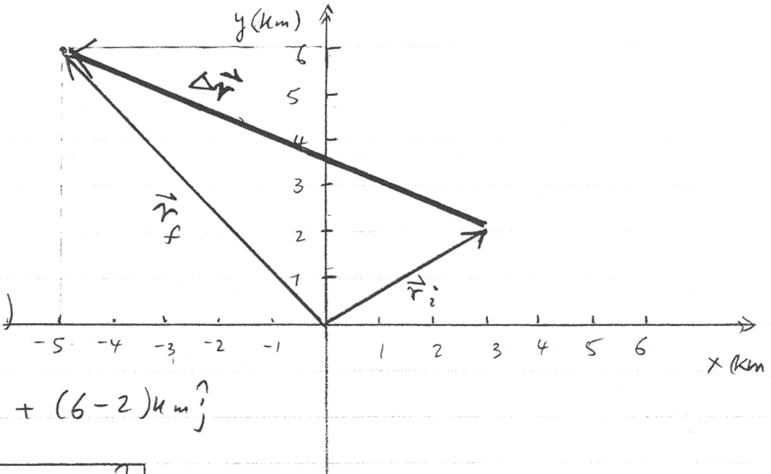
Solutions – HW # 2 PHYS 161, SPG 03.

S1/

a) –



b) $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$$\Delta\vec{r} = (-5\text{km}\,\hat{i} + 6\text{km}\,\hat{j}) - (3\text{km}\,\hat{i} + 2\text{km}\,\hat{j})$$

$$= (-5-3)\text{km}\,\hat{i} + (6-2)\text{km}\,\hat{j}$$

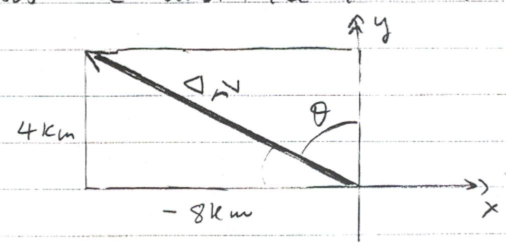
$$\Rightarrow \boxed{\Delta\vec{r} = -8\text{km}\,\hat{i} + 4\text{km}\,\hat{j}}$$

(c). The distance between the ship's initial and final positions is the magnitude of $\Delta\vec{r}$ i.e $|\Delta\vec{r}|$

$$|\Delta\vec{r}| = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(-8\text{km})^2 + (4\text{m})^2} = \underline{8.9\text{ km}}.$$

This is <u>not</u> necessarily the distance covered by the ship. This is <u>just</u> the magnitude of the ship's displacement. We don't know what path the ship took between $\vec{r}_i$ and $\vec{r}_f$ so we <u>don't</u> know the distance it covered.

d/. $\tan\theta = \dfrac{|\Delta x|}{|\Delta y|} = \dfrac{8\text{km}}{2\text{km}} = 2$

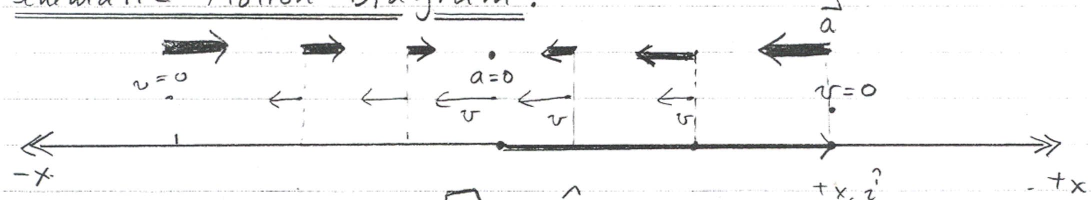


$\Rightarrow \theta = \tan^{-1}(2)$

$$\boxed{\theta \cong 63.4^\circ}$$

S2/. The problem states that $|\vec{a}| \propto |\vec{r}|$ i.e the magnitude of acceleration is propotional to the particle's displacement from the origin, i.e the farther it is the larger the magnitude of a. Also, we are told that the <u>direction</u> of $\vec{a}$ is <u>opposite</u> to that of $\vec{r}$. Altogether the two conditions imply: $\vec{a} = -k\vec{r}$, with k a constant.

