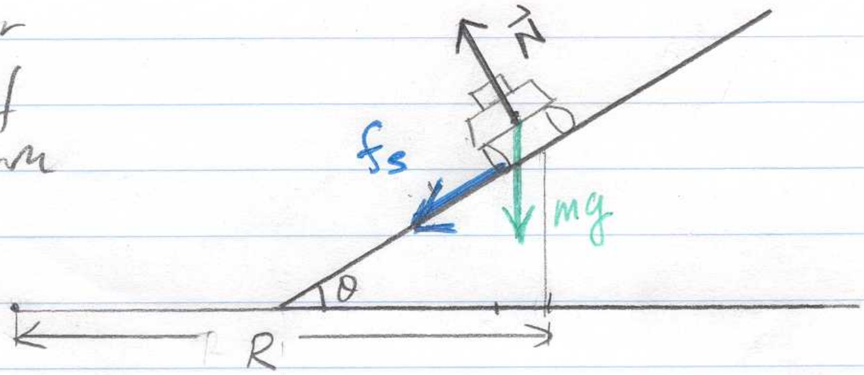
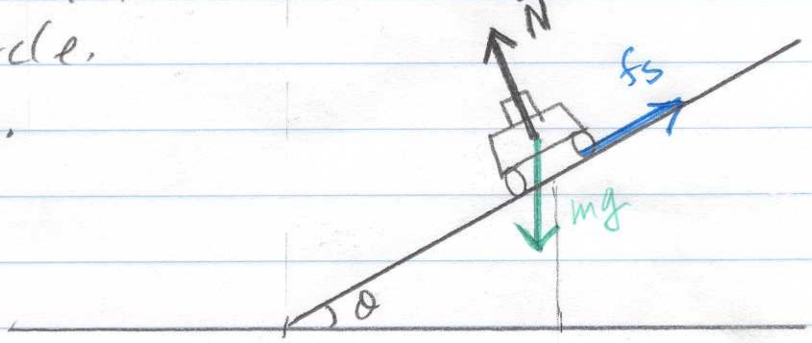
HW 5 Solutions, Phys 161, Spring 2003

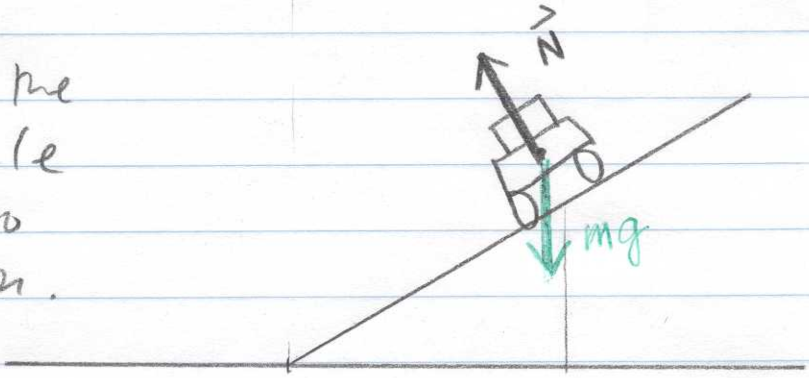
S1/. In this case the car will tend to slip out of the circle. So f_s is down the incline.



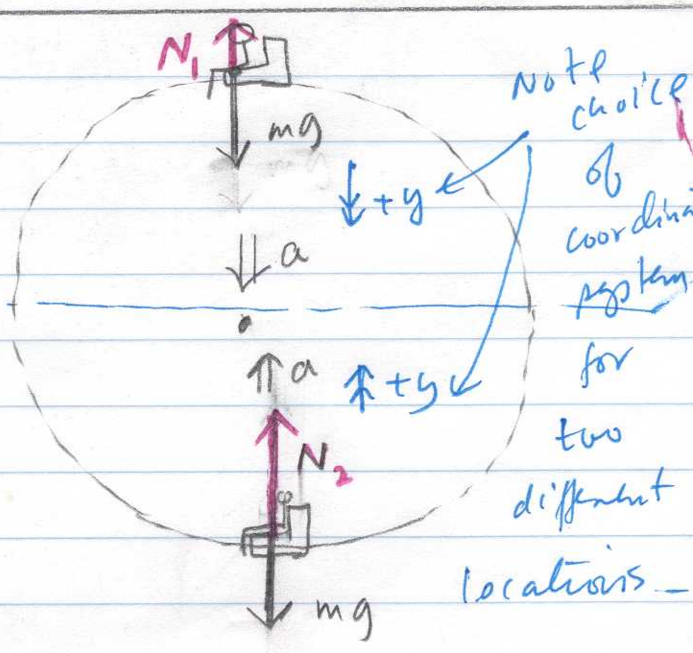
b/. In this case, the car will tend to slip into the circle. So f_s is up the incline.



c/. when $v \equiv v_{designated}$, the car goes around the circle without any tendency to slip in either direction. So $f_s = 0$.



