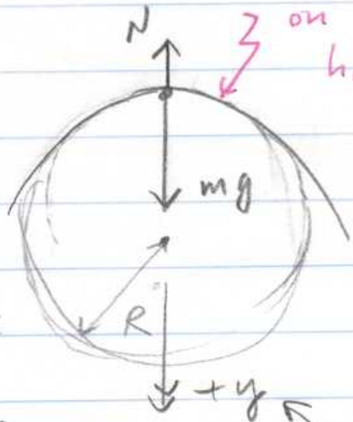
Physics 161, Homework 7 Solutions.

S1/.
Since the block is released from rest at 3.0 m, due to conservation of energy, it cannot climb up a vertical distance greater than 3.0 m so hill # 4 is the first one the block cannot climb.

(b). After failing to cross that hill, it slides back down, picking up K.E in the process and ends up at its initial starting point and then back again. If there's no friction, this process continues ad-infinitum

(c) Since we are told that the hills have identical circular tops, they have the same vertical radius R. The greatest centripetal acceleration occurs on the hill where v is the greatest. Recall $a_r = \frac{v^2}{R}$. Since the kinetic energy is the greatest on hill # 1, v is the greatest there. So a_r is the largest on hill # 1.

Zooming in on the hill top.



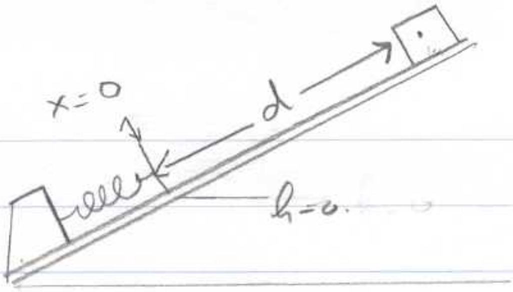
pay attention to coordinate system

(d). The larger the speed of the block, the larger the net-force pointing towards the center that must act on the block to keep it going in a circle.
So since $F_{net,y} = mg - N$, we need N to be the smallest possible when v is large.
This can also be seen from N2: $\Sigma F_{net,y} = ma_y$

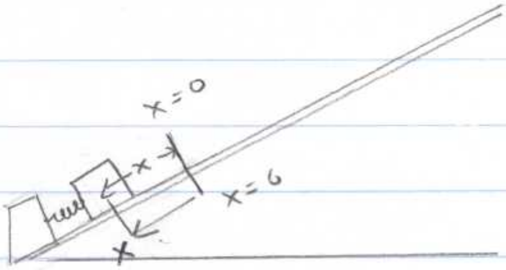
$$\Rightarrow \quad mg - N = \frac{mv^2}{R} \quad \Rightarrow \quad N = mg - \frac{mv^2}{R}.$$

Since v largest on hill # 1, that's where N is the smallest.

S21. $t_0 \to t_1$: As the block slides down to $h=0$, its gravitational P.E decreases and its K.E increases. When it reaches $h=0$, the grav. potential energy is completely converted to K.E.



$t_1 \to t_2$: Now the block starts compressing the spring. The gravitational P.E decreases further and so does its K.E. The block comes to rest at x (momentarily.)
At that point the K.E of the block is again zero.
At this point, the change in the gravitation potential energy of the block is stored as spring potential energy of the spring-block-earth system.



(spring compresses by an amount x —
note P.E$_g$ = $-mgx\sin\theta$.)

a). K.E increases from $t_0 \to t_1$, decreases to zero from t_1 to t_2.

(b). P.E$_g$ of the earth-block system decreases from t_0 to t_2.

(c) The elastic/spring potential energy is zero from t_0 to t_1 but increases from t_1 to t_2.

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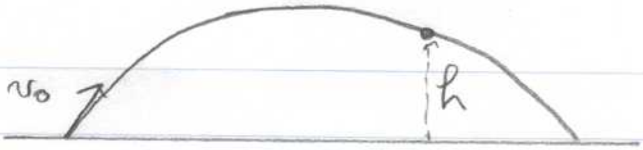
531. Consider the earth – baseball system.