Schematic Motion Diagram:



Let $\vec{r}(t) = x_0 \cos\sqrt{k}\, t\ \hat{i}$

check: $\vec{r}(t=0) = x_0 \cos(0)\,\hat{i} = +x_0\hat{i}$ ✓ check with initial conditio[ns] I.C.

$$\vec{v} = \frac{d\vec{r}}{dt} = -\sqrt{k}\, x_0 \sin\sqrt{k}\, t\ \hat{i}$$

$\vec{v}(t=0) = -\sqrt{k}\, x_0 \sin(0)\,\hat{i} = 0$ ✓ since particle starts at rest.

$$\frac{d^2\vec{r}}{dt^2} = -k\, x_0 \cos kt\ \hat{i}$$

$$\Rightarrow \quad \frac{d^2\vec{r}}{dt^2} = -k\,\vec{r}(t)\,\hat{i}.$$ ✓ checks with description of motion.

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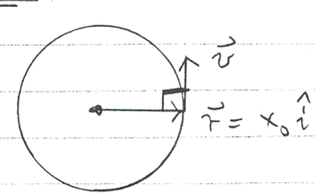
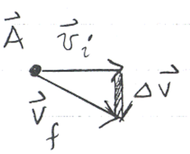
b). The object moves in a circle. Even though it travels with constant speed, since its velocity is always changing directions, its acceleration is <u>not</u> zero.



S3/. Note that since $\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$ and $\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$,

$\Delta \vec{v}$ is in the same direction as $\vec{a}$.

since $\Delta \vec{v} = \vec{v}_f - \vec{v}_i \Rightarrow \vec{v}_f = \vec{v}_i + \Delta \vec{v}$.



$\Rightarrow$ $\vec{v}_f$ at a slightly later time, so

The balls moves to the right due to $\vec{v}_i$ and also down a bit due to $\vec{a}$.

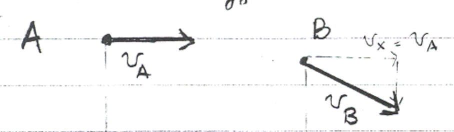
Recall that

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

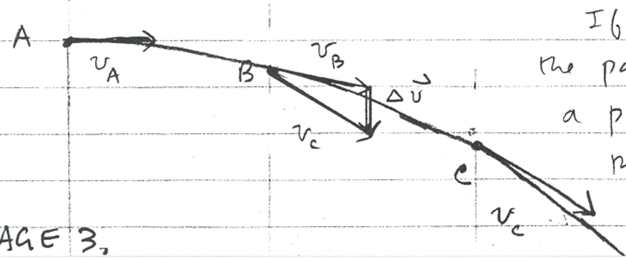
$$= (x_0 + v_{ix} t + \tfrac{1}{2} \cancel{a_x} t^2)\,\hat{i} + (y_0 + \cancel{v_{iy}} t - \tfrac{1}{2} a_y t^2)$$

$$= (x_0 + v_{ix} t)\,\hat{i} + (y_0 - \tfrac{1}{2} a_y t^2)$$

since t is very small ($t \cong 0.1\,s$), t^2 is even smaller, so the particle's y position is only a tiny bit lower than y_0. So:



From B to C, however, the particle moves to the right and also a significant amount downwards because at B it has a non-zero velocity in the y-direction. So,



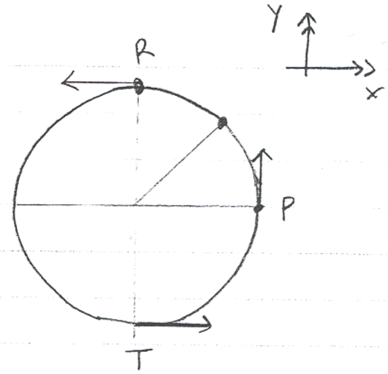
If we continue this process the particle would trace out a parabola. Notice that the instantaneous velocity is tangential to the path.

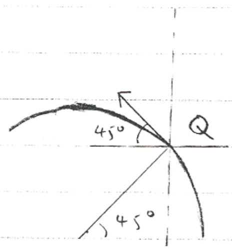
S4/. Recall that the instantaneous velocity is the tangent to the trajectory. The rate at which $\vec{v}$ changes is the acceleration. Along a tighter curve, $\vec{v}$ obviously changes direction faster so the inst. acceleration at B must be larger in magnitude than at A.