S2/. rotating at constant rate means ω is constant which in turn means v is constant since $v = R\omega$. This implies that the magnitude of the radial acceleration is constant.



At the top: $\Sigma F_y = m a_y$.

$\Rightarrow \quad mg - N_1 = ma$

$$\boxed{mg - N_1 = \frac{mv^2}{R}} \text{ eq 1.} \quad \left(\text{I've used } a = \frac{v^2}{R}\right)$$

At the bottom: $\quad \Sigma F_y = ma_y$

$$N_2 - mg = ma$$

$$\Rightarrow \quad \boxed{N_2 = mg + \frac{mv^2}{R}} \text{ eq 2.}$$

Strategy: We know mg & N_1. To find N_2, we need to know $\frac{mv^2}{R}$. So use eq 1 to find $\frac{mv^2}{R}$.

Note that a lb (pound) is a unit of force. We needn't do any conversions here at all.

From eq 1: $\quad \frac{mv^2}{R} = mg - N_1 =$

$$= (150\,lb) - (110\,lb)$$

$$\frac{mv^2}{R} = 40\,lb.$$

$$\Rightarrow \quad N_2 = mg + \frac{mv^2}{R}$$

$$= 150\,lb + 40\,lb$$

$$\Rightarrow \quad \boxed{N_2 = 190\,lb}$$

S3/. (a) see fig S3.

(b) From fig S3, the net force on the water is downwards. From Newton's second law then it has an acceleration vertically downwards at the top of the circle. The water doesn't spill because it has a velocity tangent

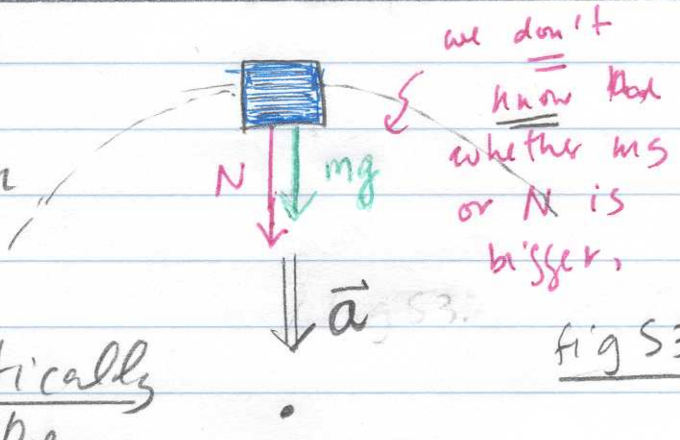


fig S3

to the circle, which at the top is completely horizontal. Having $\vec{a}$ vertically downwards then simply ensures that a moment later, the velocity of the water will change in a manner such that it is again tangential to the circle. Let's recall our vector addition & subtraction rules; & also that $\vec{a}_{avg}$ is in the direction of $\Delta\vec{V}$ since $\vec{a}_{avg} = \frac{\Delta\vec{V}}{\Delta t}$. Then

$$\vec{V}_f = \vec{v}_i + \Delta\vec{V}$$

so, a moment later, the water's velocity will be in the direction given by

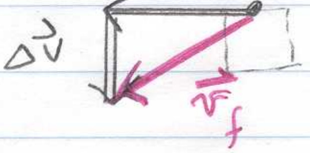
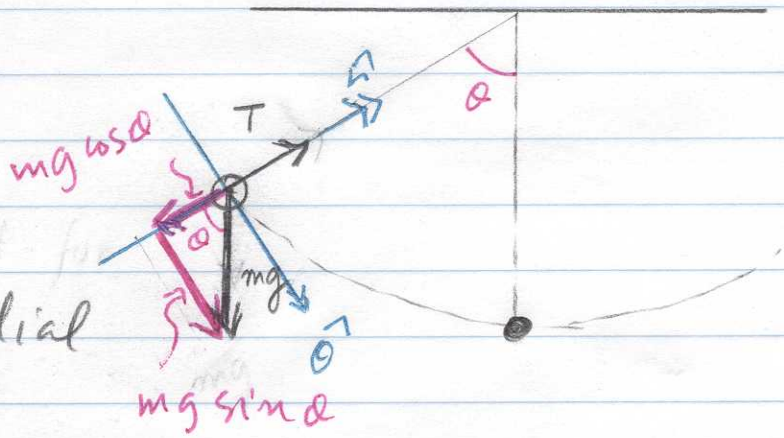


fig S3 b.

fig S3 b and <u>not</u> vertically downwards — which would be the case if the water were to spill downwards.