Since $W_{ext} = 0$,



$$\Delta E = \cancel{W_{ext}}^{\,0}$$

$$\Rightarrow \Delta E = 0 \Rightarrow E_f - E_i = 0$$

$$\Rightarrow E_f = E_i$$

$$\frac{1}{2}\cancel{m} v_0^2 = \cancel{m} g h + \frac{1}{2}\cancel{m} v_f^2$$

$$\Rightarrow v_f = \sqrt{v_0^2 - gh}$$ – Does <u>NOT</u> depend on θ.

b). To solve for v_f at $y = h$, we would need to do the following: use $y_f = y_i + v_{oy} t - \frac{1}{2} g t^2$

$$h = 0 + v_0 \sin\theta \, t - \frac{1}{2} g t^2$$

Solve for t here & then use

$$v_{fy} = v_{iy} - gt \Rightarrow v_{fy} = v_0 \sin\theta - gt.$$

So we would <u>need</u> to know the angle θ.

<u>Note</u>: $v_f = \sqrt{v_{fy}^2 + v_{fx}^2}$, but $v_{fx} = v_{ox}$ since $a_x = 0$

c). The kinematic eq'ns give us information about exactly how the system behaves as a function of time – in other words, they tell us how a system evolves in time.

The energy relationships are more convenient to use if all we're intered in are the <u>initial</u> and <u>final</u> states of a system. For example, in the above problem of the baseball, the energy conservation equations tell us what the speed is at $y_f = h$, if v_i & h_i are given.

P11. The work needed to lift 10 people by some height h is: $W = Mgh$ since the work we do gets stored as the gravitational P.E of the people-earth system. (Assuming $\Delta K.E = 0$ which it is for an elevator ride.)

Assume each person weighs ~ 50N (that's ~ 110 lbs.)

so total weight $mg = (10 \times 50N) = 500N$.

weight elevator $\cong (500kg)(10 m/s^2) = 5000N$

Assume $h \cong 10m$

so $W = Mgh = (5500N)(10m)$

$= 55{,}000 N.m$

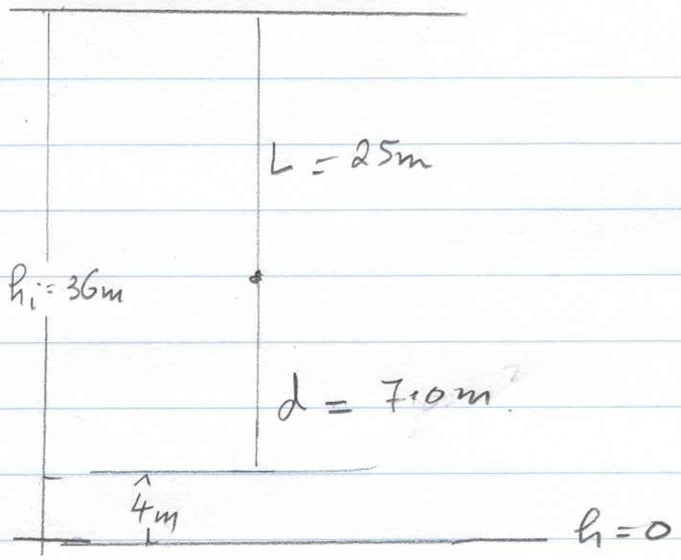
$$\boxed{W = 55\,kJ}$$

To light a 100 watt bulb for an hour the energy needed is $(100 J/s)(3600 s) = \boxed{360\,kJ.}$

So it takes roughly 6.5 times more energy to keep a light bulb lit for an hour than to transport 10 people up to the 4th floor.

* Important Note: This is assuming the ideal case in which there are no frictional forces present that convert part of the energy expended into Thermal Energy.

P21. S&B Problem 20.



I set the river to be the h = 0 level. Let's take the system to be the bungee cord + earth + jumper. Then

$$\Delta E = \cancel{W_{ext}}^{\,0}$$

$$\Delta P.E_g + \Delta P.E_{sp} + \Delta K.E = 0$$

($v_i = v_f = 0$ so $\Delta K.E = 0$)

$$\Rightarrow \quad P.E_{fg} + P.E_{f\,sp} = P.E_{ig} + \cancel{P.E_{i\,sp}}^{\,0}$$

