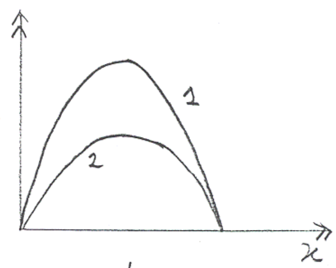
Homework 3 Solutions – Physics 161, SPG 03

S1/. a/. Since $h_{max} = \frac{v_{oy}^2}{2g}$, we can definitely say that $v_{oy1} > v_{oy2}$.



b/. No. Flight time depends on v_{oy} – as we saw $t_{flight} = \frac{2 v_{oy}}{g}$ – so $t_{flight\,1} > t_{flight\,2}$.

c/. Since $t_{flight\,1} > t_{flight\,2}$ but $x_{max\,1} = x_{max\,2}$

& $x_{max} = v_{ox} t_{flight} \Rightarrow v_{ox1} < v_{ox2}$.

It is possible for the two projectiles to have been thrown with the same initial speed.

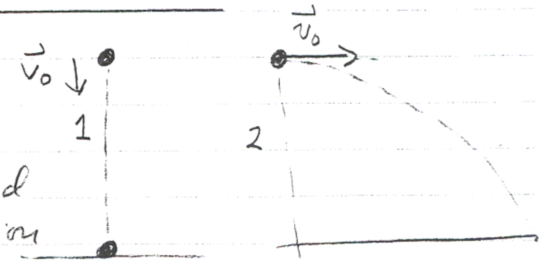
$$x_{max_1} = x_{max_2} \Rightarrow \frac{\cancel{2} v_{oy1} v_{ox1}}{g} = \frac{\cancel{2} v_{oy2} v_{ox2}}{g}$$

$v_{01}^2 \sin 2\theta_1 = v_{02}^2 \sin 2\theta_2$. If $v_{01} = v_{02}$, we find $\sin 2\theta_1 = \sin 2\theta_2$.

Since $v_{oy_1} \neq v_{oy_2}$, $\theta_1 \neq \theta_2$. It turns out that there is a solution for $\theta_1 \neq \theta_2$. If $\theta_1 = 45° + \alpha$ and $\theta_2 = 45° - \alpha$, we can still have $x_{max_1} = x_{max_2}$.

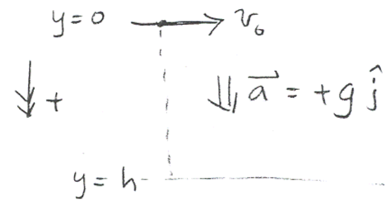
(Also see P2, part c)

S2/. a/ The ball that is thrown vertically downwards wins the race because as far as the vertical motion is concerned only velocity in the y direction matters. Therefore, the first ball has an advantage because it starts out with some



initial velocity in the y-direction and picks up more speed due to acceleration due to gravity.
As far as the vertical motion is concerned, ball 2 starts out with $v_{oy} = 0$. So it loses the race.

(b). <u>Ball 2</u>: $v_{oy} = 0$.

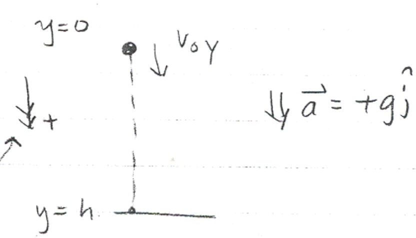


$$y = y_0 + v_{oy} t + \frac{1}{2} a_y t^2$$

$$h = 0 + 0 + \frac{1}{2} g t^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}}.$$

<u>Ball 1</u>:



$$y = y_0 + v_{oy} t + \frac{1}{2} a_y t^2$$

$$h = 0 + v_{oy} t + \frac{1}{2} g t^2$$

$$\Rightarrow \quad 0 = -h + v_{oy} t + \frac{1}{2} g t^2$$

$$\Rightarrow \quad t^2 + \frac{2 v_{oy}}{g} t - \frac{2h}{g} = 0$$

$$\Rightarrow \quad t = \frac{-\frac{2v_{oy}}{g} \pm \sqrt{\frac{4 v_{oy}^2}{g^2} + \frac{8h}{g}}}{2}$$

