Review 9/10/08

General Solution to SHM

$$x(t) = x_0 \cos(\omega t + \phi)$$

$$\omega = \sqrt{k/m}$$

$$\theta(t) = \theta_0 \cos(\omega t + \phi)$$

$$\omega = \sqrt{g/l}$$

Small angle approximation

$$\sin\theta \approx \theta - \frac{\theta^3}{3!} + \dots$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2!} + \dots$$

$$e^{i\theta} \approx 1 + i\theta + (i)^2 \frac{\theta^2}{2!} + (i)^3 \frac{\theta^3}{3!} + \dots$$

$$1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \dots$$

$$\Rightarrow e^{i\theta} = \cos\theta + i\sin\theta$$

Energy

$$\mathcal{E} = \frac{1}{2} m \omega^2 x_0^2$$

$$K = \frac{1}{2} k x_0^2 \sin^2(\omega t + \phi)$$
$$U = \frac{1}{2} k x_0^2 \cos^2(\omega t + \phi)$$

Pendulum

$$\mathcal{E} = \tfrac{1}{2} mgl\,\theta_0^2$$

$$K = \tfrac{1}{2} mgl\,\theta_0^2 \sin^2(\omega t + \phi)$$

$$U = \tfrac{1}{2} mgl\,\theta_0^2 \cos(\omega t + \phi)$$

Damped harmonic motion

$$m \frac{d^2x}{dt^2} + \Gamma \frac{dx}{dt} + kx = 0$$

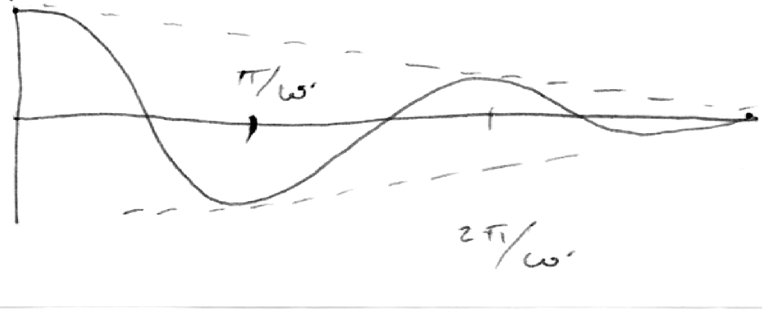
(use technique from 1^{st} day)

$$x(t) = x_0 e^{-\gamma t/2} \cos(\omega' t + \phi)$$

$$\gamma = \frac{\Gamma}{m}$$

$$\omega' = \sqrt{\omega_0^2 - (\Gamma/m)^2}$$

where $\omega_0 = \sqrt{k/m}$



$$Q = 2\pi \frac{\mathcal{E}}{\Delta\mathcal{E}} = \frac{\omega'}{\gamma}$$