$$mgh_f + \frac{1}{2}kd^2 = mgh_i$$

$$\Rightarrow \quad \frac{1}{2}kd^2 = mg(h_i - h_f)$$

$$\Rightarrow \quad k = \frac{2mg(h_i - h_f)}{d^2}$$

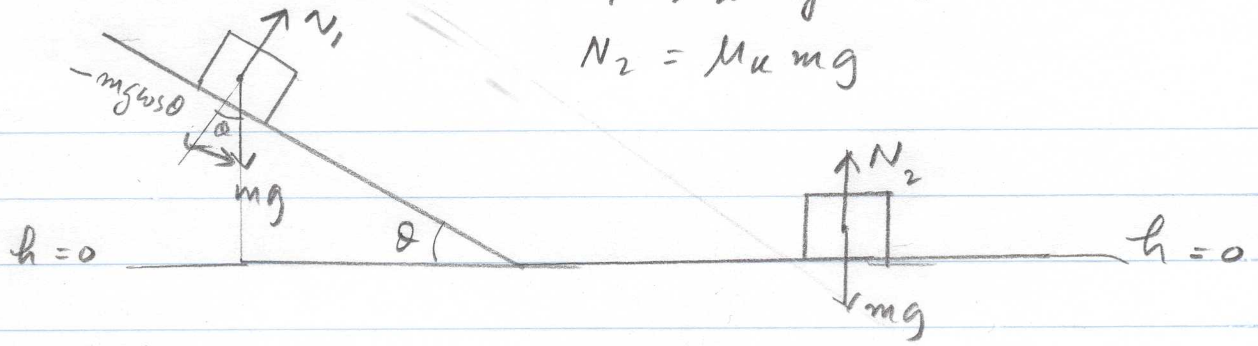
$$= \frac{2(700\,N)(32\,m)}{(7\,m)^2}$$

$$\Rightarrow \quad \boxed{k = 914\ N/m}$$

P31 S&B problem 39.

$N_1 = \mu_k mg\cos\theta$

$N_2 = \mu_k mg$



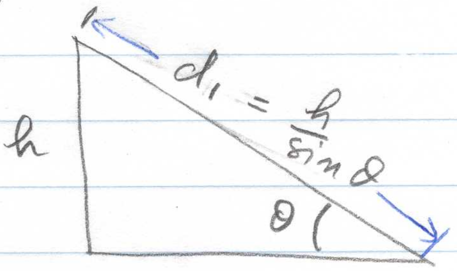
System = Earth + block.

The normal force is ⊥ to the displacement so it does

Since F_{k1} on the ramp ≠ F_{k2} on the horizontal floor the total change in thermal Energy

$$\Delta E_{thermal} = f_{k1} d_1 + f_{k2} d_2.$$

$v_i = 0,\ v_f = 0 \implies \Delta K.E = 0.$



So using $\Delta E = W_{ext}^{\nearrow 0}$

$$\Delta P.E_g + \cancel{\Delta K.E}^{\,0} + \Delta E_{thermal} = 0$$

$$\implies (\cancel{P.E_f}^{\,0} - P.E_i) + f_{k1} d_1 + f_{k2} d_2 = 0$$

$$-mgh + \mu_k mg\cos\theta\, d_1 + \mu_k mg\, d_2 = 0$$

$$\implies d_2 = \frac{mgh - \mu_k mg\cos\theta\,(h/\sin\theta)}{\mu_k mg} = 1.96\,m.$$

Note you also do this problem by splitting it into two parts. 1st calculate the K.E of the block at the bottom of the ramp using $\Delta P.E_g + \Delta K.E + \Delta E^{(1)}_{thermal} = 0$ Then use $\Delta K.E + \Delta E^{(2)}_{thermal} = 0$ on the horizontal floor. where now $K.E_f = 0$ and $K.E_i$ is the K.E of the block at the bottom of the ramp.

$\Delta E^{(2)}_{thermal}$ for this part is just $\mu_k mg d_2$.

