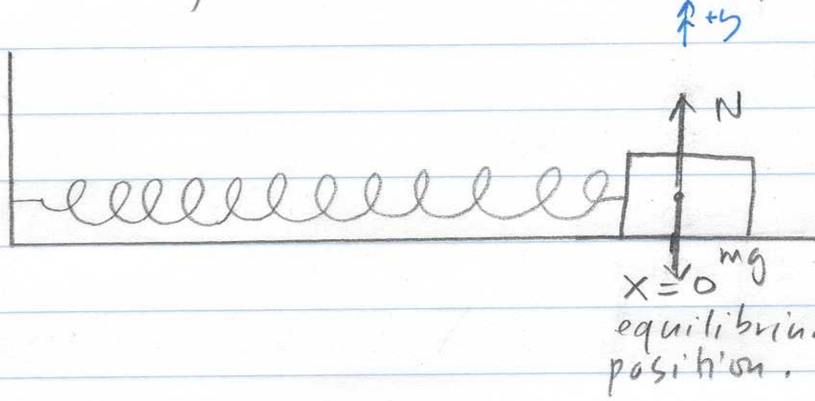
Lecture Outline (Friday, March 21)

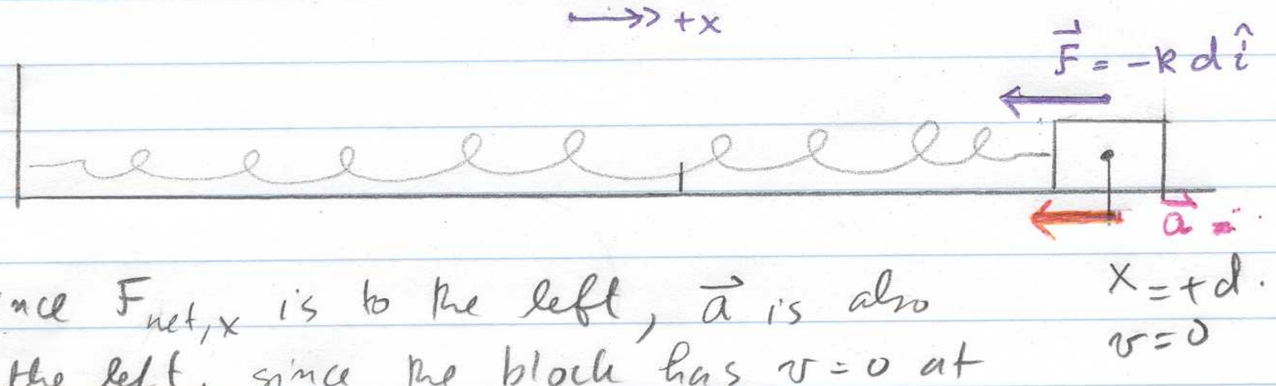
* introduced one more force to the list of forces we've compiled so far – namely, the spring force.

when a spring is compressed or extended the spring resists the compression/extension by a force given by $F_s = -kx$, where x represents the displacement of the spring from its equilibrium position. (k is called the spring constant.)

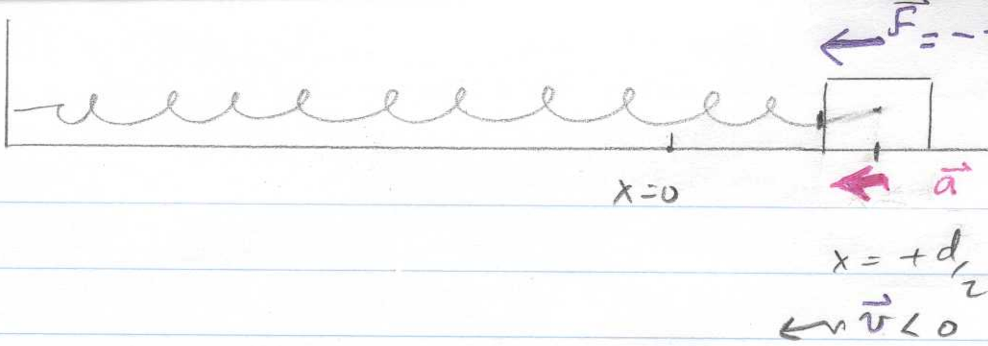
Examples: Block of mass m attached to a spring, displaced by an amount $x = +d$, and released at rest at $t=0$ (i.e $v = 0$ at $t = 0$). we then analyzed the force(s) acting on the block as it oscillates back and forth about its equilibrium position.



