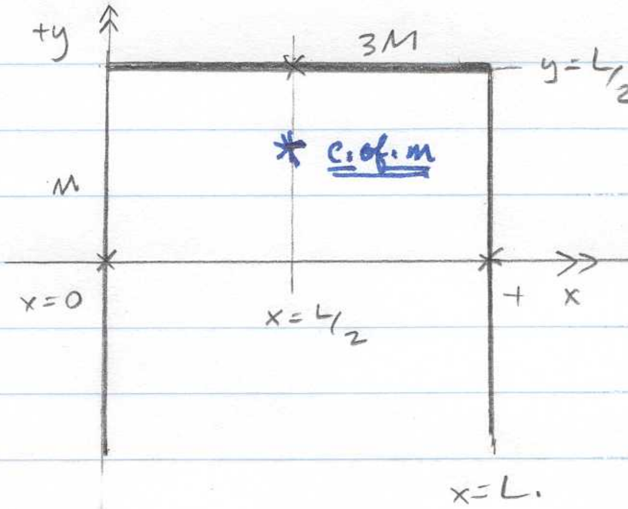
Homework 9 Solutions, Physics 161, Spring 2003.



S1/. The cross-marks represent the location of the center of mass of each individual rod.

$$x_{c.m} = \frac{M(0) + (3M)(L/2) + M(L)}{5M}$$

$$\Rightarrow x_{c.m} = \frac{5\cancel{M}}{5\cancel{M}}(L/2) \Rightarrow \boxed{x_{c.m} = L/2}$$

$$Y_{c.m} = \frac{M(0) + (3M)(L/2) + M(0)}{5M}$$

$$\Rightarrow \boxed{Y_{c.m} = \frac{3}{10} L}$$

S2/. constant $\omega \Rightarrow$ no tangential acceleration — Only radial acc.

$$a_{rad} = \frac{v^2}{R} = R\omega^2. \Rightarrow \frac{a_r}{R} = \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{11(9.8m/s^2)}{14m}} \Rightarrow \boxed{\omega = 2.77 \ rad/s}$$

(b). $\omega_i = 0$, $\omega_f = 2.77 rad/s$, $\Delta t = 120s$.

use $\omega_f = \omega_i + \alpha t \Rightarrow \frac{\omega_f}{t} = 0 + \alpha$

$$\Rightarrow \alpha = \frac{\omega_f}{t} \Rightarrow \alpha = \frac{2.77 rad/s}{120s} \Rightarrow \alpha = 0.023 rad/s^2$$

$$a_{tangential} = R\alpha$$

$$\Rightarrow \boxed{a_{tang.} = 0.324 \ rad/s^2}$$

S31.

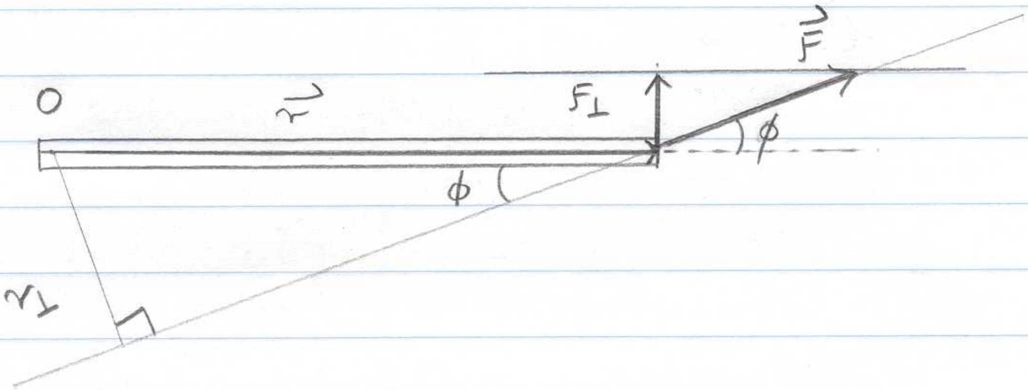
$I_a > I_b$ since more mass is distributed farther from the pivot point in case (a) than in case (b). Since the torque $\tau = LF$ is the same in both cases, we have