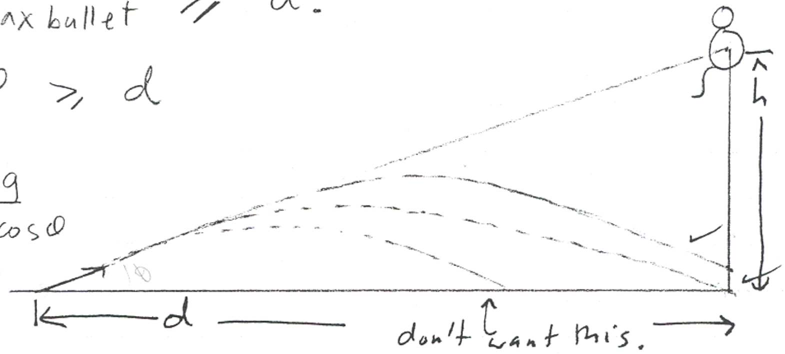
$$= -\frac{v_{oy}}{g} \pm \frac{v_{oy}}{g}\sqrt{1 + \frac{2hg}{v_{oy}^2}}$$

The question is, which sign should we keep. t must be greater than zero. Since the second term is larger, we must have

$$t = \frac{v_{oy}}{g}\left[\sqrt{1 + \frac{2hg}{v_{oy}^2}} - 1\right]$$

S3/. The point is that we don't want the bullet to bite the dust before it gets to the monkey i.e we want

$$x_{max\ bullet} \geq d.$$



$$\Rightarrow 2\frac{v_0^2 \sin\theta\cos\theta}{g} \geq d$$

$$v_0 \sin\theta \geq \frac{dg}{2v_0\cos\theta}$$

But since
$x_b = v_0 \cos\theta\, t$,

$\frac{d}{v_0\cos\theta}$ is the time $t = t^*$ it would take the bullet to get to $x = d$. The time t^* on the other hand is less than or equal to the time it takes the monkey to drop to the ground i.e

$$t^* \leq \sqrt{\frac{2h}{g}}$$ so

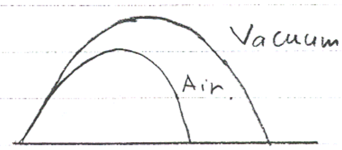
(In the book, $h \equiv d$).

$$v_0 \sin\theta \geq \frac{t^* g}{2}$$

$$\geq \sqrt{\frac{2h}{g}} \cdot \frac{g}{2} \Rightarrow \boxed{v_0 \sin\theta \geq \sqrt{\frac{hg}{2}}}$$

S4/.

Since the horizontal component of velocity decreases gradually and the vertical component drops to zero, and since the drag force is $\propto v^2$, the effect of drag is greater on the range than on the maximum height.



Problem S5:

Note that the two things we're concerned about here is the initial velocity in the y-direction as well as the acceleration in the y-direction.

Student 1:

Since S1 is moving with constant speed w.r.t Damian, we have

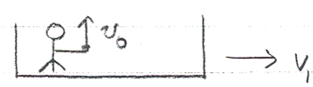
$$\vec{V}_{BD} = \vec{V}_{BS_1} + \vec{V}_{S_1D}$$

$$v_{BDY} = +v_0 + V_1$$

$$\vec{a}_{BD} = \vec{a}_{BS_1} + \cancel{a_{S_1D}}^{\,0} \Rightarrow \vec{a}_{BD} = g$$

$$\Rightarrow \underset{\text{according to Damian}}{t_{max1}} = \frac{v_{oy}}{g} = \frac{v_0 + V_1}{g}, \quad \underset{\text{according to } S_1}{t_{max1}} = \frac{v_0}{g}.$$

Student 2: $\vec{V}_{BD} = \vec{V}_{BS_2} + \vec{V}_{S_2D}$

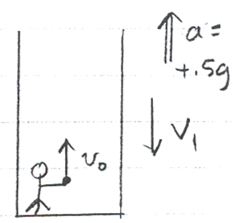


$$v_{BDoy} = V_{BS_2oy} + V_{S_2Doy}$$

$$v_{BDoy} = v_0, \quad \text{and } \vec{a}_{BD} = g \text{ as above}$$

so
$$\boxed{\underset{\text{according to Damian}}{t_{max2}} = \frac{v_0}{g}}; \quad \underset{\text{according to } S_2}{t_{max2}} = \frac{v_0}{g},$$