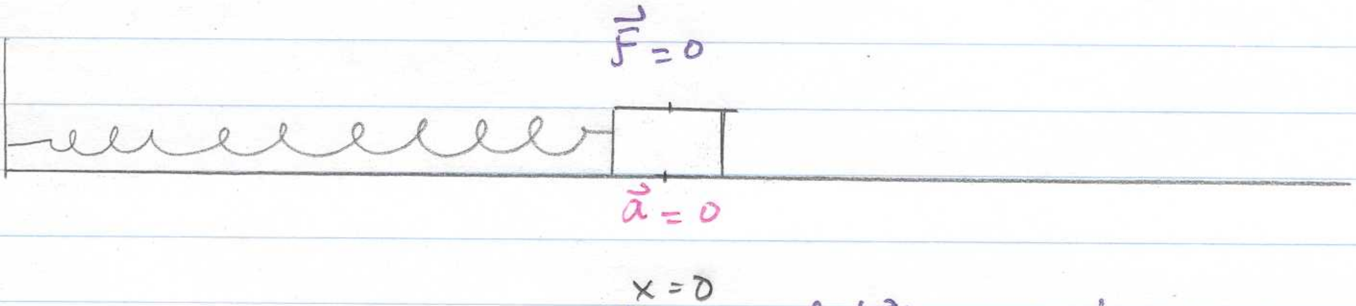
since $a_y = 0$, $N - mg = 0$ so $N = mg$. For the rest of the diagrams, I will ONLY show forces in the x-direction since forces in the y-direction are the same as here.



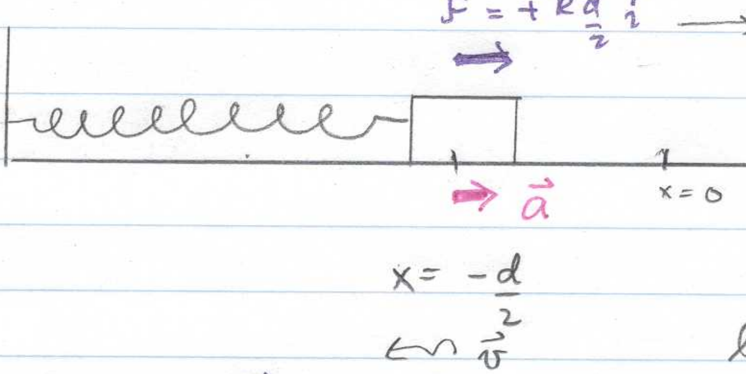
since $F_{net,x}$ is to the left, $\vec{a}$ is also to the left. since the block has $v = 0$ at this point, at a slightly later time $\vec{v}$ is to the left.



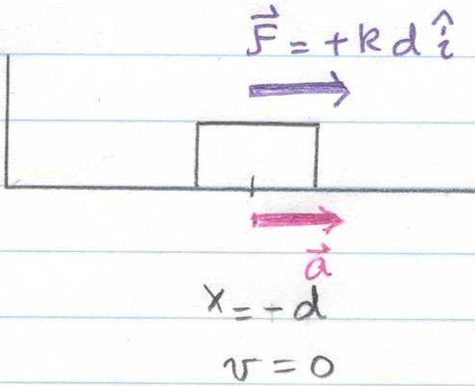
• Notes both $\vec{F}$ & $\vec{a}$ smaller than before in magnitude.



$\vec{v} < 0$ and $|\vec{v}| =$ maximum here.



• Now force is to the right since $\vec{F} = -k\vec{x}$ and $\vec{x} < 0$. Now the block is slowing down because the velocity is to the left but $\vec{a}$ is to the right



• At $x = -d$, the block comes to rest momentarily. Since $\vec{F}_{net}$ is to the right, by N2 so is $\vec{a}$. Since $v = 0$, a slightly later time the block will be moving with a velocity $\vec{v} > 0$ to the right.

The block speeds up now as it moves back towards the equilibrium position, where $\vec{a} = 0$ & $v = v_{max}$ and slows down as it moves towards $x = +d$, momentarily coming to rest there and then repeating the process all over again. If there is no friction, ~~then~~ ~~the~~ block will continue to oscillate back and forth forever and ever.

