PHYSICS 161, Spring 2003

Lecture Quiz, Due begining of class, Monday, Feb 24

This Lecture Quiz is to be done in groups of two minimum and three maximum. Turn in ONE solution per group.

Special Note: It is dishonest to put your name on work to which you did not contribute. Consider that cheating. No Make-Ups.

For this Lecture Quiz, you will need to read Ch 4, Sections 4.4-4.6

Q1) This quetsion will help you with problem P5 on HW3. Notice that on page 94, Fig 4.18a Serway & Beichner introduces a new orthogonal coordinate system $(\hat{r}, \hat{\theta})$. (Orthogonal means perpendicular. When the axes of a coordinate system are all perpendicular to each other, it is called an orthogonal coordinate system.) It turns out that this coordinate system is much more convenient to use to describe circular motion than the $(\hat{i}, \hat{j})$ coordinate system. Since $\hat{\theta}$ points along the tangent to the circle at every point on the circle, and since the velocity of an object moving along the circle also points along the tangent at every point on the circle, the instantaneous velocity vector can simply be written as $\vec{v}(t) = v(t)\hat{\theta}$.

Circular Motion with Constant Speed: If the object goes around the circle with constant speed then the velocity vector at each point on the circle can simply be written as $\vec{v}(t) = v_0\,\hat{\theta}$. (Compare this with problem S5 on HW 2 where we were using $\hat{i}, \hat{j}$ and how cumbersome it was to figure out the velocity vector at each point separately). For the case of constant speed around a circle, the instantaneous acceleration always points towards the center of the circle, i.e., along the radial direction. So the acceleration vector at all points along the circle can simply be written as $\vec{a}(t) = -a_r\,\hat{r} = -(v_0^2/r)\,\hat{r}$. (*Why the minus sign???*)

Circular Motion with Increasing or Decreasing Speed:

If the object speeds up or slows down, then obviously its velocity which is always in the tangential direction *changes*. Therefore, it has an acceleration in the tangential direction in addition to the centripetal acceleration in the radial direction. The *total* acceleration, then is the vector sum of the radial and the tangential accelerations. This is depicted in Fig 4.18 b and expressed in Eq. 4.19.

Q1.) The purpose of this exercise is to get used to the $(\hat{r}, \hat{\theta})$ coordiante system and its relationship to the $(\hat{i}, \hat{j})$ coordiante system.

a). Show that:

$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

b). Note an interesting fact: $\hat{i}$ and $\hat{j}$ are constant unit vectors because they always point in the same direction no matter where the object is on the circle. However, as the object moves around the circle and the angle θ its position vector makes with the $x-$ axis changes, $\hat{r}$ and $\hat{\theta}$ change directions! Therefore, even though, $d\hat{i}/dt = 0$ and $d\hat{j}/dt = 0$, $d\hat{r}/dt \neq 0$ and $d\hat{\theta}/dt \neq 0$. Now, assume that as the object moves around the circle, the angle θ its position vector makes with the x-axis changes at a rate of $d\theta/dt = \omega$. Find $d\hat{r}/dt$ and $d\hat{\theta}/dt$. This will help you do problem P5 on HW 3.

Relative Motion: Read Section 4.6. Below, I am going to give a slightly more useful form of Eqs 4.20, 4.21 and what should be Eq 4.22.

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

$$\vec{a}_{PA} = \vec{a}_{PB} + \vec{a}_{BA}$$

English translation: (1) position of particle P with respect to observer A is equal to the vector sum of the position of particle P with respect to B and the position of B with respect A.

Similarly, the second equation reads: velocity of particle P with respect to observer A is equal to the vector sum of the velocity of particle P with respect to B and the velocity of B with respect A.

Now read Example Problem 4.9 and 4.10 and see if they don't make more sense. After that turn to HW 3, Short Answer Question S5. In lecture we will apply these equations to understand how to solve S5.