Student 3. $\vec{V}_{BD} = \vec{V}_{BS_2} + \vec{V}_{S_2D}$



$$V_{BDoy} = V_{BS_2oy} + V_{S_2Doy}$$

$$V_{BDoy} = v_0 - V_1$$

$\vec{a}_{BD} = g \Rightarrow t^{(3)}_{max \text{ according to Damian}} = \dfrac{v_0 - V_1}{g}$

$\vec{a}_{BS_3} = ?$ Use $\vec{a}_{BS_2} = \vec{a}_{BD} + \vec{a}_{DS_2}$

$= \vec{a}_{BD} - \vec{a}_{S_2D}$

$\Rightarrow a_{BS_2y} = -g - (+\tfrac{1}{2}g)$

$\boxed{a_{BS_2y} = -\tfrac{3}{2}g} \Rightarrow t_{max\, S_3} = \dfrac{v_0}{\tfrac{3}{2}g} = \dfrac{2}{3}\dfrac{v_0}{g}$

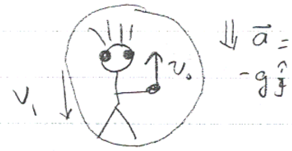
student 4. Now we've got the hang of it, so

$t_{max \text{ Damian}} = \dfrac{v_0 + V_1}{g}$

$t_{max\, S_4} = \dfrac{v_0}{\tfrac{3}{2}g} \Rightarrow t_{max\, S_4} = \dfrac{2}{3}\dfrac{v_0}{g}$.

student 5. Freely falling Student.

$t_{flight \text{ Damian}} = \dfrac{v_0 - V_1}{g}$



$\vec{a}_{BS_5} = \vec{a}_{BD} + \vec{a}_{DS_5}$

$= \vec{a}_{BD} - \vec{a}_{S_5D}$

$a_{BS_5y} = -g - (-g)$

$a_{BS_5y} = 0$!

This implies that the ball does not accelerate

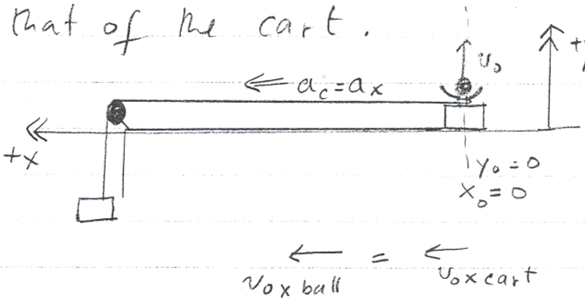
with respect to the freely falling student. It keeps moving upwards with velocity v_0. So there is no instant where the ball comes momentarily to rest w.r.t S_5. So t_{max} is not applicable here!

so $t_{max\,S_1} = t_{max\,S_2} > t_{max\,S_3} = t_{max\,S_4}$.

$t^{(1)}_{max\,Dam} = t^{(4)}_{max\,Dam} > t^{(2)}_{max\,Dam} > t^{(3)}_{max\,Dam} = t^{(5)}_{max\,Dam}$.

Problem P1.

(a). The key difference is that in case I, the ball has's acceleration does not match the acceleration of the cart in the x-direction. However in case II, even when the ball is air-borne, its acceleration i.e $\vec{a} = -g\hat{j}$ has a component parallel to the incline whose magnitude <u>and</u> direction is the same as the cart's acceleration along the incline. Since the ball has the same speed along the incline when the ball leaves the cart, <u>and</u> its accel. along the incline is also the same, it is <u>always</u> directly above the cart. So it falls back in the cart when its vertical position matches that of the cart.