S41.

a) At A, $v = 0$, so $a_r = \frac{v^2}{R} = 0$. This implies that the net force in the radial direction must be zero.



$$\Sigma F_{net,\hat{r}} = m a_r^{\nearrow 0} \text{ at A.}$$

$$T - mg\cos\theta = 0$$

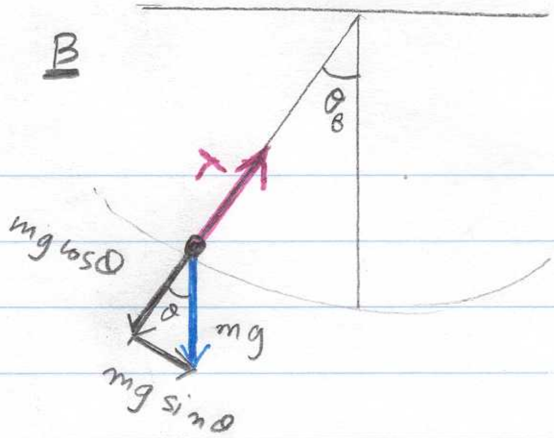
$$\Rightarrow \boxed{T = mg\cos\theta}$$

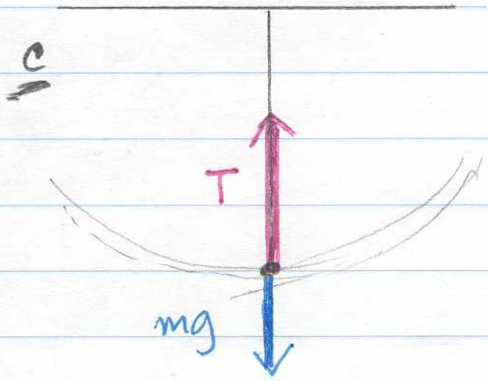
The net force is then in the $\hat{\theta}$ direction.

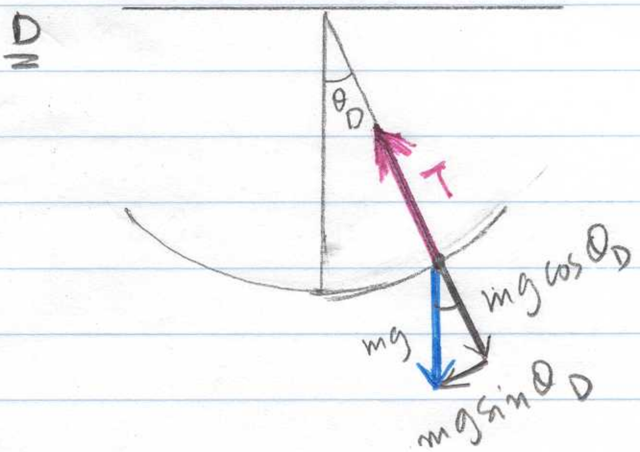
$\Sigma F_{net,\hat{\theta}} = m a_{tang.}$

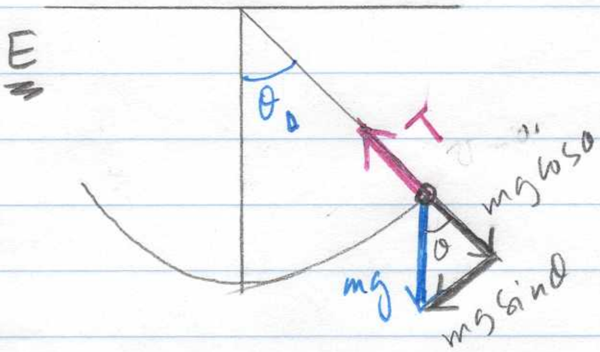
$\cancel{m} g \sin\theta = \cancel{m} a_{tang}$

