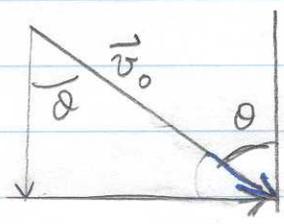


# Homework 8 Solutions, Phys 161, Spring 03

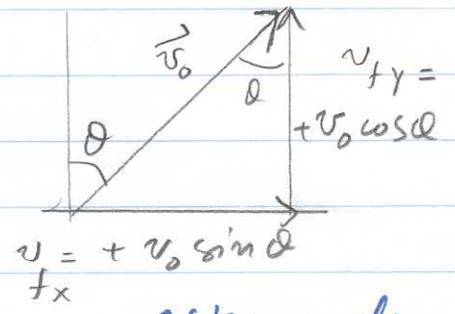
S11.

$$v_{iy} = -v_0 \cos \theta$$



$$v_{ix} = +v_0 \sin \theta$$

Initially.



Afterwards.

First note that  $\Delta p_x = p_{fx} - p_{ix}$   
 $= (m v_0 \sin \theta) - (m v_0 \sin \theta)$

$$\Rightarrow \boxed{\Delta p_x = 0}$$

$$\Delta p_y = p_{fy} - p_{iy}$$

$$= +m v_0 \cos \theta - (-m v_0 \cos \theta)$$

$$\boxed{\Delta p_y = +2m v_0 \cos \theta}$$

so if  $\theta$  is increased: (a)  $\Delta p_x$  remains zero.

(b).  $\Delta p_y = 2m v_0 \cos \theta$  decreases because  $\cos \theta$  decreases as  $\theta$  increases.

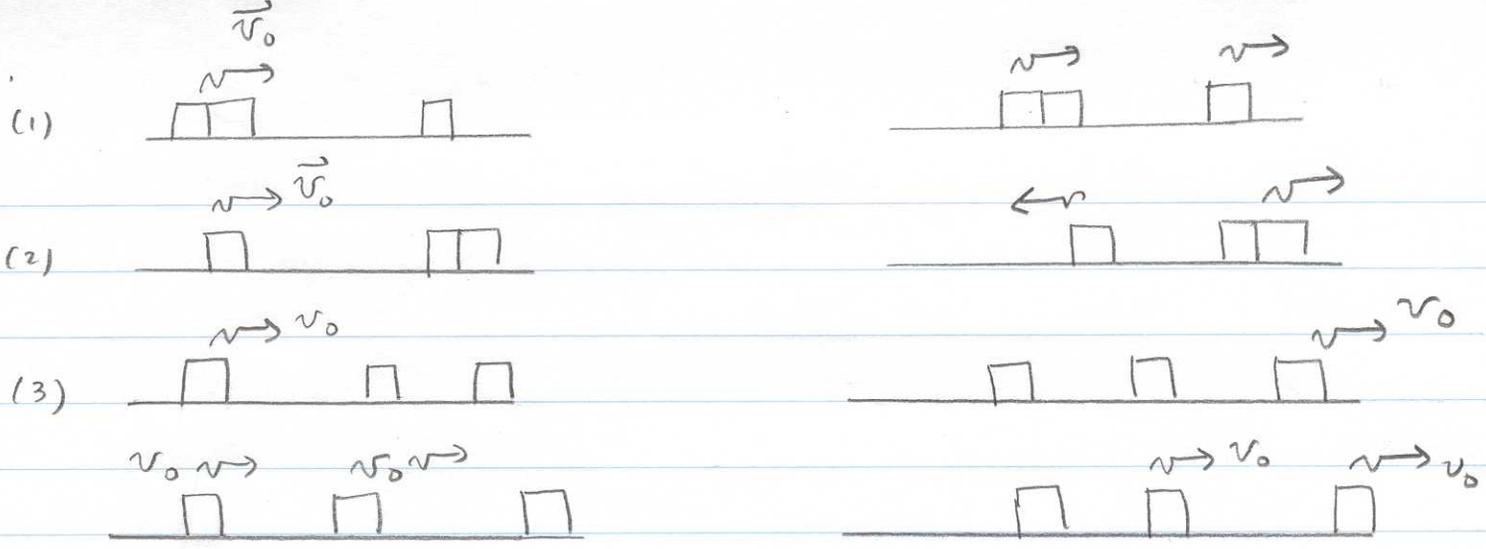
(c). The magnitude of  $\Delta \vec{p}$  i.e.  $|\Delta \vec{p}| = \sqrt{(\Delta p_x)^2 + (\Delta p_y)^2} = \Delta p_y$  in this case so  $|\Delta \vec{p}|$  decreases as  $\theta$  increases.