$$\tau_a = I_a \alpha_a \Rightarrow FL = I_a \alpha_a$$
$$\tau_b = I_b \alpha_b \Rightarrow FL = I_b \alpha_b.$$

$$\tau_a = \tau_b$$
$$I_a \alpha_a = I_b \alpha_b$$

$$\Rightarrow \frac{I_b}{I_a} = \frac{\alpha_a}{\alpha_b} \quad \text{since } I_b < I_a \Rightarrow \alpha_a < \alpha_b.$$

S41.



a). $F_{\perp r} = F \sin \varphi = (28N)(0.5)$

$\tau = r F_{\perp r} = (0.20m)(14N) = 2.8 N.m.$

(b). $r_\perp = r \sin \phi = (0.20m)(\sin 30^\circ) = 0.10 m.$

$\tau = F r_\perp = (28N)(0.10) = 2.8 N.m.$

(c) $\tau = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}| \sin \theta$ where θ is the angle

$= (0.20m)(28N) \sin 30^\circ$ between $\vec{r}$ & $\vec{F}$ in

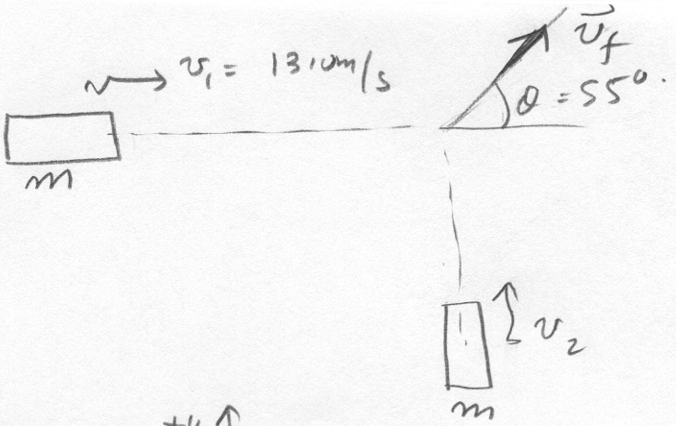
$|\vec{\tau}| = 2.8 N.m$ this case it's $\varphi = 30^\circ$.

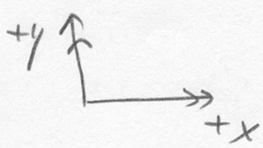
P1/. ch 9, #31.

$P_{ix} = P_{fx}$

$m v_1 = (m+m) v_f \cos\theta$

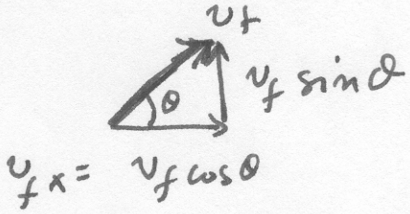
$\Rightarrow \boxed{\dfrac{v_1}{2\cos\theta} = v_f}$ ← eq1.





$P_{iy} = P_{fy}$

$m v_2 = (m+m) v_f \sin\theta$



$v_2 = 2 v_f \sin\theta$

using eq1:

$v_2 = 2 \cdot \dfrac{v_1}{2\cos\theta} \sin\theta \Rightarrow v_2 = v_1 \tan\theta$

$= (13.0\,m/s)(\tan 55°)$

$= 18.6\,m/s$

$= 41.5\,mi/hr$

$\Rightarrow$ vehicle #2 is lying!

P2. Ch10, #3. $\omega_i = 2000\,rad/s$

$\alpha = -80.0\,rad/s$

(a) $\omega_f = \omega_i + \alpha t$

$= (2000\,rad/s) - (80.0\,rad/s)(10.0s)$

$= 1200\,rad/s$.