$\Rightarrow \boxed{a_{tang} = g \sin\theta}$



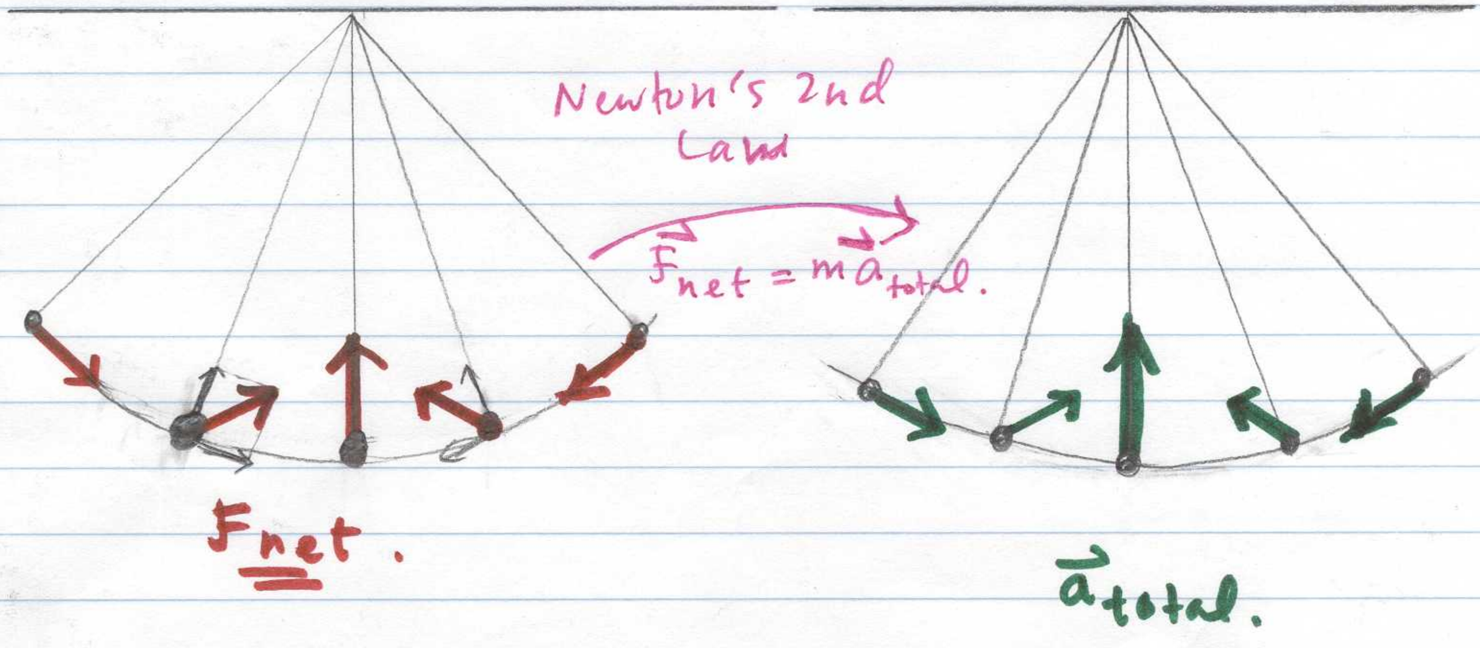






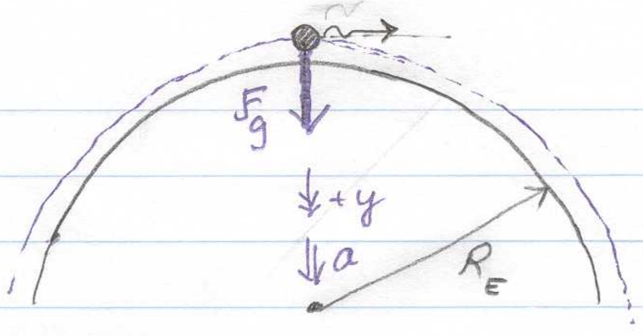
$v_E = 0. \Rightarrow a_r = 0 \Rightarrow T = mg\cos\theta$

and $a_{tan} = g\sin\theta$ as at A.



P1/. $\Sigma F_{net,y} = m_s a_y$

$$F_g = m_s \frac{v^2}{R_E}$$



$$\frac{G m_E \cancel{m_s}}{R_E^{\cancel{2}}} = \cancel{m_s} \frac{v^2}{\cancel{R_E}}$$

$$\Rightarrow v^2 = \frac{G m_E}{R_E}$$

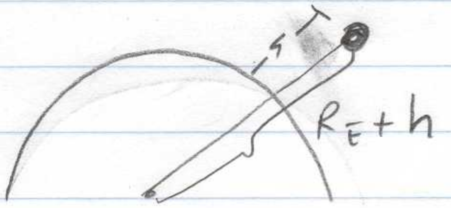
$$\Rightarrow \boxed{v = \sqrt{\frac{G m_E}{R_E}}}$$

Please note that $\frac{G m_E}{R_E^2}$ is what we call g, the acceleration due to gravity. so $\boxed{v = \sqrt{R_E g}}$.

$$T = \frac{2\pi R_E}{v} \Rightarrow \boxed{T = 2\pi \sqrt{\frac{R_E}{g}}} \text{ or } T = 2\pi \sqrt{\frac{R_E^3}{G m_E}}$$

(b). If the satellite is some height h above the surface of the earth, then the distance between the center of the satellite & the center of the earth is $R_E + h$.