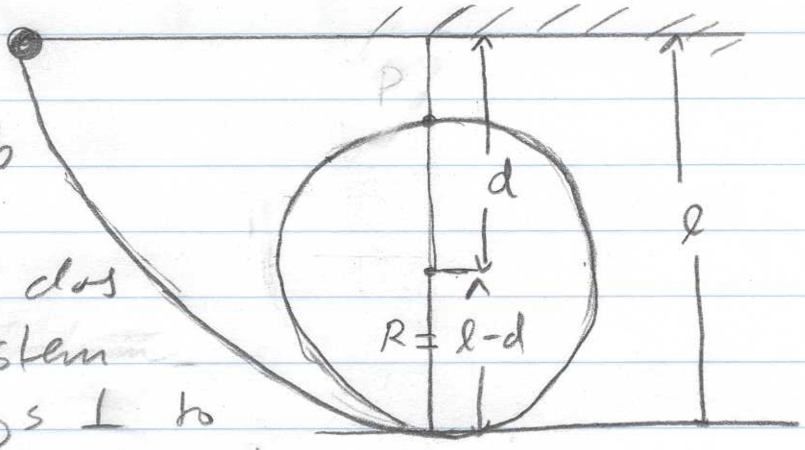
and $\Delta E^{(1)}_{thermal}$ is $\mu_k mg\cos\theta\, d_1$

P4/ S&B P70.

a/ Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg. So the ball swings equally high on both sides.

b/. Using Energy conservation for system: pendulum bob + earth

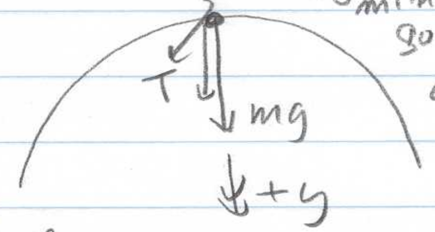
Note the string tension T does No work on the system because T is always $\perp$ to the tangential displacement.



So $\Delta E = \cancelto{0}{W_{ext}}$

$\Delta P.E_g + \Delta K.E = 0$

$\Rightarrow \boxed{P.E_f + K.E_f = P.E_i + \cancelto{0}{K.E_i}}$



(where $P.E_f$ and $K.E_f$ refer to P.E & K.E at the top of the swing where we want the bob to have minimum speed for it to go in a circle)

$\Rightarrow \boxed{2mg(l-d) + \frac{1}{2}mv^2 = mgl}$ eq 1.

Using $\Sigma F_{net,r} = ma_r \Rightarrow \cancelto{0}{T} + mg = \frac{mv^2}{R}$ set T=0 to find min. speed to go in a circle.

$\Rightarrow \frac{\cancel{m}v^2}{R} = \cancel{m}g \Rightarrow \boxed{v^2 = Rg}$ substitute $(R = l-d)$ in eq 1.

$\Rightarrow 2\cancel{mg}(l-d) + \frac{1}{2}\cancel{mg}(l-d) = \cancel{mg}\,l$

$\Rightarrow \frac{5}{2}(l-d) = l \Rightarrow \boxed{d = 3/5\, l}$

$\frac{5}{2}l - l = \frac{5}{2}d$

PS1. Consider the block-track-earth system.

$W_{ext} = 0$ so $\Delta E = 0$.

$\Rightarrow \Delta K.E + \Delta P.E = 0$

$\Rightarrow \Delta K.E = -\Delta P.E$

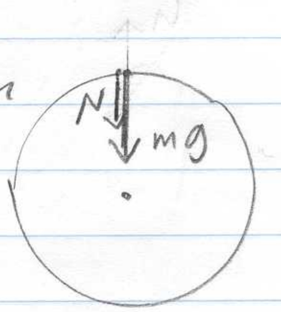
$K.E_f - \cancel{K.E_i}^{0} = -(P.E_f - P.E_i)$

$\Rightarrow K.E_f = -(mg(2R) - mgh)$

$= mgh - 2mgR$

$\boxed{K.E_f = mg(h-2R)}$ eq 1.

b/. cannot have a tangential acceleration at the top since both the normal force and F_g point in the radial direction at the top. There is no force in the tangential direction (assuming no friction.)