(b). Case I:

$$\vec{r}_c = (\cancel{x_0} + v_{0xc}t + \frac{1}{2}a_{xc}t^2)\hat{i} + (\cancel{y_0} + \cancel{v_{0yc}t} + \frac{1}{2}\cancel{a_{yc}}t^2)\hat{j}$$

$$\Rightarrow \vec{r}_c = (v_{0xc}t + \frac{1}{2}a_{xc}t^2)\hat{i}$$

$$\vec{r}_{ball} = (\cancel{x_0} + v_{0xb}t + \frac{1}{2}a_{xb}t^2)\hat{i} + (\cancel{y_{0b}} + v_{0yb}t + \frac{1}{2}a_{yb}t^2)\hat{j}$$

$$= (v_{0xb}t)\hat{i} + (v_{0yb}t - \frac{1}{2}gt^2\hat{j})$$

For the ball to fall back in the cart at some time $t = t^*$, we must have $\vec{r}_{cart}(t^*) = \vec{r}_{ball}(t^*)$

$$\Rightarrow x_c(t^*) = x_b(t^*) \;\&\; y_c(t^*) = y_b(t^*).$$

$$y_c = y_b$$

$$\Rightarrow 0 = v_{0yb}t - \frac{1}{2}gt^2 \Rightarrow \boxed{t^* = \frac{2v_{0y}}{g}}$$

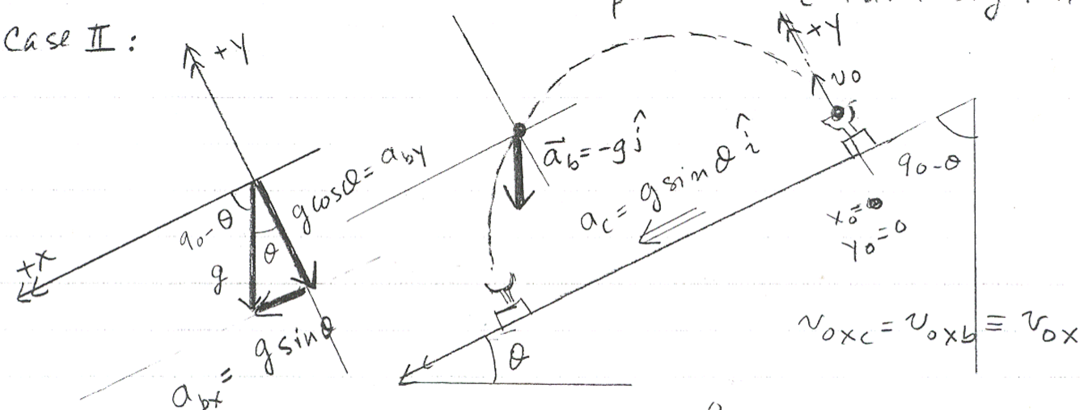
But imposing $\quad x_c(t^*) = x_b(t^*)$

$$\Rightarrow \quad v_{oxc}\, t + \frac{1}{2} a_{xc} t^2 = v_{oxb}\, t$$

since $v_{oxc} = v_{oxb}$, we find $\quad \frac{1}{2} a_{xc} t^2 = 0.$

i.e the only solution for $a_{xc} \neq 0$ is $t = 0$. So the ball does not ever meet up with the cart again.

Case II:



$$\vec{r}_{cart} = \left(v_{oxc} t + \frac{1}{2} a_{xc} t^2\right)\hat{i} + \left(\cancel{v_{oyc}}^{\,0} t + \frac{1}{2} \cancel{a_{yc}}^{\,0} t^2\right)\hat{j}$$

$$\vec{r}_{cart} = \left(v_{ox} t + \frac{1}{2} g\sin\theta\, t^2\right)\hat{i}$$

$$\vec{r}_{ball} = \left(v_{oxb} t + \frac{1}{2} a_{xb} t^2\right)\hat{i} + \left(v_{oyb} t + \frac{1}{2} a_{yb} t^2\right)\hat{j}$$

$$= \left(v_{ox} t + \frac{1}{2} g\sin\theta\, t^2\right)\hat{i} + \left(v_{oyb} t - \frac{1}{2} g\cos\theta\, t^2\right)\hat{j}$$

