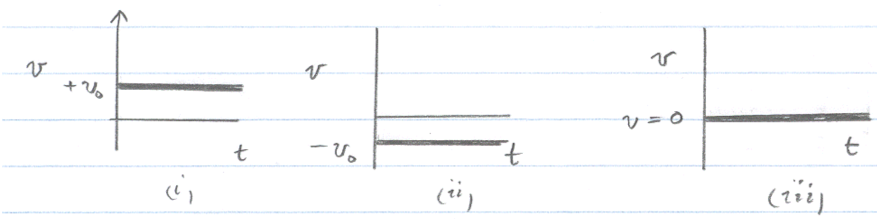
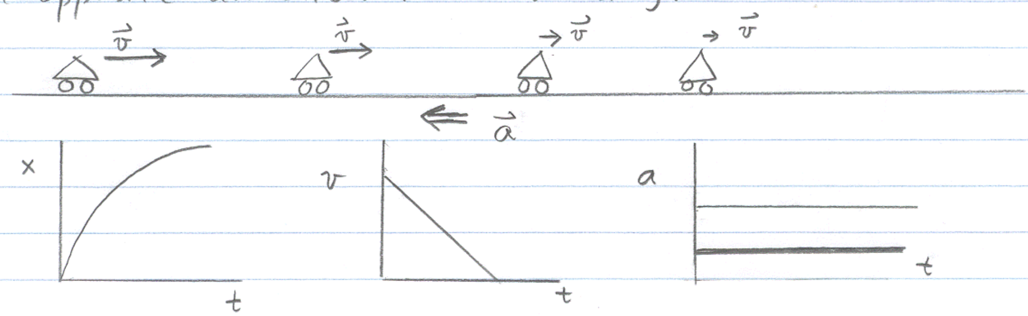
Solutions to H.W. # 1

Physics 161, Spring 2003.

S1/ a. If $\vec{a} = 0$, $\vec{v}$ = constant and $\vec{v}$ could be (i) $\vec{v} > 0$, (ii) $\vec{v} < 0$ or (iii) $v = 0$.

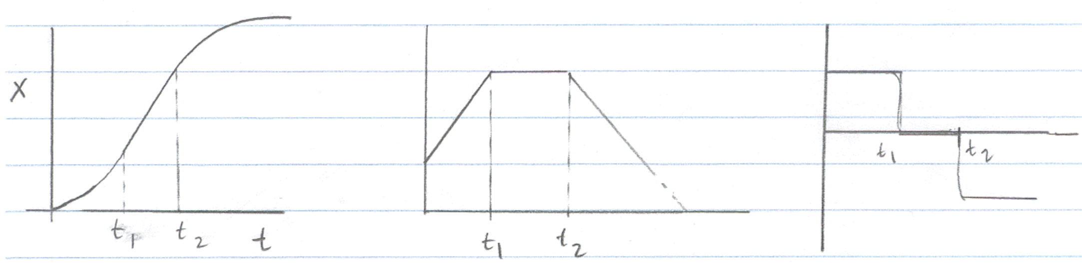


b). If the car is slowing down, then the acceleration is in the opposite direction to the velocity.