(d).  $F_x = \frac{\Delta p_x}{\Delta t} = 0$  and remains zero.

(e)  $F_y = \frac{\Delta p_y}{\Delta t}$  decreases as  $\theta$  increases assuming  $\Delta t$  remains the same.

(f).  $|\vec{F}| = \sqrt{F_x^2 + F_y^2} = F_y \Rightarrow |\vec{F}|$  also decreases as  $\theta$  increases.

S2/.



(a), since  $P_i = P_f$ , we need only calculate  $P_i$ .  
 $(P_{1i} = 2m v_0) = (P_{4i} = 2m v_0) > (P_{2i} = P_{3i} = m v_0)$

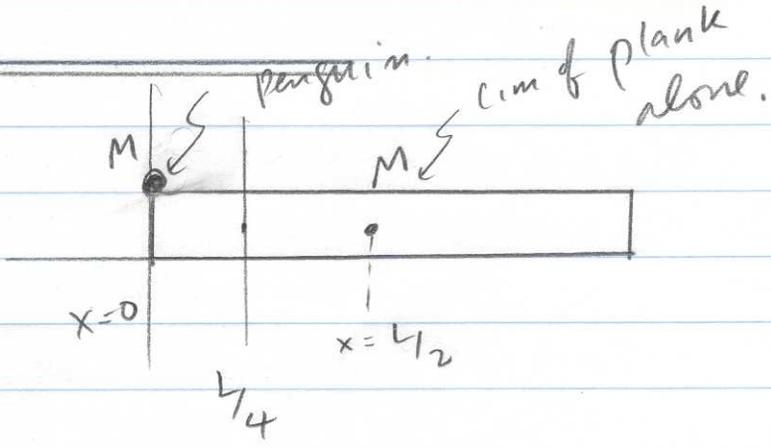
(b), The velocity of the rightmost block is greatest in case 1, 3 & 4 tie, in case 2 the velocity of the rightmost block is the least.

Intuitively, we expect this because in case 2, the block on the left is colliding with something twice as massive — since  $|\Delta p|$  should be the same for both objects, and initial momentum is just  $m v_0$ , the rightmost block moves away with velocity less than  $v_0$ .

You can also arrive at the above results by actually carrying out the calculations —

S41. The center of mass of the plank alone is at  $L/2$ . So, for the penguin-plank system:

$$x_{cm} = \frac{M(0) + M(\frac{L}{2})}{2M} \Rightarrow$$



$$\Rightarrow x_{\text{cm}} = \frac{ML}{4M} = L/4.$$

b). since

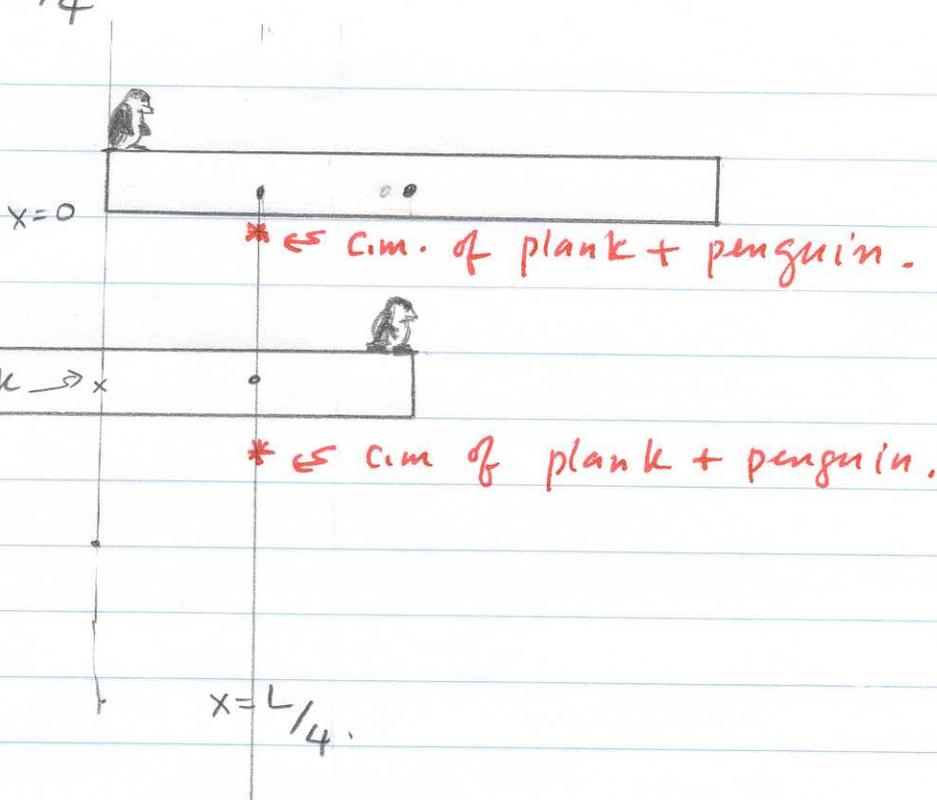
$$F_{\text{net, ext}} = 0$$

The center of mass does

not accelerate.

The center of mass remains at rest

at  $x = L/4$  at all times.



c). The center of mass of the plank is now to the left of the center of mass of the plank-penguin a distance  $L/4$  away.

d). Relative to the center of mass of the plank-penguin system, the penguin moves to the right a distance  $L/4$  away. So relative to the penguin-plank cm, the penguin moves a total distance of  $L/2$ .

- 9.58 Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M + m)v_f$$

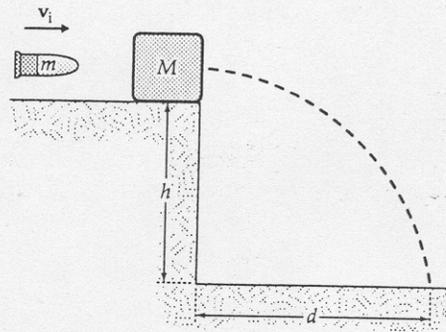
$$\text{or } v_i = \left(\frac{M + m}{m}\right)v_f \quad (1)$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = vt \quad \text{and} \quad h = \frac{1}{2}gt^2$$

$$\text{Thus, } t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d \sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

$$\text{Substituting into (1) from above gives } v_i = \left(\frac{M + m}{m}\right) \sqrt{\frac{gd^2}{2h}}$$



- 9.60 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when  $m_1$  leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

$$\text{or } (3.00 \text{ kg})v_{\text{wedge}} + (0.500 \text{ kg})(+4.00 \text{ m/s}) = 0$$

$$\text{so } v_{\text{wedge}} = \boxed{-0.667 \text{ m/s}}$$

- (b) Using conservation of energy as the block slides down the smooth (frictionless) wedge, we have

$$[K_{\text{block}} + U_{\text{block}}]_i + [K_{\text{wedge}}]_i = [K_{\text{block}} + U_{\text{block}}]_f + [K_{\text{wedge}}]_f$$

$$\text{or } [0 + m_1gh] + 0 = \left[\frac{1}{2}m_1(4.00)^2 + 0\right] + \frac{1}{2}m_2(-0.667)^2$$

which gives

$$\boxed{h = 0.952 \text{ m}}$$

- 9.6 (a) For the system of two blocks  $\Delta p = 0$ ,

$$\text{or } p_i = p_f$$

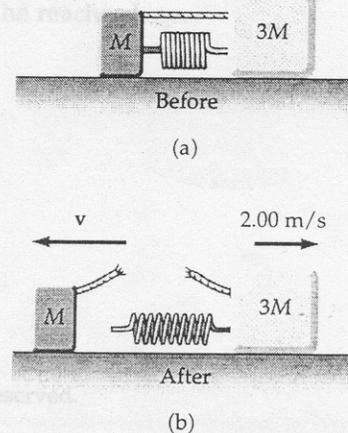
Therefore,

$$0 = Mv_m + (3M)(2.00 \text{ m/s})$$

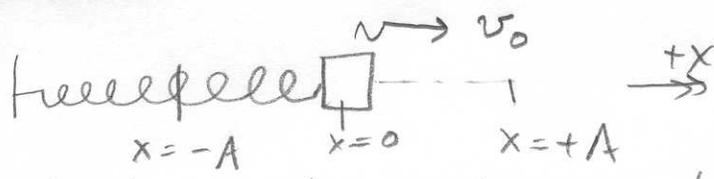
Solving gives

$$v_m = \boxed{-6.00 \text{ m/s}} \quad (\text{motion toward the left})$$

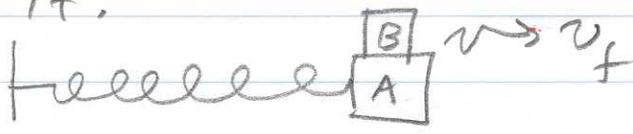
$$(b) \quad \frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = \boxed{8.40 \text{ J}}$$



P41.