To have $\vec{r}_{cart}(t) = \vec{r}_{ball}(t)$, we must have

$$\Rightarrow \quad x_c(t=t^*) = x_b(t=t^*) \quad \& \quad y_c(t=t^*) = y_b(t=t^*)$$

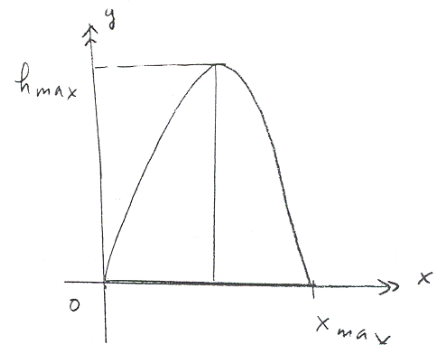
1 $\quad x_c = x_b$

$$\Rightarrow \quad v_{ox} t + \frac{1}{2} g \sin\theta\, t^2 = v_{ox} t + \frac{1}{2} g \sin\theta\, t^2$$

So $x_c = x_{ball}$ all the time! So when $y_b = y_c = 0$ the ball falls back in the cart at $\boxed{t^* = \frac{2 v_{oyb}}{g\cos\theta}}$

P2 / . (a). Need $x_{max} = h_{max}$.

Recall at $y = h_{max}$, $v_y = 0$. So



$$v_{fy}^2 = v_{oy}^2 + 2a\Delta y$$

$$0 = v_{oy}^2 - 2g(y_f - y_o)$$

$$0 = v_{oy}^2 - 2g(h_{max} - 0)$$

$$\Rightarrow \quad h_{max} = \frac{v_{oy}^2}{2g}$$

$$t_{max} = \frac{v_{oy}}{g} \Rightarrow t_{flight} = \frac{2v_{oy}}{g} = \frac{2v_0 \sin\theta}{g}$$

$$x_{max} = (v_0 \cos\theta)\, t_{flight} = v_0 \frac{\cos\theta \cdot 2v_0 \sin\theta}{g}$$

$$= \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

$$h_{max} = x_{max} \Rightarrow \frac{\cancel{v_0^2}\sin^{\cancel{2}}\theta}{2\cancel{g}} = \frac{2\cancel{v_0^2}\sin\cancel{\theta}\cos\theta}{\cancel{g}}$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = 4 \Rightarrow \tan\theta = 4$$

$$\Rightarrow \theta = \tan^{-1}4 \doteq 76^\circ.$$

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b/. $x_{max} = \frac{1}{2} h_{max}$

$$\frac{2\cancel{v_0^2}\sin\cancel{\theta}\cos\theta}{\cancel{g}} = \frac{\cancel{v_0^2}\sin^{\cancel{2}}\theta}{4\cancel{g}} \Rightarrow \tan\theta = 82.9^\circ.$$

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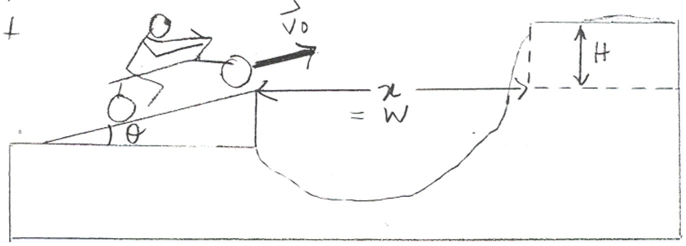
c/ $$x_{max}(\theta = 45^\circ \pm \alpha) = \frac{2v_0^2 \sin 2\theta}{g} = \frac{2v_0^2 \sin(90^\circ \pm 2\alpha)}{g}$$

$$= \frac{2v_0^2}{g}\left[\sin 90^\circ \cos 2\alpha \pm \cancel{\cos 90}^{\,0} \sin 2\alpha\right] = \frac{2v_0^2 \cos 2\alpha}{g}.$$

so for both $\theta = 45^\circ + \alpha$ & $45^\circ - \alpha$, $\boxed{x_{max} = \frac{2v_0^2\cos 2\alpha}{g}}$

Problem P4.

a/. The upper limit for H corresponds to the limit when $V_0 \to \infty$. In that case, the trajectory of the motorbike is essentially a straight line. (See Fig P4b.)