Then $F_g = \frac{G m_E m_s}{(R_E + h)^2}$



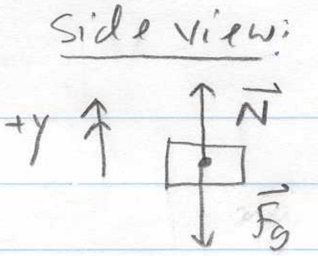
$$\Rightarrow v = \sqrt{\frac{G m_E}{R_E + h}}.$$

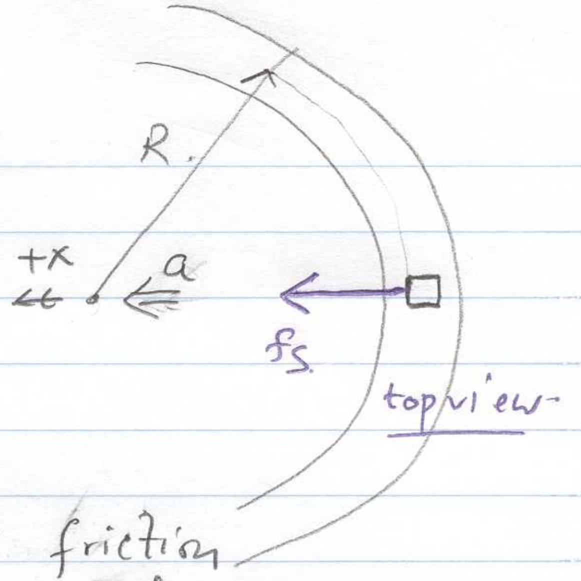
So the speed required for a circular orbit is <u>less</u>. which in turn implies that $T = \frac{2\pi (R_E + h)}{v}$

$\Rightarrow T = 2\pi \sqrt{\frac{(R_E+h)^3}{G m_E}}$ i.e T increases. (smaller speed & longer distance).

(c) v & T don't depend on the mass of the satellite at all. So the answers to parts a & (b) will remain the same if m_s is doubled.

P3/.





$$\Sigma F_x = m a_x$$

$$f_s = m \frac{v^2}{R}$$

Recall that the magnitude of static friction is such that $0 \leq f_s \leq \mu_s N$. To find the <u>minimum</u> value of μ_s, we must have

$$f_{s,max} = m \frac{v^2}{R}$$

$$\Rightarrow \mu_s N = m v^2 / R$$

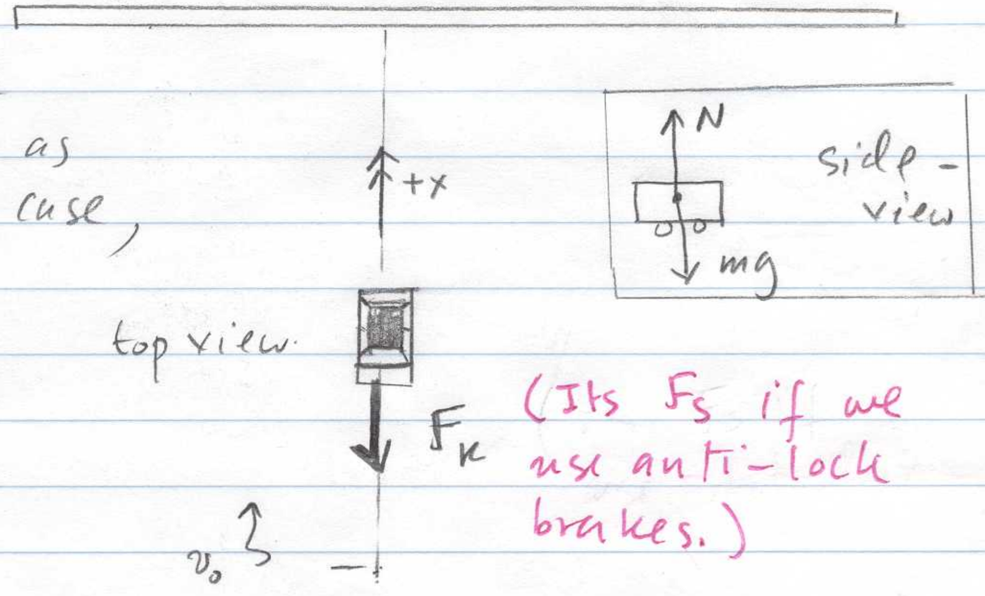
To find N, we use $\Sigma F_y = m a_y$

$$\Rightarrow N - mg = 0 \Rightarrow \boxed{N = mg}, \text{ so}$$

$$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{R} \Rightarrow \boxed{\mu_s = \frac{v^2}{Rg}}$$

P4/. <u>Braking</u>:

Let's consider the option of simply braking as hard as we can first. In this case, there will be a rearward force F_K on the car due to kinetic friction. If we use antilock brakes, it will be F_s.



(Its F_s if we use anti-lock brakes.)