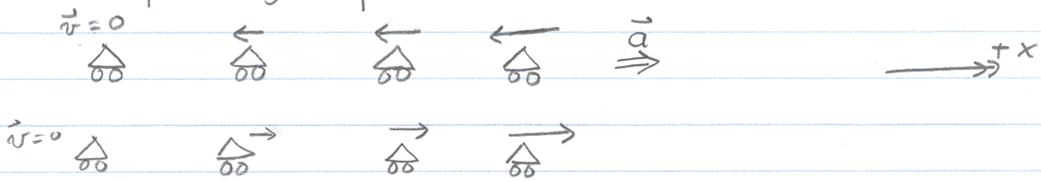


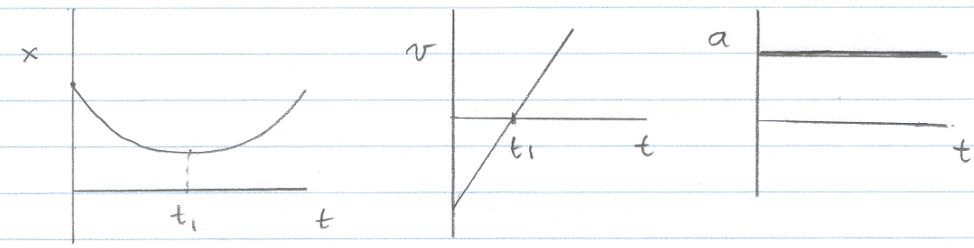
In the example shown above, the car is accelerating at a constant rate. But that doesn't have to be the case. The acceleration can also be non-uniform.

c). If the car first speeds up, then coasts at constant velocity for a while and then slows down.

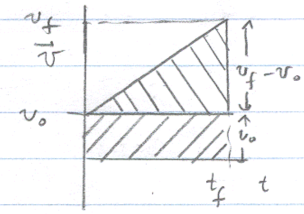


d/. Suppose we pick a coordinate system in which the +x-axis is to the right. Then a car travelling in the negative x-direction but slowing down has negative velocity but positive acceleration.
Let the car stop momentarily and then go in reverse speeding up:





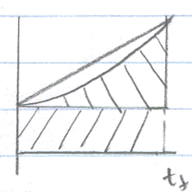
S2./ $\vec{v}_{avg} \equiv \dfrac{\Delta \vec{x}}{\Delta t}$. The area under the $\vec{v}$ vs t curve gives us Δx. So $\Delta x = v_0 \Delta t + \frac{1}{2}(v_f - v_0)\Delta t$.



$\Rightarrow v_{avg} = \dfrac{\Delta x}{\Delta t} = \left(\dfrac{v_0 + v_f}{2}\right)\dfrac{\cancel{\Delta t}}{\cancel{\Delta t}}$

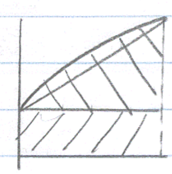
$\Rightarrow \boxed{v_{avg} = \dfrac{v_0 + v_f}{2}}$

(2)



(2) Here, area under the curve is <u>less</u> that is (1) _ so Δx is less but Δt is the same so $v_{avg} < \dfrac{v_0 + v_f}{2}$.

(3)

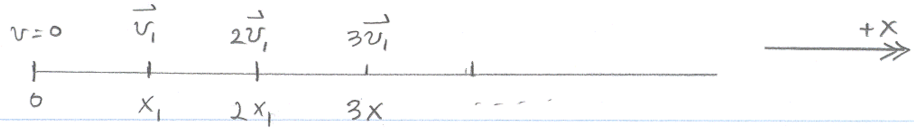


(3). Here,

$$v_{avg} > \frac{v_0 + v_f}{2}.$$

Area is <u>greater</u> than in (1).

S3.



Caution: Note that in this problem, we are <u>not</u> told that the acceleration is constant, so NONE of the equations in Table 2.2 are valid here. DO NOT EVEN ATTEMPT TO USE THEM HERE. Rather, let's argue conceptually.

(1). Let's label the time intervals from $x=0$ to x_1 as Δt_1, $x_1 \to 2x_1$ as Δt_2, $2x_1 \to 3x_1$ as Δt_3, ... etc.. Since the car has a higher velocity as it goes along we know that $\Delta t_1 > \Delta t_2 > \Delta t_3 > \ldots$.

