

Simple Harmonic Motion

- Many forces in nature are either time or position dependent rather than constant such as gravity

Such forces give rise to oscillatory motion, in which objects move back and forth.

Demo G1-52

- These oscillations appear all around us in nature and in many engineering applications.

In this section our goal is

1. Analyze the kinematics of oscillatory motion

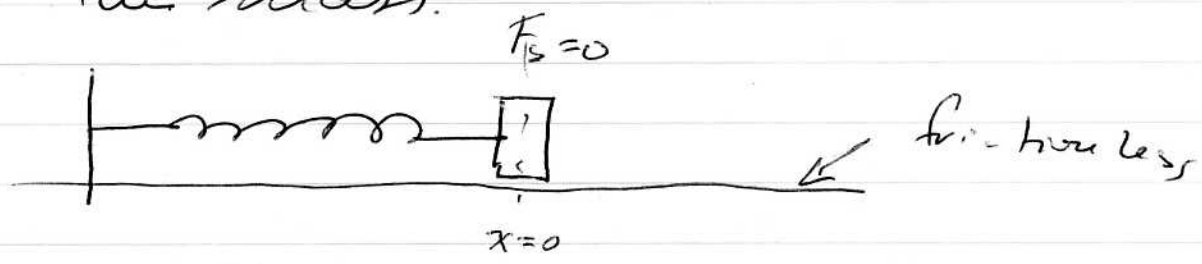
2. Examine the forces that give rise to linear oscillations

3. Examine the torques that give rise to rotational oscillations

4. Consider energy conservation as applied to oscillations

Demo 91-35 Mass on Spring

Lets consider the forces acting on this mass.



Sliding on a table

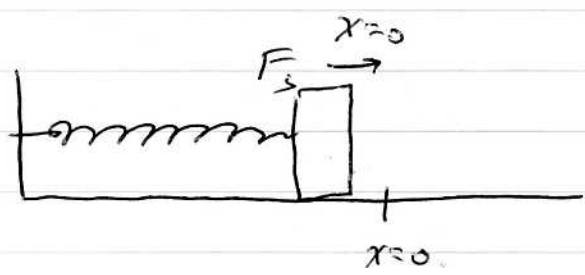
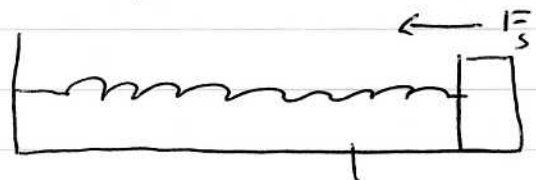
initially the mass is at rest and the spring is at its equilibrium length

→ What are the forces on the block?

None - other wise block would move!

↓ gravity down but table up.

Now lets move the mass to the right so that $x > 0$



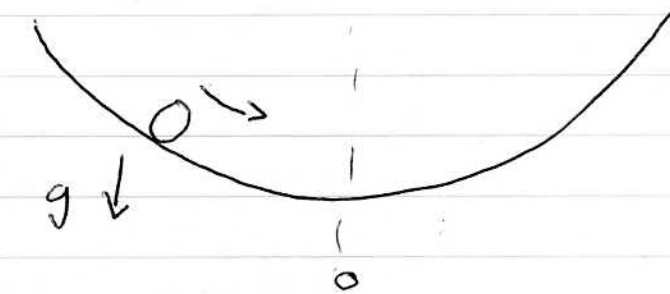
Spring ~~exerts~~ exerts a force $F_s < 0$ (i.e., to the left!)

Note regardless of which way we move the mass away from its equilibrium location the force due to the spring is always in the opposite direction.

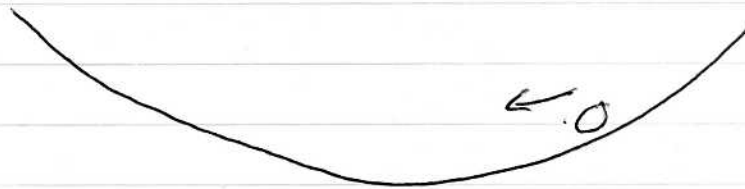
This force is called a restoring force.