(b). $0 = \omega_i + \alpha t$

$t = \dfrac{\omega_i}{-\alpha} = \dfrac{2000\,rad/s}{-(-80.0\,rad/s)} \Rightarrow \boxed{t = 25.0s.}$

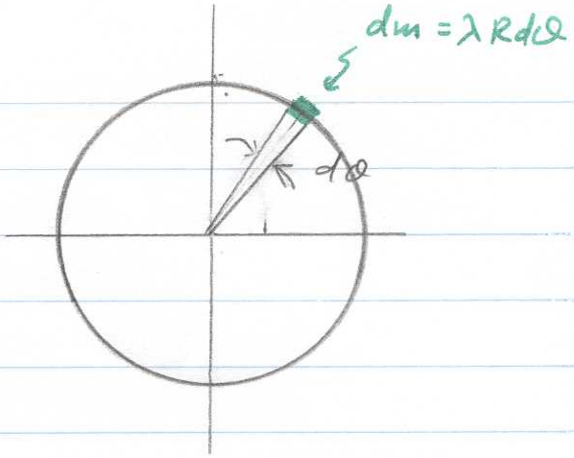
P3. $X_{c.m} = \frac{1}{M}\int x\,dm$

Since the mass density is uniform,

$$\lambda = \frac{M}{L} = \frac{M}{2\pi R}$$

then $dm = \lambda\,ds = \lambda R\,d\theta$.

& $x = R\cos\theta$

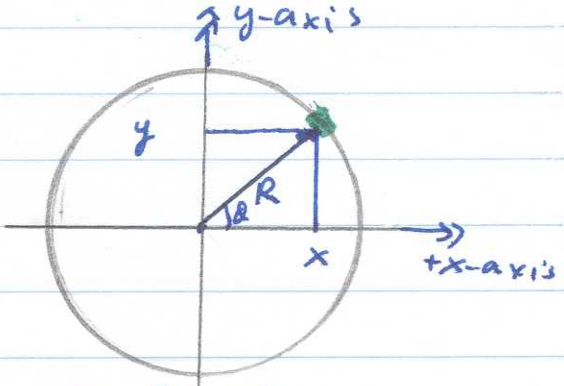


$$\Rightarrow X_{c.m} = \frac{1}{M}\int_0^{2\pi}(R\cos\theta)(\lambda R\,d\theta)$$

$$= \frac{\lambda R^2}{M}\int_0^{2\pi}\cos\theta\,d\theta$$

$$= +\frac{\lambda R^2}{M}\sin\theta\Big|_0^{2\pi}$$

$$\boxed{X_{c.m} = 0}$$



$x = R\cos\theta$

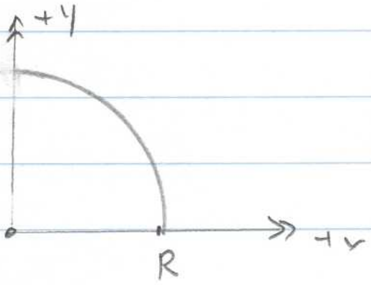
$y = R\sin\theta$.

$$Y_{c.m} = \frac{1}{M}\int y\,dm$$

$$= \frac{1}{M}\int_0^{2\pi}(R\sin\theta)(\lambda R\,d\theta) = \frac{\lambda R^2}{M}\int_0^{2\pi}\sin\theta\,d\theta$$

$$= -\frac{\lambda R^2}{M}\cos\theta\Big|_0^{2\pi} = -\frac{\lambda R^2}{M}\underbrace{[\cos 2\pi - \cos 0]}_{[1 \;\; -1]} = 0.$$

$\Rightarrow$ $\boxed{Y_{c.m} = 0}$ so the center of mass is smack in the center.



b). In this case, only the limits on the θ-integration change. Everything else is the same. Now θ goes from $0 \to \pi/2$.

