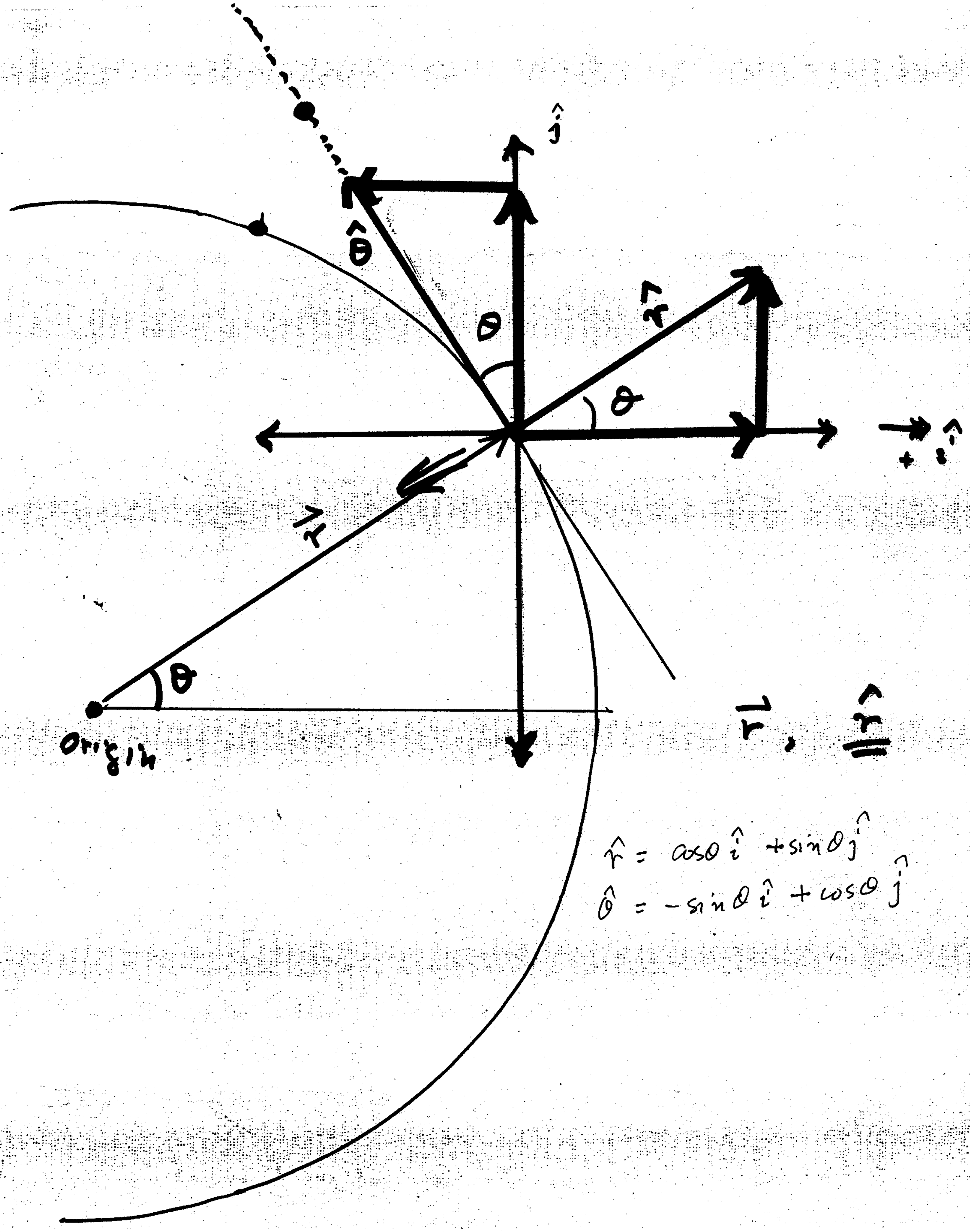


$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\hat{r} = \underline{\cos\theta}\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}.$$

$$\theta(t) = \omega t \quad - \quad \frac{d\theta}{dt} = \omega.$$

$$\frac{d\hat{r}}{dt} = \frac{d\hat{r}}{d\theta}\frac{d\theta}{dt}$$

$$\frac{d\hat{r}}{dt} = -\sin\theta \cdot \frac{d\theta}{dt}\,\hat{i} + \cos\theta\,\frac{d\theta}{dt}\,\hat{j}$$

