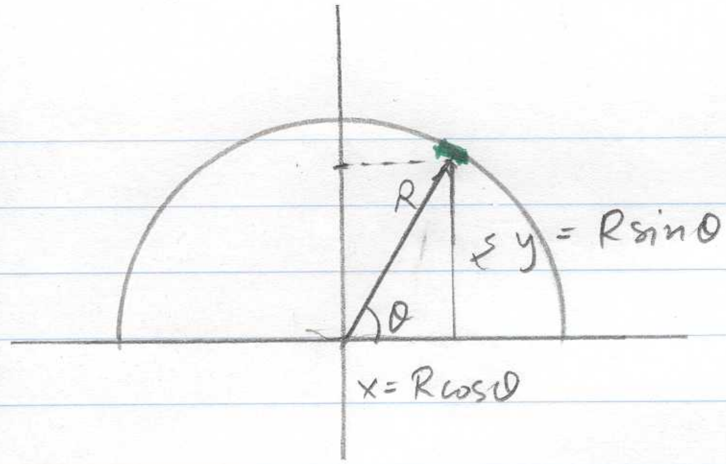
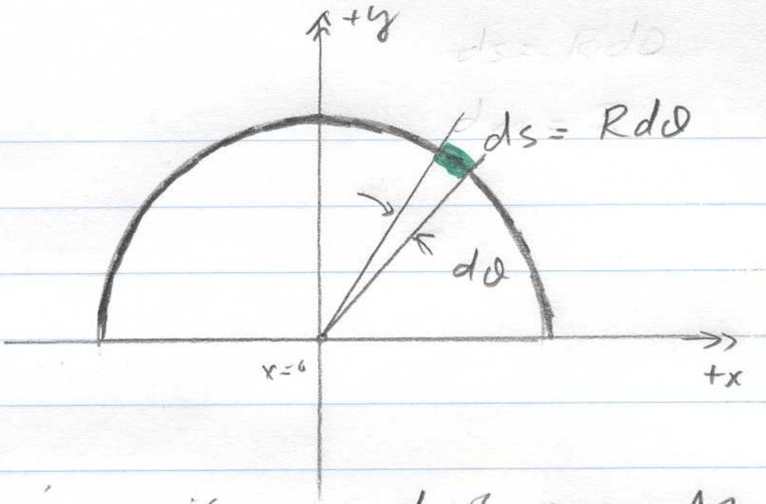
Sample Problem, SP1.





Goal: Find the center of mass of a uniform rod of mass M bent into a half-circle.

$$x_{cm} = \frac{1}{M}\int x\,dm, \qquad Y_{cm} = \frac{1}{M}\int y\,dm.$$

$dm \equiv$ mass of a tiny chunk of the rod – we must express dm in terms of R & θ somehow since x & y can be easily written in terms of R & θ as shown above. The goal then would be to integrate over θ.

Let the density of the rod be $\lambda = \frac{M}{L} = \frac{M}{\pi R}$.

Then $dm = \lambda\,ds$, where ds is the length of the tiny chunk of the rod.

$$\Rightarrow dm = \lambda R\,d\theta$$

$$\Rightarrow \quad x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^{\pi}(R\cos\theta)(\lambda R\,d\theta)$$

$$= \frac{\lambda R^2}{M}\int_0^{\pi}\cos\theta\,d\theta = \frac{\lambda R^2}{M}\,\sin\theta\Big|_0^{\pi} = 0.$$

$$Y_{cm} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^{\pi}(R\sin\theta)(\lambda R\,d\theta) = \frac{R^2\lambda}{M}\int_0^{\pi}\sin\theta\,d\theta$$

$$= -\frac{R^2\lambda}{M}\cos\theta\Big|_0^{\pi} = \frac{2R^2\lambda}{M}, \quad \text{using } \lambda = \frac{M}{\pi R},$$

we find $Y_{cm} = \frac{2R^2}{M}\cdot\frac{M}{\pi R} \Rightarrow \boxed{Y_{cm} = \frac{2}{\pi}R}$; $\boxed{x_{cm} = 0}$