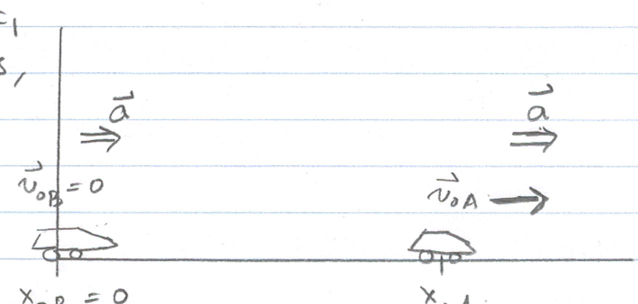
Since $\Delta \vec{v}_1 = v_1 - 0 = v_1$, $\Delta \vec{v}_2 = 2v_1 - v_1 = v_1$, $\Delta \vec{v}_3 = 3v_1 - 2v_1 = v_1$, i.e since $\Delta \vec{v}_1 = \Delta \vec{v}_2 = \Delta \vec{v}_3 = \ldots$ we know that

$$\frac{\Delta v_1}{\Delta t_1} < \frac{\Delta v_2}{\Delta t_2} < \frac{\Delta v_3}{\Delta t_3} \quad \text{so}$$

$$\Rightarrow \quad a_{1avg} < a_{2avg} < a_{3avg} -$$

So the car's acceleration is <u>not</u> uniform, its acceleration is <u>decreasing</u>.

S4/. At time $t=t_1$ when car B starts, car A has already been accelerating so it must have



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acheived some velocity $\vec{v}_{0A}$ and its at some distance x_{0A} away from car B. Let's rename the time $t = t_1$ to be $t=0$. Then:

$$x_A = x_{0A} + v_{0A} t + \frac{1}{2} a t^2$$

$$x_B = \cancel{x_{0B}}^{\,0} + \cancel{v_{0B}}^{\,0} t + \frac{1}{2} a t^2 = \frac{1}{2} a t^2$$

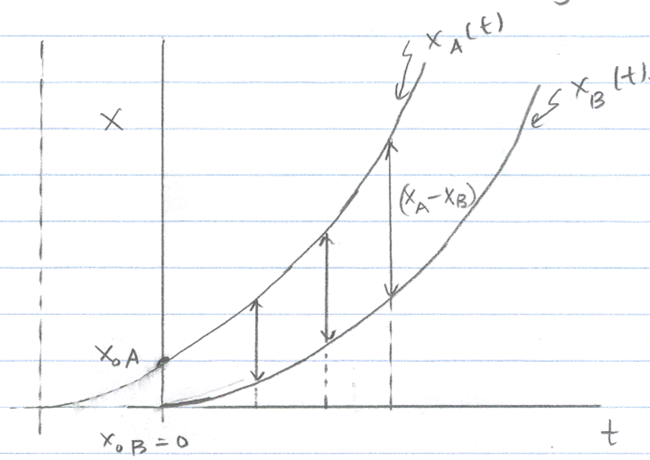
The spacing between them is given by

$$x_A - x_B = \left(x_{0A} + v_{0A} t + \frac{1}{2} a t^2\right) - \left(\frac{1}{2} a t^2\right)$$

$$\Rightarrow \boxed{x_A - x_B = x_{0A} + v_{0A} t}$$

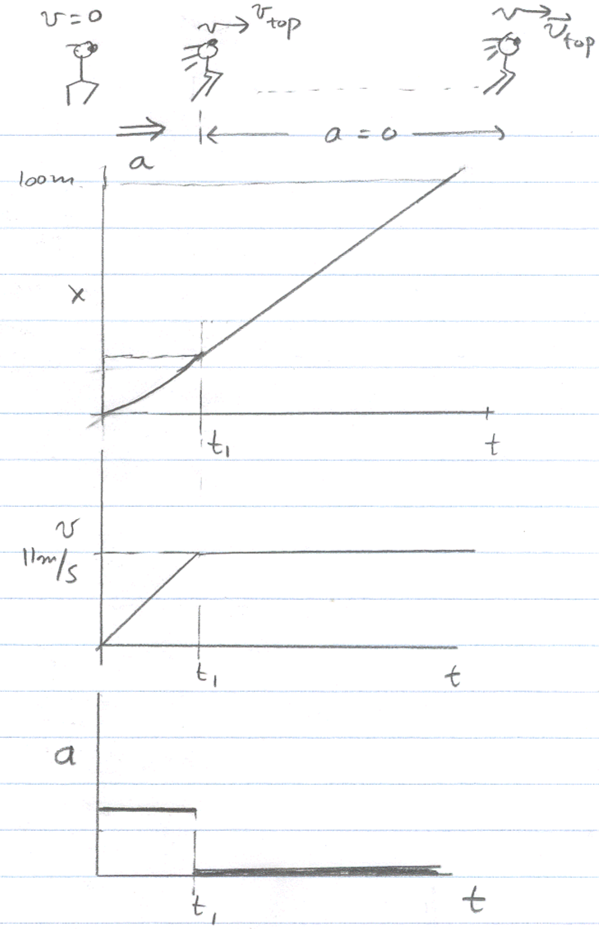
So the spacing between them is increasing with time.