Restoring Force - It always pushes or pulls a displaced object back to its equilibrium position. At the equilibrium position it exerts No Force.

Back to §1-52



The component of g || to surface acts as the restoring force.



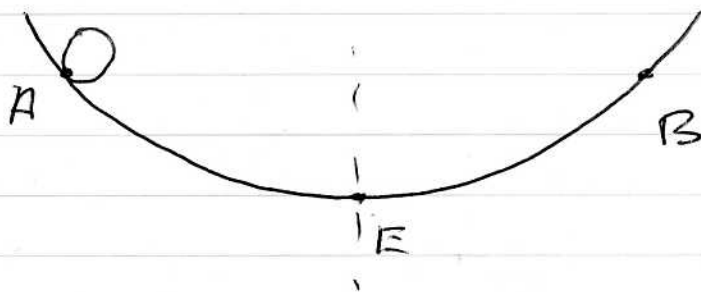
Note the further the ball is displaced from the bottom the larger the force.

Important!

the term "restoring force" does not refer to a new type of force like gravity or friction or the spring force, rather it indicates what the force does.

- It is ~~pretty~~ pretty clear to see how this restoring force leads to oscillatory motion.

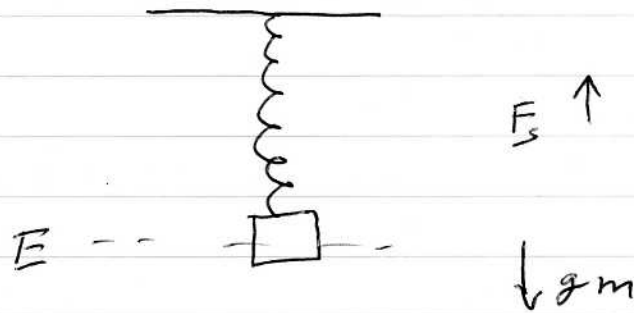
~~When ball rolls down~~



- At A the restoring force accelerates the ball back towards E
- When it reaches E it is in motion and its inertia (its tendency to remain in motion) keeps it moving through E, since there are no forces acting on it at E. Eventually the ball rolls up to B where it stops momentarily and reverses and rolls back down.

- Again in presence of F and oscillation is established
- If it weren't for friction this oscillation would continue endlessly.

Now let's consider G1-35 Vertical spring



Now we have stable k for F_s is \propto the displacement but in the opposite direction