Suppose that block A is moving to the right with velocity  $+v_0$  at the instant block B is dropped on top of it.



Then, the velocity of the block A+B combo will be positive. We find its magnitude by noting that since there is no net external force on the block-cart-spring system in the x-direction,

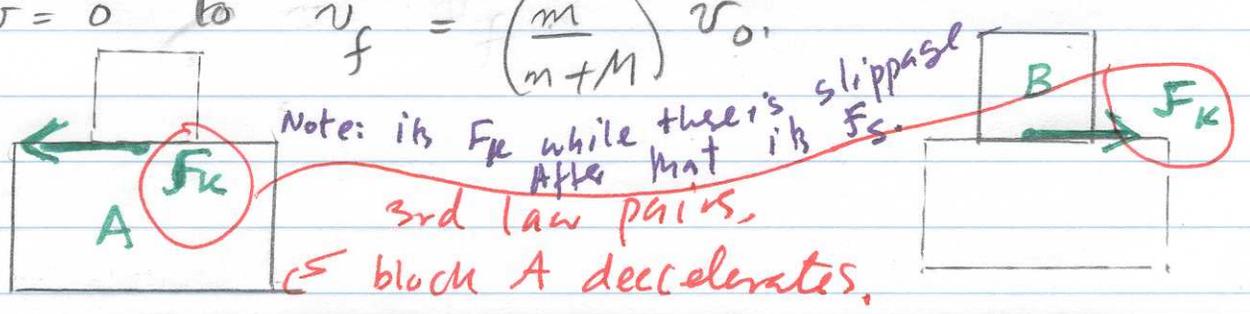
$$P_{ix} = P_{fx}$$

$$\Rightarrow m_A v_0 = (m_A + m_B) v_f$$

$$\Rightarrow v_f = \left( \frac{m_A}{m_A + m_B} \right) v_0 \quad \text{Note that } v_f < v_0.$$

This is an inelastic collision.  $K_i E_f < K_f E_i$ ; - as shown below

by. If there were no friction, block B will simply stay at  $x=0$  and block A would continue moving to the right without any loss in energy. In the presence of friction, block B initially slides backwards relative to block A so the kinetic frictional force on it is in the  $+x$ -direction. It is this frictional force that accelerates block B to the right relative to the ground. from  $v=0$  to  $v_f = \left( \frac{m}{m+M} \right) v_0$ .



Note: it's  $F_k$  while there's slippage. After that it's  $F_s$ .  
 3rd law pairs,  
 block A decelerates.

$$K.E_i = \frac{1}{2} m_A v_0^2 + \frac{1}{2} m_B \cdot 0^2 = \frac{1}{2} m_A v_0^2$$

$$K.E_f = \frac{1}{2} (m_A + m_B) v_f^2$$

$$= \frac{1}{2} \frac{(m_A + m_B) \cdot m_A^2}{(m_A + m_B)^2} v_0^2$$

$$K.E_f = \frac{1}{2} \frac{m_A^2}{m_A + m_B} v_0^2 \Rightarrow K.E_f = \frac{1}{2} \frac{m_A \cdot m_A v_0^2}{(m_A + m_B)}$$

$$\Rightarrow \boxed{K.E_f < K.E_i}$$

df. with block B on top of block A, the kinetic energy at  $x=0$  is less than what it was before the collision, so now the maximum extension of the spring-block system would be less than A. To find it, use  $E_i = E_f$

$$K.E_i = P.E$$

$$\frac{1}{2} \frac{m_A^2}{m_A + m_B} v_0^2 = \frac{1}{2} k x_{\max}^2$$

$$\Rightarrow x_{\max}^2 = \left( \frac{m_A^2}{m_A + m_B} \right) \frac{v_0^2}{k}$$

To compare with  $x_{\max} = A$  before the collision, note that before the collision