You can see that $x_A - x_B$ is increasing.



Problems:

P1. Let's first find the time she takes to accelerate to 11.0 m/s. But we need a.



$$v_f^2 = v_0^2 + 2a\Delta x$$

$$v_f^2 = 0 + 2a\Delta x$$

$$\Rightarrow \quad a = \frac{v_f^2}{2\Delta x}$$

$$a = \frac{(11\,m/s)^2}{2(12.0\,m)} = 5\,m/s^2.$$

Step 2:

$$v_f = v_i + at$$

$$v_f = 0 + at$$

$$\Rightarrow \quad t = \frac{v_f}{a} = \frac{11\,m/s}{5\,m/s^2}$$

$$\Rightarrow \quad \boxed{t = 2.2\,s}$$

She covers the rest of the 88 m at constant speed, so $\Delta x = v\Delta t \Rightarrow \Delta t = \frac{\Delta x}{v} = \frac{88\,m}{11\,m/s} = 8\,s$

$$\Rightarrow T_{total} = 2.2\,s + 8\,s \Rightarrow \boxed{T_{total} = 10.2\,s}$$

(b). Now we need to find the distance over which she accelerates such that $t_1 + t_2 = 10$ seconds. Her new acceleration is:

$a = \frac{v_f^2}{2\Delta x}$; the time to reach v_f is $t_1 = \frac{v_f}{a}$

$\Rightarrow \quad t_1 = \dfrac{v_f}{a} = \dfrac{v_f}{v_f^2/2\Delta x} = \dfrac{2\Delta x}{v_f} \quad \Rightarrow \quad \boxed{t_1 = \dfrac{2\Delta x}{v_f}}$

The time to cover the rest of the distance i.e $(100\,m - \Delta x)$ is $t_2 = \dfrac{(100\,m - \Delta x)}{v_f}$.

$T_{total} = t_1 + t_2 = 10\,s.$

$\Rightarrow \quad \dfrac{2\Delta x}{v_f} + \dfrac{(100\,m - \Delta x)}{v_f} = 10\,s$

$\Rightarrow \quad 2\Delta x + 100\,m - \Delta x = (10\,s)\,v_f.$

$\Rightarrow \quad \Delta x + 100\,m = (10\,s)v_f \Rightarrow \Delta x = 10\,s\, v_f - 100\,m.$

$\Rightarrow \quad \Delta x = (10\,s)(11\,m/s) - 100\,m$

$\Rightarrow \quad \boxed{\Delta x = 10\,m}$

(c). Assuming that the acceleration is uniform — the $a = a_{avg} = \dfrac{\Delta v}{\Delta t}$.