$$F_{\text{restor}} = -kx$$

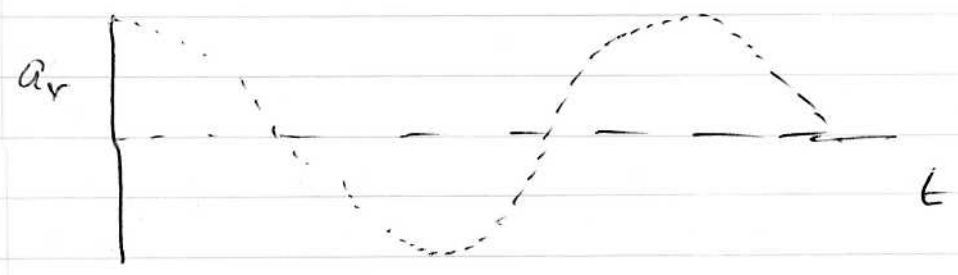
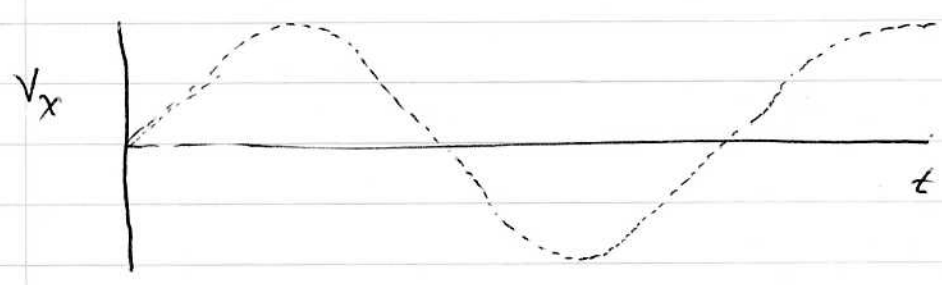
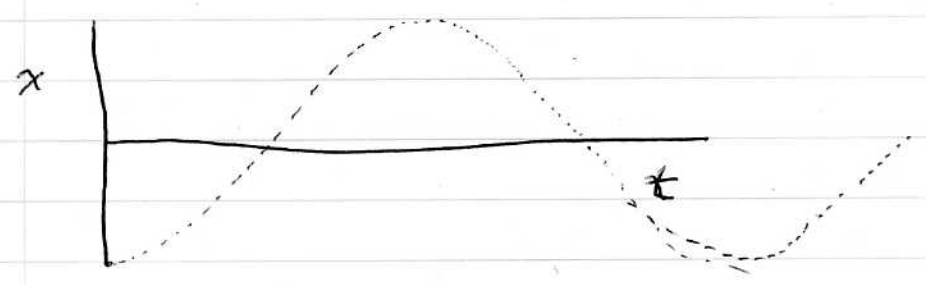
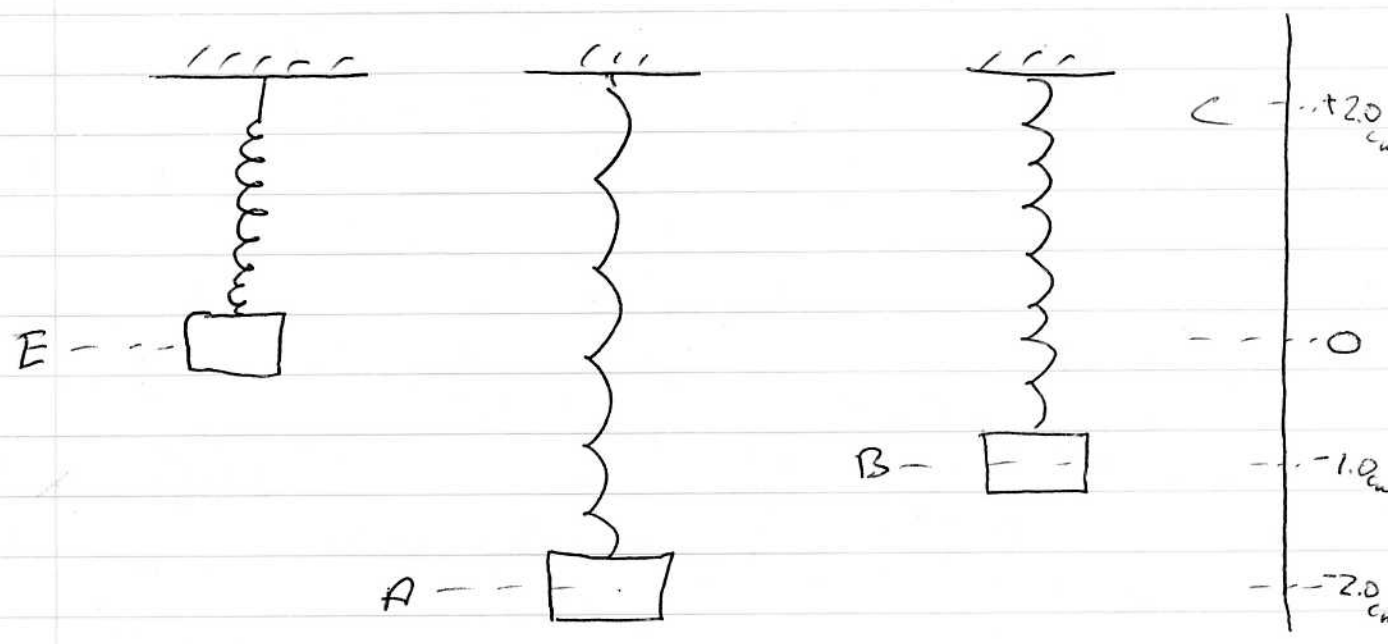
When k is a constant, and considered > 0 positive