Therefore $\dfrac{H}{W} = \tan\theta$

or $H = W\tan\theta$.

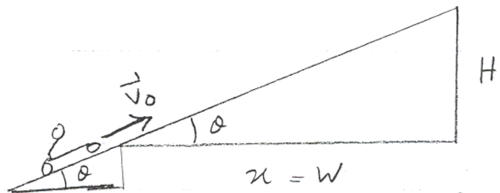
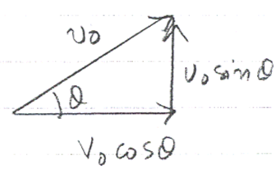


Fig P4b.

In real life, $V_0 \to \infty$ simply means V_0 is very very large. But what does it mean to say very very large? It must be very large compared to something, we will see that shortly.

b/. $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$



$$H = 0 + v_0\sin\theta\, t - \frac{1}{2}gt^2$$

At the time that the motorbike is at $y = H$, we want it to be at $x = W$.

$$x = \cancel{x_0}^{\,0} + V_{0x}t + \frac{1}{2}\cancel{a_x}^{\,0}t^2$$

$$W = V_0\cos\theta\, t \Rightarrow t = \frac{W}{V_0\cos\theta}$$

$$\Rightarrow H = \cancel{V_0}\sin\theta \cdot \frac{W}{\cancel{V_0}\cos\theta} - \frac{1}{2}g\left(\frac{W}{V_0\cos\theta}\right)^2$$

$$H = W\tan\theta - \frac{1}{2}g\frac{W^2}{V_0^2\cos^2\theta}$$

[Note that when $V_0^2 \ggg gW$, $H \to W\tan\theta$.

That means that $v_0 \to \infty$ is physically the same thing as saying that v_0 is much much larger than $\sqrt{gW}$. Note that $\sqrt{gW}$ has dimensions of speed. $\left[\sqrt{\frac{L}{T^2} \cdot L} = \sqrt{\frac{L^2}{T^2}} = L/T.\right]$.

$$\Rightarrow \quad \frac{W^2}{v_0^2 \cos^2\theta} = \frac{2}{g}(W\tan\theta - H)$$

$$\Rightarrow \quad v_0^2 = \frac{gW^2}{2(W\tan\theta - H)\cos^2\theta}.$$

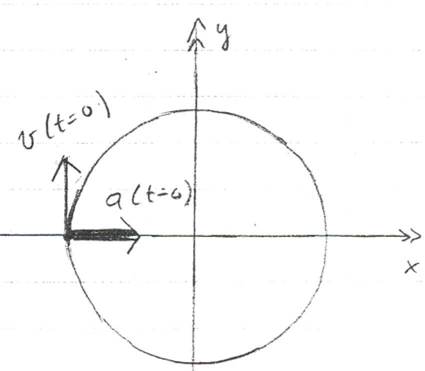
when $\theta = 0$ $\quad v_0^2 = \frac{gW^2}{-2H}$. This is non-sensical because v_0^2 cannot be negative. But this simply indicates that if $\theta = 0$, $v_{0y} = 0$, so the motorbike cannot gain any height whatsoever.

P5/. $\vec{r}(t) = -10\text{m}\cos\omega t\,\hat{i} + 10\text{m}\sin\omega t\,\hat{j}$

(a) $|\vec{r}(t)| = \sqrt{r_x^2 + r_y^2}$

$= \sqrt{(-10\text{m}\cos\omega t)^2 + (10\text{m}\sin\omega t)^2}$

$= \sqrt{(10\text{m})^2(\cos^2\omega t + \sin^2\omega t)}$



$|\vec{r}(t)| = 10\text{m}$. Since the magnitude of the position vector is independent of time, i.e. constant, it implies that the object moves in a circle.