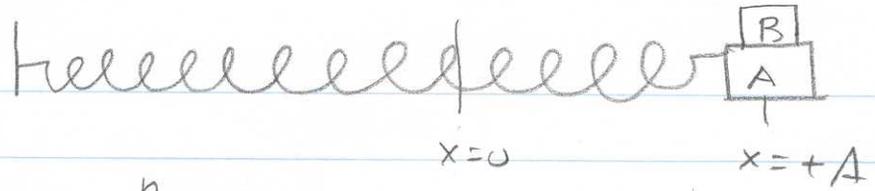
$$\frac{1}{2} m_A v_0^2 = \frac{1}{2} k A^2$$

$$\Rightarrow \boxed{\frac{m_A v_0^2}{k} = A}$$

$$\Rightarrow x_{\max} \text{ after the collision} = \frac{m_A}{m_A + m_B} \cdot \frac{m_A v_0^2}{k} = \frac{m_A}{m_A + m_B} A$$

→  $x_{\max}$  after the collision is less than  $A$ .

ef. when block B is dropped on block A at  $x = A$ , block A is momentarily at rest so there is no relative slippage of the two blocks at that point. So no frictional losses.



As block A moves to the left, block B will be accelerated to the left by static friction. [Of course, the force of static friction must be large enough to accelerate block B with the same acceleration as block A.]

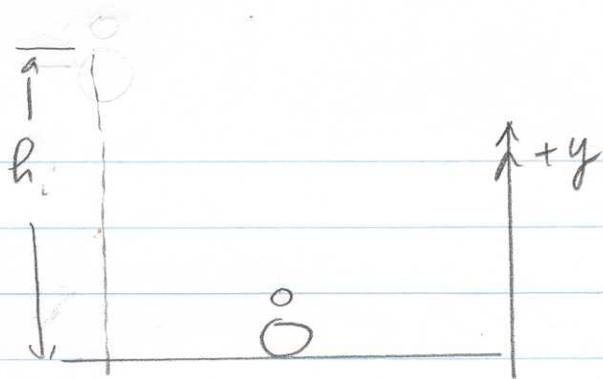
Therefore, the blocks (A+B) + spring system keeps oscillating back & forth between  $x = +A$  and  $x = -A$ . At  $x = 0$ ,  $K.E_{\max}$  is the same as before and since the total mass now is bigger, the speed at  $x = 0$  will be smaller than before.

---

---

P6. Before the collision:

If the initial height of the balls is  $h$ , then their speeds as they get close to the ground is  $v_0 \approx \sqrt{2gh}$  (see Assumption 2.)



The basketball collides with the floor elastically, (see assumption 3). Therefore, it bounces off the floor with the same speed i.e.  $v_0 \approx \sqrt{2gh}$ . Right before the baseball - basketball collision, therefore:

$$\vec{v}_{\text{basketball}} = +v_0 = +\sqrt{2gh}$$

$$\vec{v}_{\text{baseball}} = -v_0 = -\sqrt{2gh}$$

b). When the two balls collide, it's true that gravity is still acting on them, but the forces they apply on each other during that short interval of time are so large that we can assume that  $F_{\text{net, ext}} \approx 0$ . In that case, momentum of the two balls is conserved.  $\Rightarrow P_{ix} \approx P_{fx}$ .

$$\boxed{-mv_0 + Mv_0 = mv_{f1} + Mv_{f2}} \quad \text{eq 1.}$$

using

$$v_{1z} - v_{2z} = -(v_{f1} - v_{f2})$$

$$\Rightarrow \begin{aligned} -v_0 - v_0 &= -v_{f1} + v_{f2} \\ -2v_0 &= -v_{f1} + v_{f2} \end{aligned}$$