$v_f = 45\,mi/hr = 45 \times (1.61)\,\dfrac{km}{hr} \cong 20\,m/s.$

$a = \dfrac{20\,m/s}{2.5\,s} = 8\,m/s^2.$

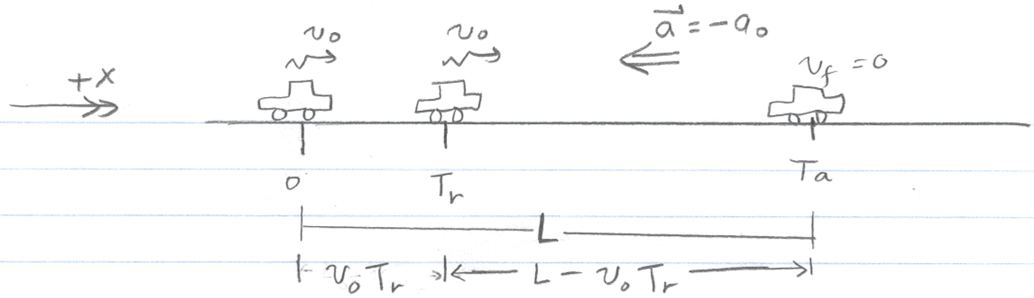
Distance covered during this time $\Delta x = \dfrac{v_f^2}{2a} = 25\,m.$

The time needed to cover the remaining 75 m is

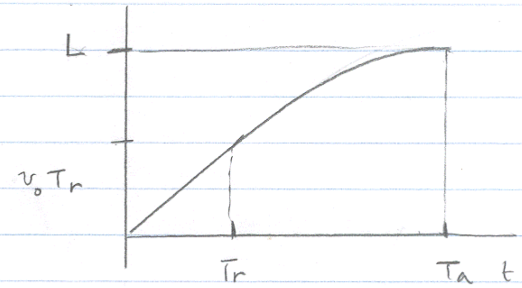
$t_2 = \dfrac{(75\,m)}{v_f} = \dfrac{75\,m}{20\,m/s} = 3.75\,s.$

$\Rightarrow \quad T_{total}$ for Cheetah $= t_1 + t_2 = \underline{\underline{6.25\,s}}.$

P2.1



(b). The Eq's in Table 2.2 are valid only if a = const. So we can apply them separately in time intervals $0 - T_r$, and $T_r - T_a$.



(c). $v_f^2 = v_i^2 + 2a\Delta x$

Here Δx is the stopping distance. The maximum it can be for safety is: $\Delta x = L - v_o T_r$.

since $v_f = 0$, and $\vec{a} = -a_o$,

$$0 = v_o^2 - 2a_a \Delta x$$

$$\Rightarrow \quad a_o = \frac{v_o^2}{2\Delta x} \quad \Rightarrow \quad \boxed{a_o = \frac{v_o^2}{2(L - v_o T_r)}}$$

numerically $\boxed{a_o = 1.89 \, m/s^2}$

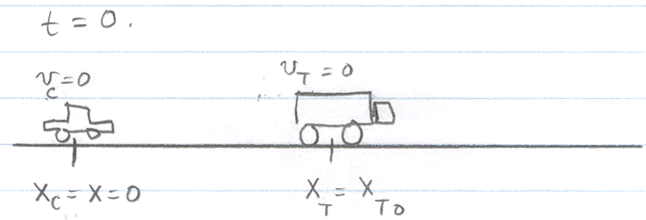
(d). To find v_{avg} we use the basic definition

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t} = \frac{L}{T_a + T_r}.$$

P3./ Car catching up with a Truck:

a) at $t = t^*$ the car catches up and passes the truck.

(b). The car passes the truck when the truck has travelled 48m.



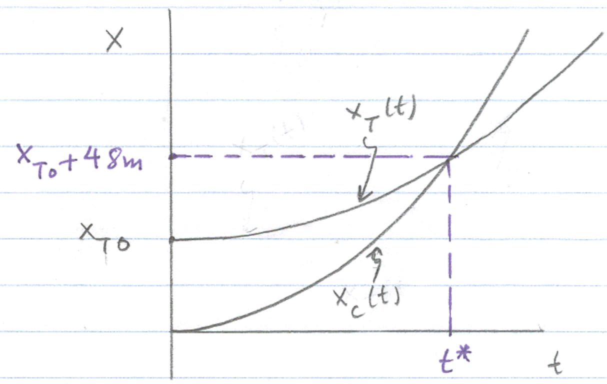
$$X_T(t) = X_{T0} + v_{0T}t + \frac{1}{2}a_T t^2$$

$$X_T(t) - X_{T0} = 0 + \frac{1}{2}a_T t^2$$

$$48\,m = \frac{1}{2}a_T t^2$$

$$\Rightarrow\ t^2 = \frac{2(48m)}{a_T}$$

$$\Rightarrow\ \boxed{t^* = 8.9s}$$



c)