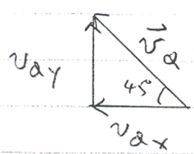
S5/. $\vec{v}_P = v_0 \hat{j}$;

$\vec{v}_R = -v_0 \hat{i}$;

$\vec{v}_T = +v_0 \hat{i}$



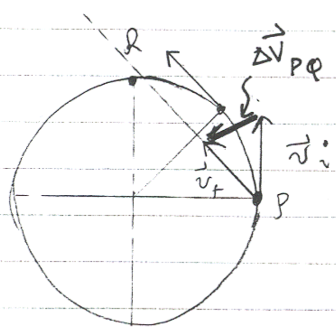




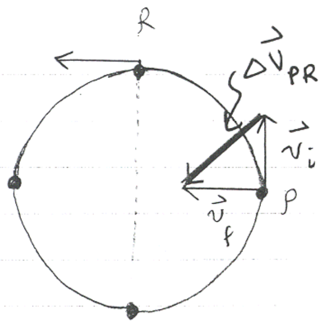
$$\vec{v}_Q = -v_0 \cos 45° \hat{i} + v_0 \sin 45° \hat{j}$$

$$\vec{v}_Q = -\frac{1}{\sqrt{2}} v_0 \hat{i} + \frac{1}{\sqrt{2}} v_0 \hat{j}$$

b/.



Since $\vec{a}_{avg} = \dfrac{\Delta\vec{V}_{PQ}}{\Delta t_{PQ}}$, the direction of $\vec{a}_{avg}$ is the <u>same</u> as the direction of $\Delta\vec{V}_{PQ}$.



Again, $\vec{a}_{avg\,PR}$ is in the <u>same</u> direction as $\Delta\vec{V}_{PR}$

e) $$\vec{a}_{avg\,PQ} = \frac{\Delta\vec{V}_{PQ}}{\Delta t_{PQ}} = \frac{\vec{v}_Q - \vec{v}_P}{t_Q - t_P}$$

$$= \frac{\left(-\frac{1}{\sqrt{2}} v_0 \hat{i} + \frac{1}{\sqrt{2}} v_0 \hat{j}\right) - (v_0 \hat{j})}{\Delta t_{PQ}}$$

$$\boxed{\vec{a}_{avg\,PQ} = \frac{-\frac{1}{\sqrt{2}} v_0 \hat{i} + \left(\frac{1}{\sqrt{2}} - 1\right) v_0 \hat{j}}{\Delta t_{PQ}}}$$

Note that this implies that $\vec{a}_{avg\,PQ}$ has both a negative x-component as well as a negative y component. This agrees with our graphical result on pg 4.

$$\vec{a}_{avg\,PR} = \frac{\Delta\vec{V}_{PR}}{\Delta t_{PR}} = \frac{\vec{v}_R - \vec{v}_P}{t_R - t_P} = \frac{(-v_0\hat{i}) - (+v_0\hat{j})}{\Delta t_{PR}}$$

$$\boxed{\vec{a}_{avg\,PR} = \frac{-v_0\hat{i} - v_0\hat{j}}{\Delta t_{PR}}}$$

Note that $\Delta t_{PR} = 2\Delta t_{QP}$.

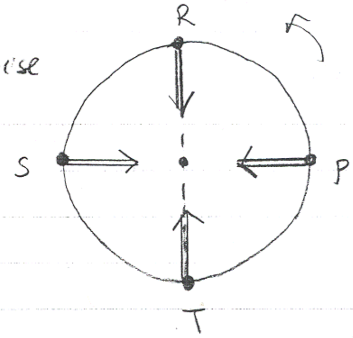
PAGE 5.

The magnitude of the average acceleration changes depending on which time interval you are consideri[ng].

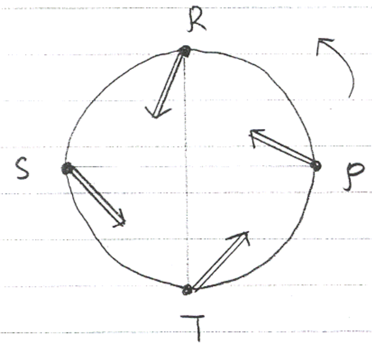
For example, $\vec{a}_{avg} = 0$ over the time interval over which the particle goes around the circle once. (Can you see why?).

d).

moving counterclockwise with constant speed.