$$\Rightarrow \boxed{2v_0 = v_{f1} - v_{f2}} \quad \text{eq 2.}$$

Using the fact that  $M=3m$ , let's solve eq's 1 & 2 simultaneously —

$$-mv_0 + 3mv_0 = mv_{f1} + 3mv_{f2} \quad \text{eq 1}$$

$$\Rightarrow 2v_0 = v_{f1} + 3v_{f2} \quad \text{eq 1.}$$

From eq 2, we have

$$-2v_0 = -v_{f1} + v_{f2}$$

$$0 = 4v_{f2}$$

$$\Rightarrow \boxed{v_{f2} = 0}$$

$$\Rightarrow 2v_0 = v_{f1} - v_{f2} \Rightarrow \boxed{v_{f1} = 2v_0}$$

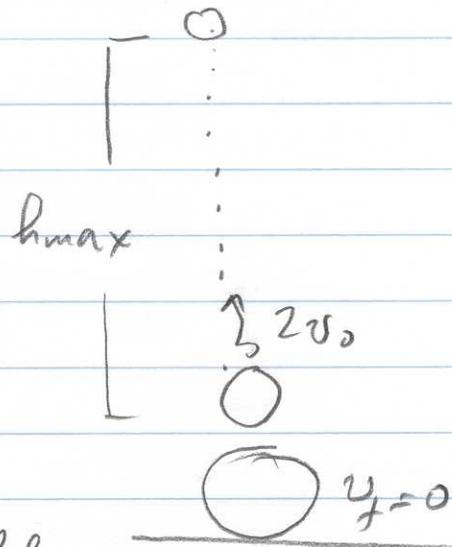
So the basketball comes essentially to rest after the collision and the baseball bounces up with  $v_f = 2v_0$ . [Note the exact similarity between this problem and the 3:1 mass ratio collision of carts moving with equal & opposite velocities discussed & shown in class.]

Q1 To find the height that the baseball bounces up to, we return back to our knowledge of kinematics from ch 2.

$$h_{\max} = \frac{v_{iy}^2}{2g} = \frac{(2v_0)^2}{2g}$$

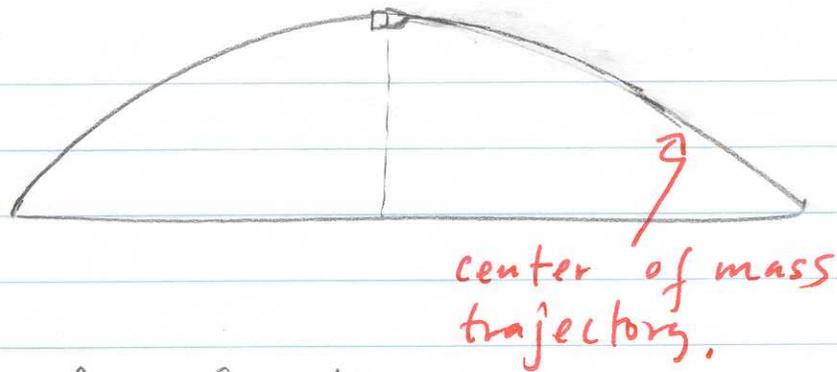
$$= \frac{4v_0^2}{2g} = 4\left(\frac{v_0^2}{2g}\right)$$

$$= \underline{\underline{4h}}$$



So, the baseball bounces off to 4 times the height it was dropped from.

PS/, Since the net external force on the center of mass is just the force of gravity - it continues along the same projectile trajectory.



The center of mass's final x-location is

$$x_{cmf} = v_{0x} t = v_0 \cos \theta t$$

To find  $t$ , we use,  $y_f = y_0 + v_{0y} t - \frac{1}{2} g t^2$

Since  $y_f = y_0$ , we have

$$0 = v_{0y} t - \frac{1}{2} g t^2 \Rightarrow v_{0y} = \frac{1}{2} g t \Rightarrow t = \frac{2 v_0 \sin \theta}{g}$$

$$\Rightarrow x_{cmf} = \frac{2 v_0^2 \sin \theta \cos \theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$$

$$= \frac{(20 \text{ m/s})^2 (\sin 40^\circ)}{9.8 \text{ m/s}^2} = 35.3 \text{ m.}$$

Since one piece falls at  $x_1 = 17.67 \text{ m}$ , we have,

using

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{(m_1 + m_2)}$$

$$x_{cm} = \frac{\frac{m}{2} x_1 + \frac{m}{2} x_2}{m}$$

$$m x_{cm} = \frac{m}{2} x_1 + \frac{m}{2} x_2$$

$$\Rightarrow (x_{cm} - \frac{1}{2} x_1) = \frac{1}{2} x_2 \Rightarrow x_2 = 2(x_{cm} - \frac{1}{2} x_1)$$

$$= 2(35.3 \text{ m} - \frac{1}{2} 17.67 \text{ m})$$

$$= 52.9 \text{ m} \approx 53 \text{ m.}$$

