitational field from ~~to~~ W_{el}
We can define a potential energy that describes the state of the system

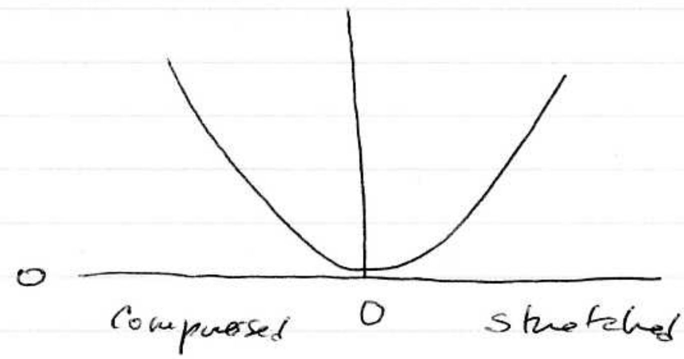
$$\underline{U = \frac{1}{2} k x^2}$$

In this case $U = 0$ is when $x = 0$ at equilibrium

potential energy of the spring

units J or N·m

~~As w/ grav. potential energies we are free to define where to take zero~~



$$\Rightarrow \underbrace{W_{el} = -\Delta U}$$

The work the spring does on the mass causes it to loose energy!

Work - Energy theorem

$W_{tot} = K_2 - K_1$ change in kinetic energy

$W_{tot} = W_{el} = U_1 - U_2$ if the elastic force is the only force acting on mass

$K_2 - K_1 = U_1 - U_2$

or

$K_1 + U_1 = K_2 + U_2$

$\frac{1}{2} m v_1^2 + \frac{1}{2} k_1 x_1^2 = \frac{m}{2} v_2^2 + \frac{1}{2} k_1 x_2^2$

$\mathcal{E}$ = conserved or const.

$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

all may be potential or kinetic

$\mathcal{E} = \frac{1}{2}kx_0^2$

↑ max stretch/compression.

Damped Oscillation

$$F = -kx \underbrace{-bv_x}$$

$\underbrace{\text{drag or friction}}$

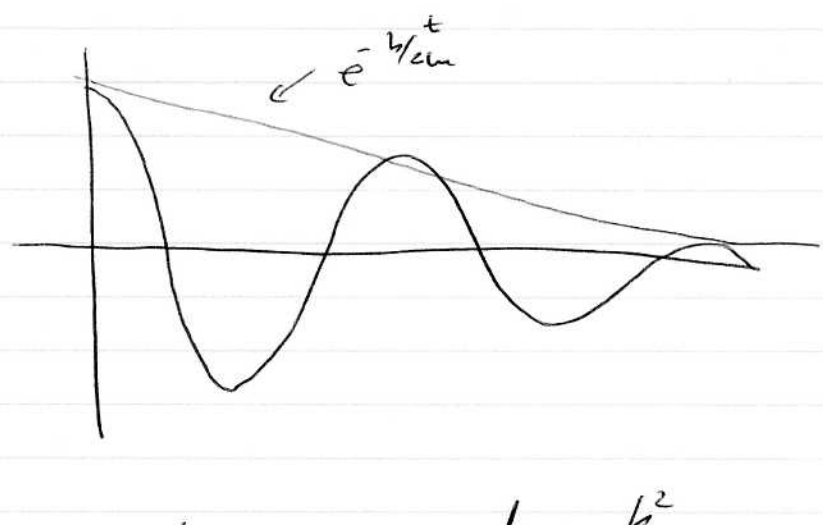
causes Energy to be lost

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$$x(t) = A e^{-\frac{b}{2m}t}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$e^{-\frac{b}{2m}t}$ causes oscillator to damp out in time

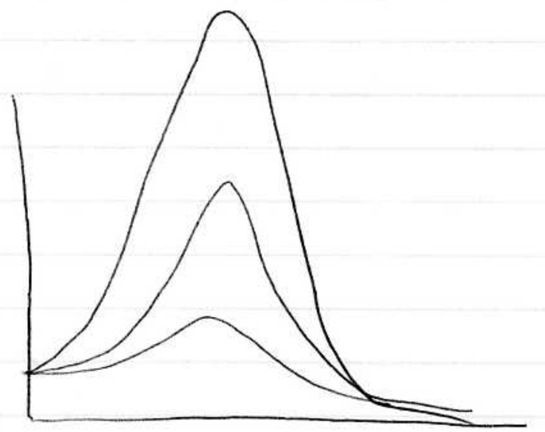


critical damping $\frac{k}{m} = \frac{b^2}{4m^2}$

$\omega' = 0$ No oscillation

principle use in Shock absorbers in Cars

Forced oscillation



Something to be avoided when building structure like bridges

Famous Tacoma Narrow bridge

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_d)^2 + b\omega_d^2}}$$

$\omega_d = \sqrt{\frac{k}{m}}$ resonant frequency.