Let's Apply N2 to the block.

$$\sum F_x = m a_x$$

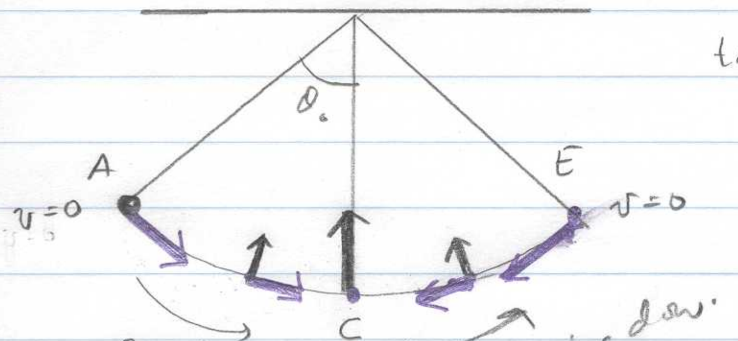
$$-kx = m a_x$$

$$\Rightarrow \quad -\frac{k}{m}x = a_x$$

or $a_x = \frac{k}{m}x$, using $a_x = \frac{d^2x}{dt^2}$, we find

S1. $\boxed{\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x}$ Equation of motion for the spring-block system.

• next we compared the block-spring system to a pendulum.



If you focus just on the tangential acceleration for a moment, you will see the incredible similarity with the spring-block system. At A, $v=0$, $\vec{a} = -g\sin\theta\,\hat{\theta}$. So the bob speed up as it moves from A to C. At C,

* going from A to E.

$a_{tangential} = 0$ (only $a_{radial} \neq 0$) – so at C, the bob has maximum tangential speed. Then as the bob moves from C to E, the bob slows down – note how the tangential acceleration has now switched directions – the bob comes to rest momentarily at E, then since $a_{tangential}$ is pointing in the $-\hat{\theta}$ direction, the bob speeds up as it moves towards C, and so on. Using N2 for the pendulum $\longrightarrow$

see next page.

$\Sigma F, \hat{\theta} = m a_{tangential}$

$-\cancel{m} g \sin\theta = \cancel{m} a_{tang}$

$\Rightarrow \boxed{a_{tangential} = -g \sin\theta}$ — eq P1.

Now $a_{tang} = \dfrac{dv}{dt}$, & $v = r\omega$ so

$= \dfrac{d(r\omega)}{dt}$, but r = length of pendulum $\boxed{r = l}$ = constant

so $a_{tang} = l \dfrac{d\omega}{dt}$

recall that $\omega = \dfrac{d\theta}{dt}$, so $\dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

$\Rightarrow a_{tang} = \dfrac{l d^2\theta}{dt}$ — substituting this on L.H.S of eq P1.,

$-g \sin\theta = l \dfrac{d^2\theta}{dt^2}$

so $\boxed{\dfrac{d^2\theta}{dt^2} = -\dfrac{g}{l} \sin\theta}$ P2.

Now, it is a fact that if we use <u>radians</u> for θ angle θ, <u>for very small angles</u>

$\boxed{\sin\theta \cong \theta}$ This is called the small angle approximation —

so, for small angles, P2 becomes —

eq. P3 $\boxed{\dfrac{d^2\theta}{dt^2} = -\left(\dfrac{g}{l}\right)\theta}$ Equation of motion for pendulum for small angles.

$\Sigma F_{,\hat{\theta}} = m a_{tangential}$

$-\cancel{m} g \sin\theta = \cancel{m}\, a_{tang}$

$\Rightarrow \boxed{a_{tangential} = -g \sin\theta}$ — eq P1.

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eq. <u>P3</u> $\boxed{\frac{d^2\theta}{dt^2} = -\left(\frac{g}{l}\right)\theta}$ Equation of motion for pendulum for small angles.

* Notice the similarity between the block-spring eq'n of motion S1. & pendulum-bob eq'n of motion P3.

$$\boxed{\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x} \quad \& \quad \boxed{\frac{d^2\theta}{dt^2} = -\left(\frac{g}{\ell}\right)\theta}$$

Note the $\frac{k}{m}$ and $\frac{g}{\ell}$ are constants. The solutions for both eq'ns above are given by sine or cosine functions.

DEMOS.

*1. showed the chaotic double-legged double pendum. behaves like a normal pendulum for small angles θ (i.e when $\sin\theta \approx \theta$) but for large angles, even if both pendula are started off with very similar initial conditions, after some time they start doing WILDLY DIFFERENT THINGS, THIS is THE DEFINITION OF CHAOS. purpose of demo was to show that when the small angle approximation is not ~~val~~ valid, the eq'n of motion (P2) is more complicated than (P3) and some very interesting phenomenon can occur.

*2. Showed the pendulum waves demo. Just a bunch of pendula with different lengths, all lined up in a row — had different periods of oscillation —

* Started talking about Work as Transfer of energy — Asked you guys to read Ch 7, Sec's 7.1 — 7.5 & study the dot product if not familiar with it already.