$\Rightarrow \Sigma F_x = m.a_x$

$\Rightarrow -F_k = m a_x \Rightarrow \boxed{a_x = -\frac{F_k}{m}}$ or $\left(-\frac{F_s}{m}\right.$ for antilock brakes.$\left.\right)$

using the kinematic equations:

$$v_f^2 = v_0^2 + 2a\Delta x$$

$$0 = v_0^2 + 2a\Delta x \Rightarrow \Delta x = -\frac{v_0^2}{2a}$$

using $a = -\frac{F_k}{m}$ we find: $\Delta x = -\frac{v_0^2}{2(-F_k/m)}$

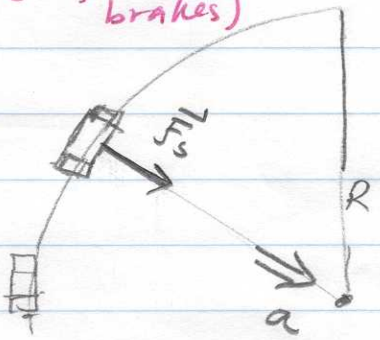
$$\Rightarrow \Delta x = \frac{1}{2}\frac{m v_0^2}{F_k}.$$

using $F_k = \mu_k N = \mu_k mg$, we find

$$\Delta x = \frac{1}{2}\frac{\cancel{m} v_0^2}{\mu_k \cancel{m} g} \Rightarrow \boxed{\Delta x = \frac{1}{2}\frac{v_0^2}{\mu_k g}}$$ stopping distance.

$\left(\Delta x = \frac{1}{2}\frac{v_0^2}{\mu_s g}\right.$ ⇐ for Anti-lock brakes$\left.\right)$

__Turning without braking:__



$\Sigma F_r = m a_r$

$F_{sF} = m\frac{v_0^2}{R}$, since $f_{sf\,max} = \mu_s N$

$$\Rightarrow \mu_s N = \frac{m v_0^2}{R}$$

$$\Rightarrow \mu_s \cancel{m} g = \frac{\cancel{m} v_0^2}{R} \Rightarrow \boxed{R = \frac{m v_0^2}{\mu_s g}}$$

Note that our clearance from the wall is $D - \Delta x$, and $D - R$, since $\Delta x_{braking} \leq \frac{1}{2}R$, assuming $\mu_s \cong \mu_k$ (though in general μ_s is a bit bigger than μ_k) we see that we have twice as much clearance if we simply brake as hard as we can without turning.

For the case where the car has anti-lock brakes, the above result is EXACT.

(b). Estimates: $D = 50\,m$. Let's assume $\mu_s \cong \mu_k \cong 0.5$.
So now we want to stop in a distance D.
Since we're solving for maximum velocity possible with which we could be driving at the instant we see the wall & still be able to stop.

So $D = \frac{1}{2}\frac{v_0^2}{\mu_s g} \Rightarrow v_0^2 = 2\mu_s g D$

$= 2(0.5)(9.8\,m/s^2)(50\,m)$

$= 490\,m^2/s^2$

$\Rightarrow \boxed{v_0 \cong 22\,m/s}$ This is corresponds to

$\sim v_0 \sim (22\,m/s)\left(\frac{2.24\,mi/hr}{m/s}\right) = 49\,mi/hr$ — so roughly

50 mi/hr.

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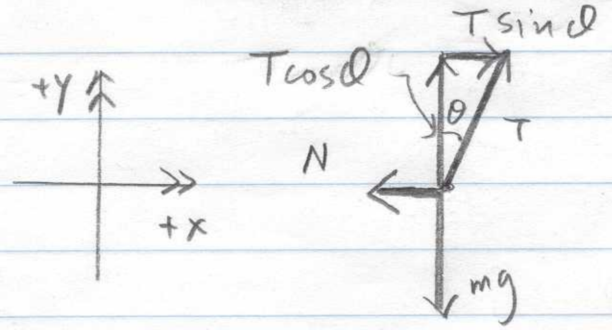
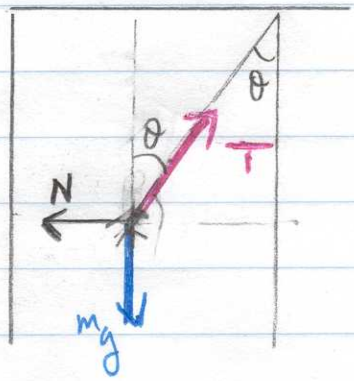
PS/





$\Sigma F_{net,y} = 0$ since man at rest

$\Rightarrow T\cos\theta - mg = 0$

$\Rightarrow \boxed{T = \frac{mg}{\cos\theta}}$

(Note $\Sigma F_x = m a_x^{\,0}$

$\Rightarrow -N + T\sin\theta = 0$

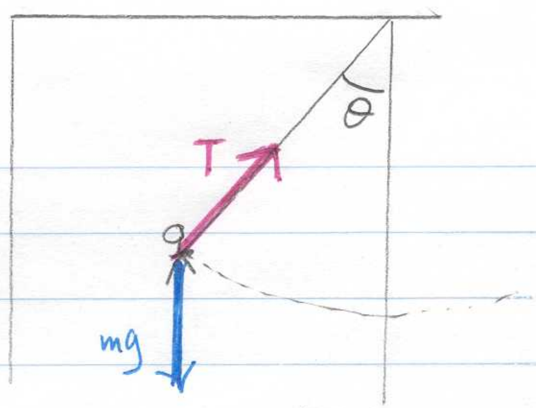
$N = T\sin\theta$

$\boxed{N = mg\tan\theta}$)

graders: this was not asked for.