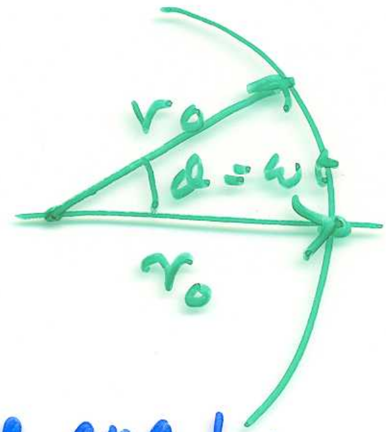
$$= \omega\,[\underbrace{-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}}_{\Downarrow\,\hat{\theta}}]$$

$$\boxed{\frac{d\hat{r}}{dt} = \omega\,\hat{\theta}}$$

$$\frac{d\hat{\theta}}{dt} = -\underline{\omega}\,\hat{r}$$

without ω, the dimensions <u>don't</u> match on both sides of the equation.

$$\vec{r}(t) = r_0[\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}]$$



$$\vec{v}(t) = d\vec{r}/dt$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt}$$

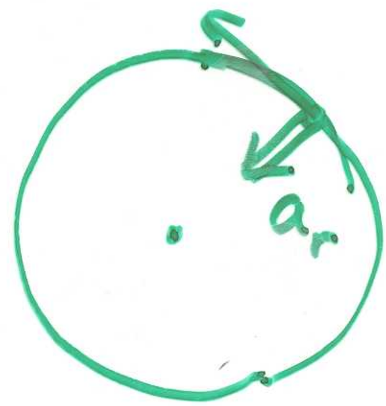
Case I: ω constant – [ω is the angular speed]

$$\vec{r}(t) = r_0\,\hat{r}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = r_0\frac{d\hat{r}}{dt} = \underline{\omega}\,r_0\,\hat{\theta}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \omega r_0\frac{d\hat{\theta}}{dt} = \omega r_0(-\omega\hat{r})$$

$$\vec{a}(t) = -\underbrace{r_0\,\omega^2}\,\hat{r}$$



$$\omega = \frac{v}{r_0}$$

$$\Rightarrow \boxed{v = r_0\,\omega} \Leftrightarrow$$

$$\boxed{\vec{a}(t) = -\frac{v^2}{r}\,\hat{r}}$$ radial.

Footnote: If ω = constant, $\omega = \frac{2\pi}{T}$ where T is the time to go around the circle once. If v is constant then $v = \frac{2\pi}{T}r_0 \Rightarrow \boxed{\omega r_0 = v}$

Now consider $\dfrac{d\omega}{dt} \neq 0$ — $\omega \neq$ constant.

$$\vec{r}(t) = r_0 \hat{r}$$

$$\vec{v}(t) = r_0 \frac{d\hat{r}}{dt} = r_0 \underbrace{\omega}\,\underbrace{\hat{\theta}}$$

$$\frac{d\vec{v}}{dt} = r_0 \frac{d\omega}{dt}\hat{\theta} + r_0 \omega \frac{d\hat{\theta}}{dt}$$

$$\vec{a}(t) = \boxed{r_0 \overset{\alpha}{\left(\frac{d\omega}{dt}\right)}}\,\hat{\theta} - \underbrace{r_0 \omega^2}\,\hat{r}$$

$$\vec{a}(t) = a_t \hat{\theta} + \underbrace{-\left(\frac{v^2}{r_0}\right)\hat{r}}_{a_r \hat{r}}$$

$$\vec{a}(t) = a_t \hat{\theta} + a_r \hat{r}$$

$$\vec{a}(t) = a_t \hat{\theta} + \underbrace{\left(-\frac{v^2}{r_0}\right)}_{a_r}\hat{r}$$