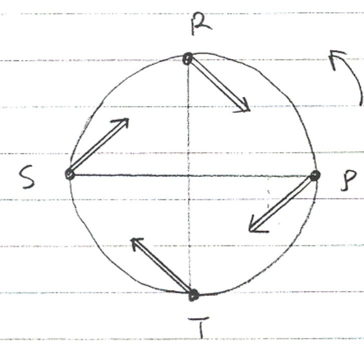


e).



moving counterclockwise & speeding up.

(f)



moving counterclockwise & slowing down.

3.37 (a) $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$\mathbf{F} = 120 \cos(60.0°)\mathbf{i} + 120 \sin(60.0°)\mathbf{j} - 80.0 \cos(75.0°)\mathbf{i} + 80.0 \sin(75.0°)\mathbf{j}$

$\mathbf{F} = 60.0\mathbf{i} + 104\mathbf{j} - 20.7\mathbf{i} + 77.3\mathbf{j} = (39.3\mathbf{i} + 181\mathbf{j})$ N

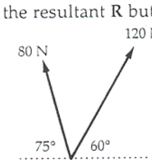
$|\mathbf{F}| = \sqrt{(39.3)^2 + (181)^2} = \boxed{185 \text{ N}}$; $\theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$

(b) $\mathbf{F}_3 = -\mathbf{F} = \boxed{(-39.3\mathbf{i} - 181\mathbf{j}) \text{ N}}$

Goal Solution

The helicopter view in Figure P3.37 shows two people pulling on a stubborn mule. Find (a) the single force that is equivalent to the two forces shown and (b) the force that a third person would have to exert on the mule to make the resultant force equal to zero. The forces are measured in units of newtons.

G: The resultant force will be larger than either of the two individual forces, and since the two people are not pulling in exactly the same direction, the magnitude of the resultant should be less than the sum of the magnitudes of the two forces. Therefore, we should expect 120 N < R < 200 N. The angle of the resultant force appears to be straight ahead and perhaps slightly to the right. If the stubborn mule remains at rest, the ground must be exerting on the animal a force equal to the resultant **R** but in the opposite direction.



O: We can find **R** by adding the components of the two force vectors.

A: $\mathbf{F}_1 = (120 \cos 60)\mathbf{i}$ N $+ (120 \sin 60)\mathbf{j}$ N $= 60.0\mathbf{i}$ N $+ 103.9\mathbf{j}$ N

$\mathbf{F}_2 = -(80 \cos 75)\mathbf{i}$ N $+ (80 \sin 75)\mathbf{j}$ N $= -20.7\mathbf{i}$ N $+ 77.3\mathbf{j}$ N

$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = 39.3\mathbf{i}$ N $+ 181.2\mathbf{j}$ N

$R = |\mathbf{R}| = \sqrt{(39.3)^2 + (181.2)^2} = 185$ N

The angle can be found from the arctan of the resultant components.

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{181.2}{39.3} = \tan^{-1}(4.61) = 77.8°$ counterclockwise from the $+x$ axis

The opposing force that the either the ground or a third person must exert on the mule, in order for the overall resultant to be zero, is 185 N at 258° counterclockwise from $+x$.

L: The resulting force is indeed between 120 N and 200 N as we expected, and the angle seems reasonable as well. The process applied to solve this problem can be used for other "statics" problems encountered in physics and engineering. If another force is added to act on a system that is already in equilibrium (sum of the forces is equal to zero), then the system may accelerate. Such a system is now a "dynamic" one and will be the topic of Chapter 5.

* ch4, P5: $\vec{v}_f = \vec{v}_i + \vec{a}t \Rightarrow \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} = 2\,\text{m/s}^2\,\hat{i} + 3\,\text{m/s}^2\,\hat{j}$

(b) $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 = (3\hat{i} - 2\hat{j})\,\text{m/s}\,t + \frac{1}{2}(2\hat{i} + 3\hat{j})\,\text{m/s}^2\,t^2$