In the case of the ball on the track, if the ball were sliding instead of rolling, the

$$k = \frac{mg}{R}$$

where R = radius of curvature of the track.

6/1/19



Let's summarize what we've learned.

- The mass gains speed at a decreasing rate as it approaches the equilibrium position E .
- It loses speed at an increasing rate when going away from E .

This allowed us to conclude the $x(t)$ looks like (-cosine) curves.

At the same time $\sin \Rightarrow$

$$v = \frac{dx(t)}{dt}$$

$\Rightarrow v(t)$ is a sine curve

$$a = \frac{dv(t)}{dt}$$

$\Rightarrow a(t) = \text{cosine curve}$

Can apply the same reasoning to
pendulum.

Show 61-17

- Characterizing the motion

Amplitude: size of the oscillation

Period: the time required for one complete oscillation

frequency: reciprocal of the period

units frequency Hz, cycles, s⁻¹

- Dependence of period on Amplitude

⇒ If we do the experiment

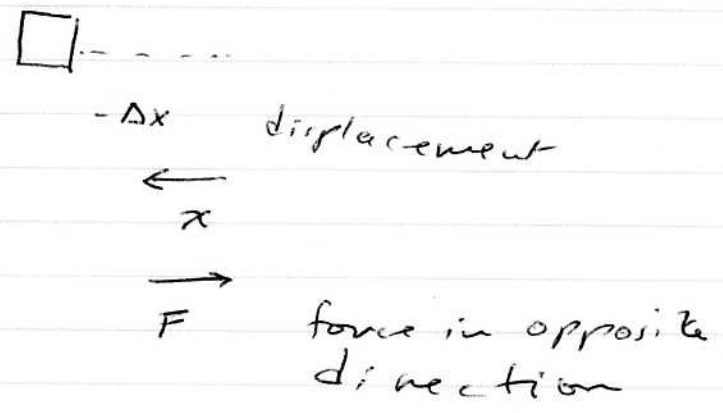
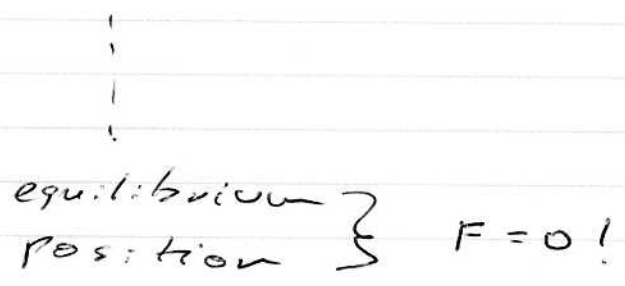
Swing of pendulum or oscillation of the mass on spring does not depend on Amplitude!

* True for small oscillations! *

09/2/09 Review

Restoring Force

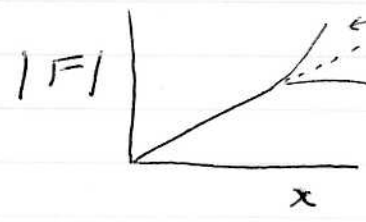
- Not a new force but indicates what the force does



We will focus on the linear problem

$$F = -kx \quad \text{Hooke's Law}$$

↑
Constant



Force becomes more linear
e.g., spring breaks ~~for~~

Snap shots of simple harmonic motion