At $t = t^*$, $X_c(t=t^*) = X_T(t=t^*)$

$$\frac{1}{2}a_c t^{*2} = X_{T0} + \frac{1}{2}a_T t^{*2}$$

$$\Rightarrow\ X_{T0} = \frac{1}{2}a_c t^{*2} - \frac{1}{2}a_T t^{*2}$$

$$= \frac{1}{2}(1.8m/s^2)(8.9s)^2 - \frac{1}{2}(1.2m/s^2)(8.9s)^2$$

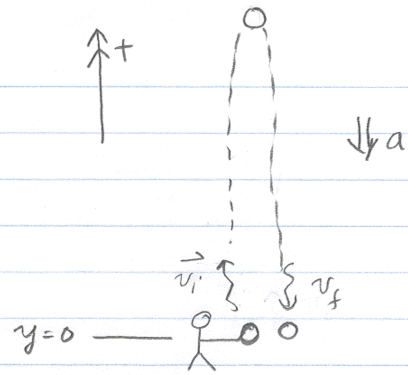
$$\Rightarrow\ X_{T0} = 72m - 48m$$

$$\boxed{X_{T0} = 24m}$$

d). $v_T = \cancel{v_{T0}}^{\,0} + a_T t \Rightarrow v_T = a_T t^* = (1.2m/s^2)(8.9s) = \underline{10.7m/s}$

$v_c = v_{c0} + a_c t \Rightarrow v_c = a_c t^* = (1.8m/s^2)(8.9s) = \underline{16.02\,m/s}$

P4/ Ball Thrown Vertically Upward.



a). $y_f = y_i = 0.$

$$y_f = y_i + v_i t + \frac{1}{2} a t^2$$

$$0 = 0 + v_i T - \frac{1}{2} g T^2$$

$$\Rightarrow \quad -v_i T = -\frac{1}{2} g T^2$$

If $T \neq 0$, $\boxed{v_i = \frac{1}{2} g T}$

(b). At maximum height, $v = 0$.

$$v_f = v_i + at \Rightarrow 0 = v_i - g t_{up} \Rightarrow \boxed{\frac{v_i}{g} = t_{up}}$$

note from above that since $T = \frac{2v_i}{g}$, & $t_{up} = \frac{v_i}{g}$

$t_{up} = T/2$.

(c) $y_f = y_i + v_i t + \frac{1}{2} a t^2$

$$h_{max} = 0 + v_i \cdot \left(\frac{v_i}{g}\right) - \frac{1}{2} g \left(\frac{v_i}{g}\right)^2$$

$$h_{max} = \frac{v_i^2}{g} - \frac{v_i^2}{2g} \Rightarrow \boxed{h_{max} = \frac{v_i^2}{2g}}$$

d). time to reach $h/2$ is not equal to the time to go from $h/2$ to h_{max}. Since the average velocity is higher in the begining $t_{0 \to h/2} < t_{h/2 \to h_{max}}$.

e). The initial velocity of the ball is the same as the velocity of the hot-air balloon, i.e

P4-e cont'd /. $\vec{v}_i = +v_b \hat{j}$
so the ball first rises, reaches a max. height & then falls to the ground.

A Thought Expt:

P5/. According to the observer located in the helicopter, at $t=0$, the ball is momentarily at rest. Then since the ball starts slowing down w.r.t A, B sees the ball moving away from it. B will see the ball move away from it with faster and faster speeds.

$$v_{\text{Ball according to A}} = v_{o\,\text{ball}} - gt$$
$$= 30\,m/s - gt.$$

$$v_{\text{helicopter according to A}} = 30\,m/s.$$

$$\Rightarrow v_{\text{Ball according to B}} = (30\,m/s - gt) - 30\,m/s$$
$$= -gt.$$

so B does not see the ball first rise, reach a max height with $v=0$ and then fall. According to B, the ball is always falling!