b/. Immediately after he releases his hold from the wall, $N = 0$ since he's no longer in contact with the wall. Now he's exactly like a pendulum bob being released at rest at an angle θ with the vertical

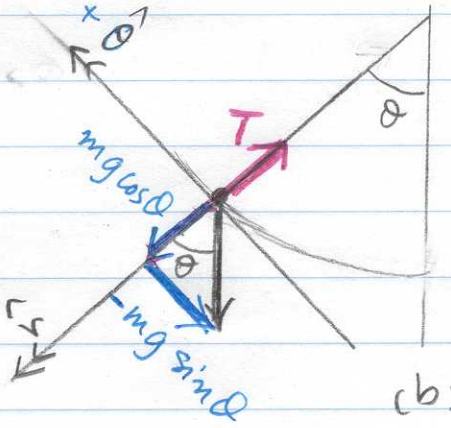
Since we now have a tangential acceleration, it's best to use the $\hat{r}, \hat{\theta}$ coordinate system.



(b)

Also recall that at the moment he lets go of the rock, he's at rest so $v=0$ and therefor $a_r = \frac{v^2}{r} = 0$.

Therefore $\sum F_{net,r} = m\cancel{a_r}^{\,0}$.



(b).

$\Rightarrow \quad -T + mg\cos\theta = 0$

$\Rightarrow \quad \boxed{T = mg\cos\theta}$

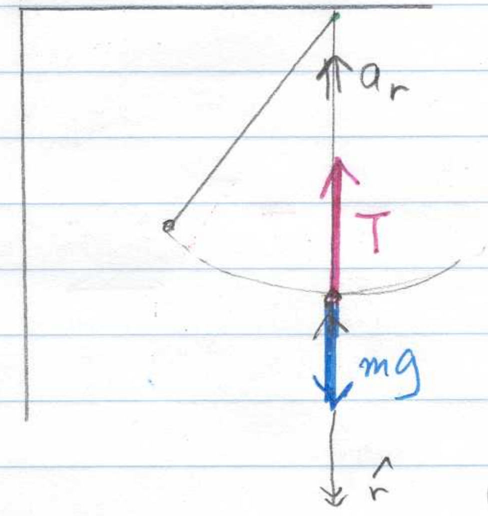
c/. At the bottom of the swing:

$\sum F_{net,r} = m a_r$

$-T + mg = -m\frac{v_B^2}{L}$

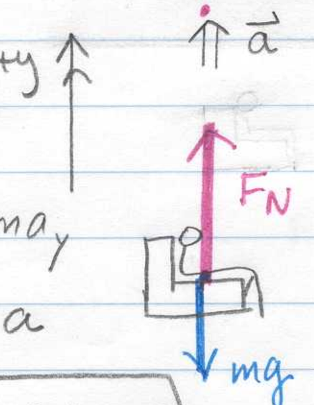
Note that these minus signs are there because $+\hat{r}$ at this point points downward. See fig (c).

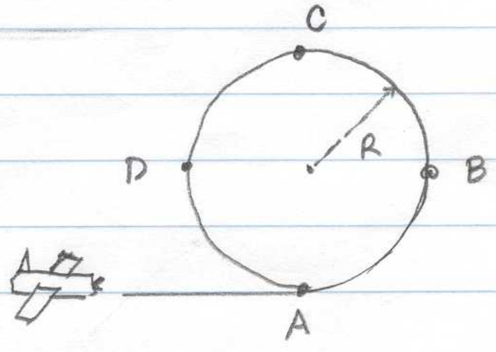
$\Rightarrow \quad \boxed{T = mg + \frac{m v_B^2}{L}}$



(c).

P6/.



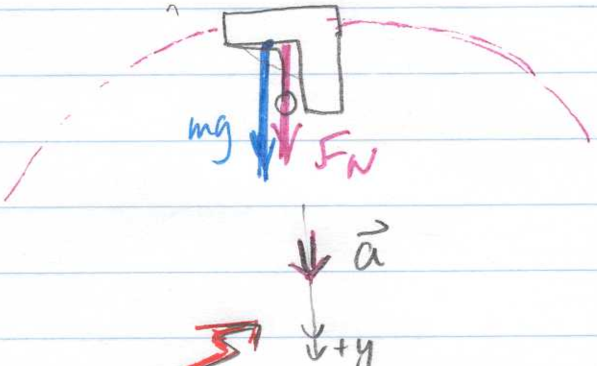


a/. $\sum F_{net,y} = m a_y$

$F_N - mg = ma$

$\Rightarrow \boxed{F_N = mg + ma}$ at point A.