$$\Rightarrow X_{c.m} = \frac{\lambda R^2}{M}\int_0^{\pi/2}\cos\theta\,d\theta = \frac{\lambda R^2}{M}\left[\sin\theta\Big|_0^{\pi/2}\right] = \frac{\lambda R^2}{M}.$$

$$X_{c.m} = \frac{\lambda R^2}{M}$$

using the fact that $\lambda = \frac{M}{L}$, where $L = \frac{2\pi R}{4} = \frac{\pi R}{2}$

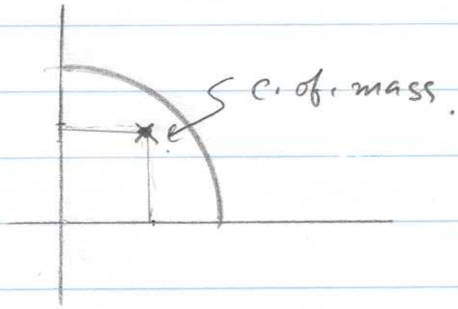
$$\Rightarrow \lambda = \frac{M}{\pi R/2} = \frac{2M}{\pi R}.$$

$$\Rightarrow X_{c.m} = \frac{2\cancel{M}}{\pi \cancel{R}} \cdot \frac{R^{\cancel{2}}}{\cancel{M}} \Rightarrow \boxed{X_{c.m} = \frac{2R}{\pi}}$$

similarly, $Y_{c.m} = \frac{\lambda R^2}{M} \int_0^{\pi/2} \sin\theta \, d\theta = -\frac{\lambda R^2}{M} \cos\theta \Big|_0^{\pi/2}$

$$\Rightarrow Y_{c.m} = +\frac{\lambda R^2}{M}$$

$$\Rightarrow \boxed{Y_{c.m} = \frac{2R}{\pi}}$$

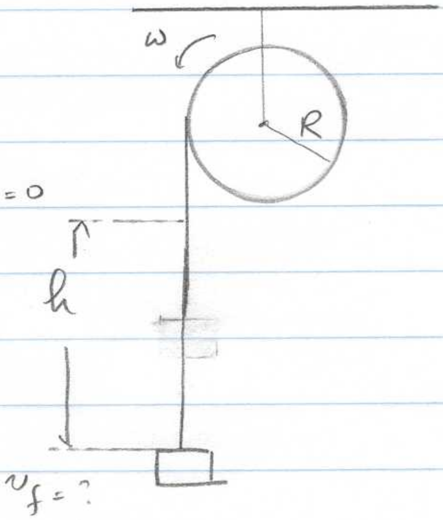


P41. a). using conservation of energy: $\Delta E = W_{ext}$

Including the earth as part of our system, $W_{ext} = 0$. So

$$\Delta E = 0 \Rightarrow E_i = E_f$$

$$P.E_i = K.E_{block} + K.E_{pulley}$$



$$Mgh = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$$

using $v = R\omega$

$$Mgh = \frac{1}{2} M v^2 + \frac{1}{2} I \frac{v^2}{R^2}$$

$$\Rightarrow Mgh = \frac{1}{2}\left(M + \frac{I}{R^2}\right) v^2$$

$$\Rightarrow v^2 = \frac{2Mgh}{M + I/R^2} \Rightarrow \boxed{v = \sqrt{\frac{2Mgh}{M + I/R^2}}}$$

b/. (i) If the pulley is a spoked wheel, with its mass concentrated on the outer rim: then $\boxed{I = mR^2}$

using this,

$$v = \frac{2Mgh}{M + \frac{m\cancel{R^2}}{\cancel{R^2}}} \Rightarrow \boxed{v_{wheel} = \sqrt{\frac{2Mgh}{M+m}}},$$

(ii) If the pulley is a solid disk, $I = \frac{1}{2}mR^2$

$$\Rightarrow v = \frac{2Mgh}{M + \frac{1}{2}\frac{m\cancel{R^2}}{\cancel{R^2}}} \Rightarrow v_{disk} = \sqrt{\frac{2Mgh}{M + m/2}}$$

$\Rightarrow v_{disk} > v_{wheel}$.