To find a_r, we need v – using eq 1 above,

$\frac{1}{2}\cancel{m}v^2 = \cancel{m}g(h-2R)$

$\Rightarrow v^2 = 2g(h-2R)$

$a_r = \frac{v^2}{R} \Rightarrow \boxed{a_r = \frac{2g(h-2R)}{R}}$

c/ For minimum speed, set $N = 0$. $\Rightarrow \sum F_{net,r} = ma_r$

$\Rightarrow \boxed{mg = \frac{mv^2}{R}}$

$P.E_i = P.E_f + K.E_f$

To find minimum h, use $mgh = 2mgR + \frac{1}{2}mv^2$

$\Rightarrow mgh = 2mgR + \frac{1}{2}mgR$

$\cancel{m}g h = \frac{5}{2}\cancel{mg}R \Rightarrow \boxed{h = 5/2\,R}$

P6/

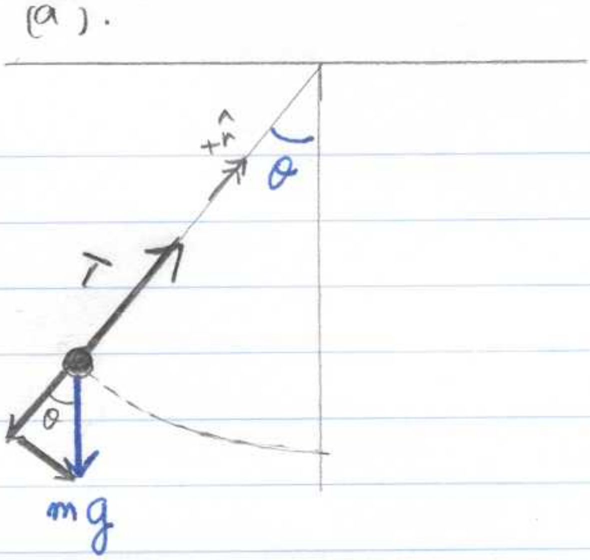
(b) $\Sigma F_{net,r} = m a_r$

$$T - mg\cos\theta = \frac{mv^2}{R}$$

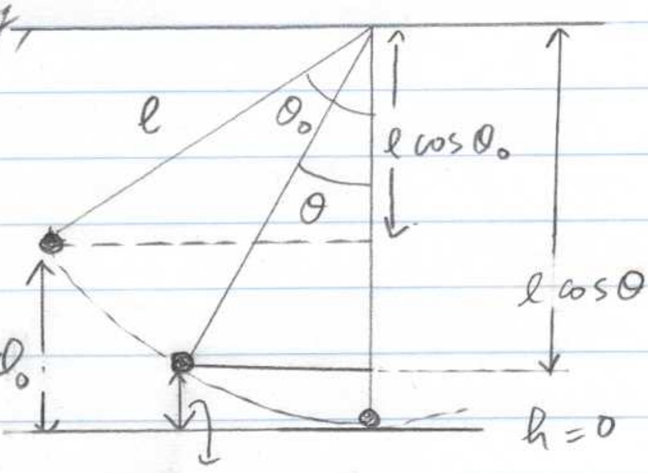
$$T = mg\cos\theta + \frac{mv^2}{R}$$

Since v increases as $\theta \to 0$ and $\cos\theta \to 1$ as $\theta \to 0$,
$\Rightarrow$ Tension T increases as the pendulum moves towards the bottom.

(a).



(C). Using conservation of energy,
$\Delta E = \cancel{W_{ext}}^{\,0}$ — considering the earth pendulum-bob system. (Note tension does no work.)



$h_f = l - l\cos\theta$

$\Delta E = 0$

$\Rightarrow \Delta P.E + \Delta K.E = 0$

$\Rightarrow P.E_i + \cancel{K.E_i}^{\,0} = P.E_f + K.E_f$

$$mgh_i = mgh_f + \frac{1}{2}mv_f^2$$

$$\cancel{m}gl(1-\cos\theta_0) = \cancel{m}gl(1-\cos\theta) + \frac{1}{2}\cancel{m}v_f^2$$

$$\Rightarrow gl - gl\cos\theta_0 = gl - gl\cos\theta + \frac{1}{2}v_f^2$$

$$\Rightarrow 2gl(\cos\theta - \cos\theta_0) = v_f^2$$

$$\Rightarrow \boxed{v_f = \sqrt{2gl(\cos\theta - \cos\theta_0)}}$$

d) $a_r = \frac{v^2}{l}$

$a_r = \frac{2g\cancel{l}(\cos\theta - \cos\theta_0)}{\cancel{l}}$

$\Rightarrow \boxed{a_r = 2g(\cos\theta - \cos\theta_0)}$

e). If $\theta_0 = 90°$

$v_f = \sqrt{2gl}$

$v_f = \sqrt{2g\Delta h}$ since in this case

$h_f = l(1-\cos\theta) = 0$ at the bottom and
$h_i = l(1-\cos\theta_0) = l(1-\cos 90°) = \underline{\underline{l.}}$

$\Rightarrow \Delta h =$