$\Rightarrow F_N = mg + \dfrac{mv^2}{R}$. F_N can only point upward at A. Also, it cannot be zero. Otherwise there is not net force pointing radially inwards.



(b). Now the pilot is upside down.

$$\Sigma F_{net,y} = ma_y$$

$$F_N + mg = \frac{mv^2}{R}$$

$$\Rightarrow F_N = \frac{mv^2}{R} - mg$$

Pay attention to the choice of coordinate system & +/- signs for forces and accelerations.

Now F_N can be zero when

$$\frac{\cancel{m}v^2}{R} = \cancel{m}g \Rightarrow \boxed{v = \sqrt{Rg}}$$

otherwise, it's directed downwards.

(c). As mentioned above, F_N can be zero at C but not at A and nowhere else. In this case, from part (b), we see that if $F_N = 0$, $\Rightarrow v = \sqrt{Rg} = \sqrt{(350\,m)(9.8\,m/s^2)}$

$$= 58.6\,m/s.$$

d). Suppose the pilot flies around with constant speed. Then, $a_{tangential} = 0$ and $a_r = \dfrac{v^2}{R}$.

$$\Rightarrow 8g = \frac{v^2}{R} \Rightarrow v^2 = 8Rg$$

$$\Rightarrow v = 2\sqrt{2}\sqrt{Rg}$$

$$\boxed{v = 165\,m/s}$$

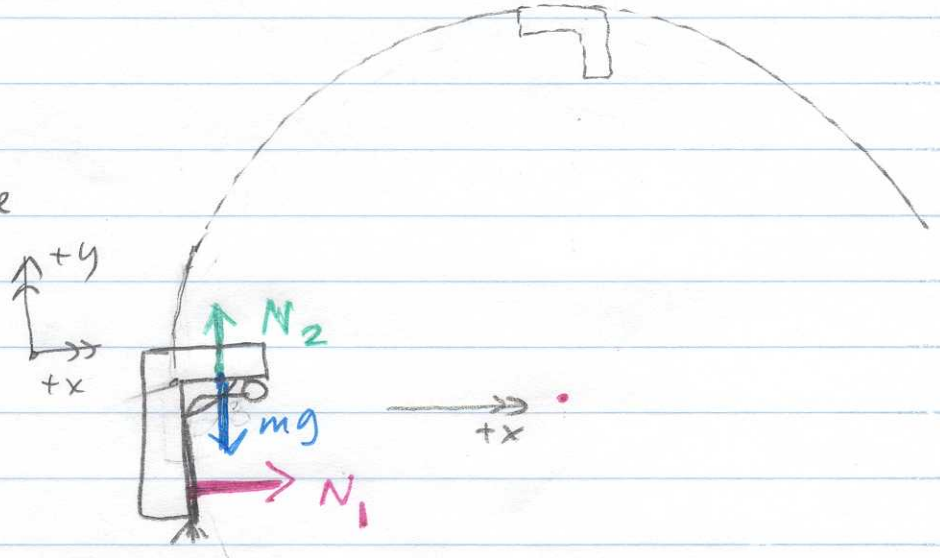
e) Since the stunt pilot is flying around at constant speed, at D (as at any other point) its tangential acceleration is zero. It has only a radial acceleration. Therefore,

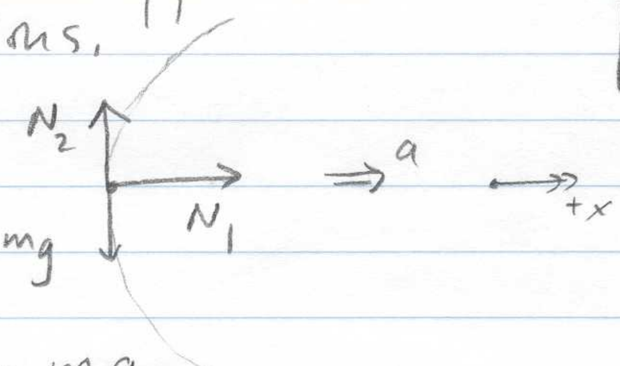
$$F_{net,y} = m\cancel{a_y}^{0}$$

$$N_2 - mg = 0$$

$$\Rightarrow \quad N_2 = mg.$$

we know this must be the case because $a_y = 0$. So $\underline{N_2}$ and $\underline{mg}$ have to point in opposite directions.





$$\Sigma F_x = m a_x$$

$\boxed{N_1 = \frac{mv^2}{R}}$ and N_1 points towards the center of the circle. Not $\vec{F}_{net} = \vec{N}_1$. since mg & N_2 add up to zero.