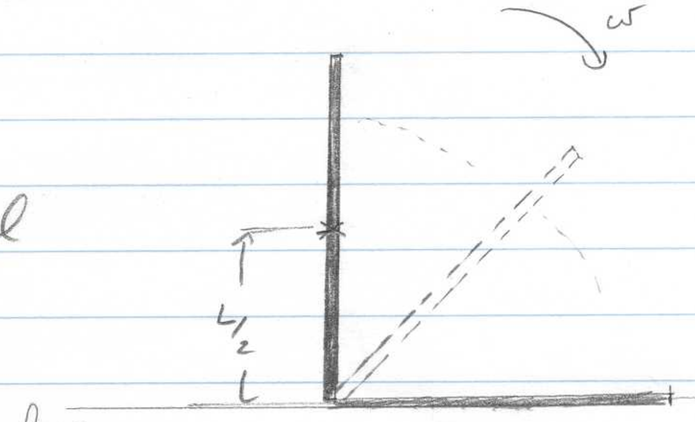
Q: why does this result make sense?

(c) If mass of the pulley $m \ll M$, then m is negligible compared to M. In that case $I \approx 0$.

$$\Rightarrow v \cong \sqrt{\frac{2\cancel{M}gh}{\cancel{M}}} \Rightarrow \boxed{v \cong \sqrt{2gh}}$$

Recall this result from the earlier part of the semester when we were assuming that m_{pulley} is negligible.

P5) For an extended object like the stick, to calculate the change in the gravitational potential energy, we must consider the height of the center of mass above (or below) the $h=0$ level.



In this case the rod being uniform, its center of mass is at $L/2$. So in the case of the meter stick, its

$\frac{1}{2}$ meter.

using Energy conservation again –

$$E_i = E_f$$

$$P.E_i = K.E_f$$

$$mg\,L/2 = \frac{1}{2} I \omega^2 ,$$ Note: here I must be about the end of the stick – $I = \frac{1}{3} mL^2$

$$\cancel{m} g \cancel{L}/\cancel{2} = \frac{1}{\cancel{2}} \cdot \frac{1}{3} \cancel{m} L^{\cancel{2}} \cdot \omega^2$$

$$\Rightarrow \omega^2 = \frac{3g}{L}$$

Now the speed of each point on the stick is related to its distance from the pivot point R – as $v = R\omega \Rightarrow \omega = v/R$

For the other end of the stick, $R = L$.

So

$$\frac{v^2}{R^2} = \frac{3g}{L},$$ using $R = L$ & simplifying –

$$\Rightarrow v^2 = 3gL$$

$$\Rightarrow \boxed{v = \sqrt{3gL}.}$$

P6/. The equation $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$ is valid in an elastic collision where both momentum & kinetic energy are conserved.

(ii). This equation <u>is</u> <u>valid</u> even when the masses of the colliding objects are not the same. If you follow the derivation in the text, you'll see the cancellation of mass – Okay, I'll repeat the derivation.

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ eq 1. momentum conservation.

$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$ — eq 2.

Rewriting eq 2 as:

$$\frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} m_2 v_{2f}^2 - \frac{1}{2} m_2 v_{2i}^2$$

and using the fact that $a^2 - b^2 = (a+b)(a-b)$

eq2' $\cancel{\frac{1}{2}} m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = \cancel{\frac{1}{2}} m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i})$

Now re-writing eq 1 as:

$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i})$$

This says that the circled terms on each side of eq 2' are equal — so they cancel.

$$\Rightarrow (v_{1i} + v_{1f}) = v_{2f} + v_{2i}$$

Rearranging once again —

$$v_{1i} - v_{2i} = -v_{1f} + v_{2f}$$

$$\Rightarrow \boxed{v_{1i} - v_{2i} = -(v_{1f} - v_{2f})}$$

(c)(i). $K.E_{Ti} = K.E_1 + K.E_2$

$$= \frac{1}{2} m v_0^2 + \frac{1}{2} m v_0^2$$

$$\boxed{K.E_{Ti} = m v_0^2}$$

$$K.E_{cm\,i} = \frac{1}{2} m_{total} v_{cm}^2$$

But $v_{cm} = \frac{m v_0 + m(-v_0)}{2m} = 0 \Rightarrow K.E_{cm\,i} = 0$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$ eq 1. momentum conservation.