(Note that at $t=0$, the object is at $r(t=0) = -10\text{m}\,\hat{i}$.)

b). $\vec{v}(t) = \frac{d\vec{r}}{dt} = +(10m)\,\omega \sin\omega t\,\hat{i} + (10m)\,\omega \cos\omega t\,\hat{j}$

$|\vec{v}(t)| = \sqrt{v_x^2 + v_y^2} = \sqrt{(10m)^2\omega^2 \sin^2\omega t + (10m)^2\omega^2\cos^2\omega t}$

$= \sqrt{100m\,\omega^2(\sin^2\omega t + \cos^2\omega t)} = \sqrt{(100m^2)\omega^2}$

$|\vec{v}(t)| = (10\,\omega)\,m$ [note $[\omega] = s^{-1}$]

$= 10 \cdot 2\,s^{-1} \cdot m$

$= 20\,m/s.$ Note, speed is constant. So $a_{tangential} = 0$.

(c). $\vec{a}(t) = \frac{d\vec{v}}{dt} = (10m)\,\omega^2\cos\omega t\,\hat{i} - (10m)\,\omega^2 \sin\omega t\,\hat{j}$.

$|\vec{a}(t)| = \sqrt{(10m)^2\omega^4[\cos^2\omega t + \sin^2\omega t]}$

$= (10\,\omega^2)\,m$

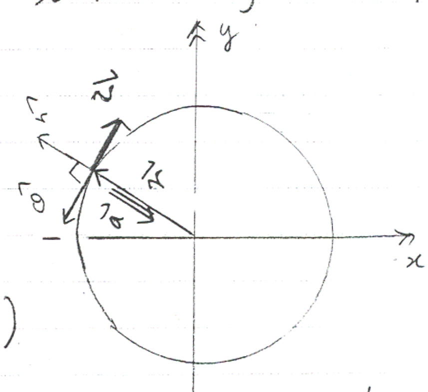
$|\vec{a}(t)| = 40\,m/s^2.$ The tangential acceleration is zero since $|\vec{v}|$ is constant.

Note also: $v(t=0) = 10m\,\omega\,\hat{j}$ and

$a(t=0) = (10m)\,\omega^2\,\hat{i}$ so radially inwards.

The object is moving clockwise.

d). $\vec{r}(t) = r_0\,\hat{r} = (10m)\,\hat{r}$.



$\vec{v}(t) = -r_0\,\omega\,\hat{\theta}$ (it's $-\hat{\theta}$ since the object is moving clockwise & $\hat{\theta}$ is normally defined + in the counterclockwise direction.)

$\vec{a}(t) = -r_0\,\omega^2\,\hat{r}$, negative sign because $\vec{a}$ points radially inwards – $+\hat{r}$ is radially outwards.

4.39 $v = (150^2 + 30.0^2)^{1/2} = \boxed{153 \text{ km/h}}$

$\theta = \tan^{-1}\left(\dfrac{30.0}{150}\right) = \boxed{11.3^\circ}$ north of west

4.40 For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$. Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{2L/c}{1 - v^2/c^2}}$$

For Beth, her cross-stream speed (both ways) is $\sqrt{c^2 - v^2}$

Thus, the total time for Beth is

$$t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \boxed{\frac{2L/c}{\sqrt{1 - v^2/c^2}}}$$

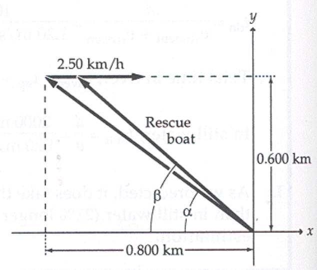
Since $1 - \dfrac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

4.41 α = Heading with respect to the shore

β = Angle of boat with respect to the shore

(a) The boat should always steer for the child at heading

$$\alpha = \tan^{-1}\frac{0.600}{0.800} = \boxed{36.9^\circ}$$



(b) $v_x = 20.0 \cos\alpha - 2.50 = 13.5$ km/h

$v_y = 20.0 \sin\alpha = 12.0$ km/h

$$\beta = \tan^{-1}\left(\frac{12.0 \text{ km/h}}{13.5 \text{ km/h}}\right) = \boxed{41.6^\circ}$$

(c) $t = \dfrac{d_y}{v_y} = \dfrac{0.600 \text{ km}}{12.0 \text{ km/h}} = 5.00 \times 10^{-2} \text{ h} = \boxed{3.00 \text{ min}}$