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$ — eq 2.

Rewriting eq 2 as :

$$\frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} m_2 v_{2f}^2 - \frac{1}{2} m_2 v_{2i}^2$$

and using the fact that $a^2 - b^2 = (a+b)(a-b)$

eq2' $$\cancel{\frac{1}{2}} m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = \cancel{\frac{1}{2}} m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i})$$

Now re-writing eq 1 as:

$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i})$$

This says that the circled terms on each side of eq 2' are equal — so they cancel.

$$\Rightarrow \quad (v_{1i} + v_{1f}) = v_{2f} + v_{2i}$$

Rearranging once again —

$$v_{1i} - v_{2i} = -v_{1f} + v_{2f}$$

$$\Rightarrow \quad \boxed{v_{1i} - v_{2i} = -(v_{1f} - v_{2f})}$$

(c)(1). $$K.E_{Ti} = K.E_1 + K.E_2$$
$$= \frac{1}{2} m v_0^2 + \frac{1}{2} m v_0^2$$

$$\boxed{K.E_{Ti} = m v_0^2}$$

$$K.E_{cm\,i} = \frac{1}{2} m_{total} v_{cm}^2$$

But $v_{cm} = \dfrac{m v_0 + m(-v_0)}{2m} = 0 \quad \Rightarrow \quad K.E_{cm\,i} = 0$

$K.E_{Tf} = 0$

$K.E_{Tcm} = 0.$

So as we discussed in class, in a perfectly inelastic collision the energy lost is

$K.E_{lost} = K.E_{Total_i} - K.E_{cm_i}$

Since in this case, $K.E_{cm_i} = 0 \Rightarrow K.E_{lost} = K.E_{Total}$

i.e. **all** the $K.E_i$ is lost to thermal energy.

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(2). This is an elastic collision.

$K.E_{Ti} = m v_0^2$, $K.E_{cm\,i} = 0.$

$K.E_{Tf} = m v_0^2$, $K.E_{cm\,f} = 0.$

So $K.E_T$ is conserved as expected.

3/. $K.E_{Ti} = \frac{1}{2} m_A v_0^2$ since B essentially at rest.

$$v_{cm,i} = \frac{m_A v_0 + m_B (0)}{m_A + m_B} \Rightarrow v_{cm,i} = \frac{m_A}{m_A+m_B} v_0$$

$$\Rightarrow K.E_{cm,i} \quad \frac{1}{2}(m_A + m_B) \cdot \left(\frac{m_A}{m_A+m_B}\right)^2 v_0^2$$

$$\boxed{K.E_{cm,i} = \frac{1}{2} \frac{m_A^2}{m_A+m_B} v_0^2}$$

After the collision, the two blocks stick together, so their velocity together is the same as the velocity of the center of mass. From HW8, we know that using momentum conservation

$$P_{ix} = P_{fx}$$

$$m_A v_0 = (m_A + m_B) v_f \Rightarrow v_f = v_{cm_f} = \left(\frac{m_A}{m_A+m_B}\right) v_0$$

So $K.E_{Tf} = \frac{1}{2}(m_A + m_B) v_f^2$

$$= \frac{1}{2} \frac{m_A^2}{m_A + m_B} v_0^2$$

$\Rightarrow K.E_{Tf} = K.E_{cm_i} = K.E_{cm_f}$.

$\Rightarrow K.E_{lost} = K.E_{Ti} - K.E_{cm}$

This example is to show you that even in a perfectly inelastic collision, K.E of the center of mass cannot be lost. Only $K.E_{Ti} - K.E_{cm}$ is lost.