Homework 10 Solutions, Physics 161, Spg 2003

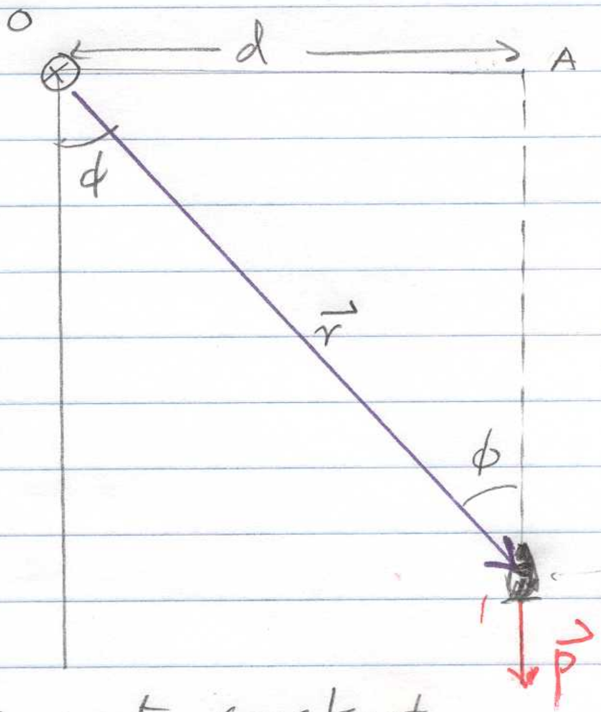
(Dr. Rama.)

S1/ $\vec{L}_0 = \vec{r} \times \vec{p}$

$\Rightarrow L_0 = rp\sin\phi$

$= \underbrace{r\sin\phi}\, p$

$\boxed{L_0 = d\,mv}$, into the page.



b/. $\vec{\tau}_0 = \vec{r} \times \vec{F}_g$

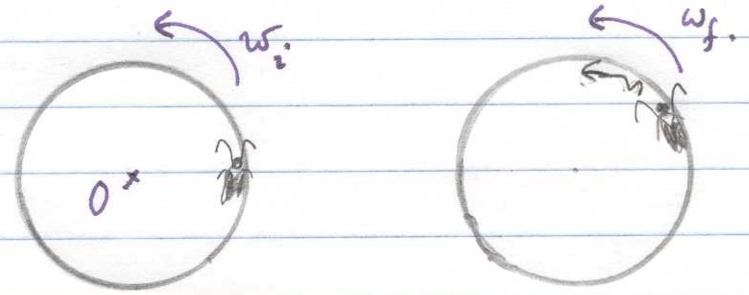
$= rF_g\sin\phi$

$\Rightarrow \boxed{\tau_0 = dmg}$, into the page.

c/. Angular momentum about O is not constant because there is torque on the penguin about point O.

d/. About point A, $L_A = \vec{r} \times \vec{p} = 0$ and $\vec{\tau}_A = \vec{r} \times \vec{F} = 0$ since $\vec{r}$ & $\vec{p}$ are parallel and so are $\vec{r}$ & $\vec{F}$.

S2/. a/ when the cockroach starts walking, there is no external torque on the system so the angular momentum of the cockroach-disk system must be conserved.



(b). The angular momentum of the cockroach increases and so does its angular velocity.

(c) To conserve the total angular momentum of the cockroach-disk system, the angular momentum of the disk must decrease and so must its angular velocity.

when the cockroach starts walking in the opposite direction, angular momentum would again be <u>conserved</u> since, again, there is no external torque on the system — angular momentum and angular velocity of the cockroach <u>decrease</u> — angular momentum & angular velocity of the disk <u>increase</u> —

S4/. $\vec{r} = r\cos 45^\circ \hat{i} + r\sin 45^\circ \hat{j}$

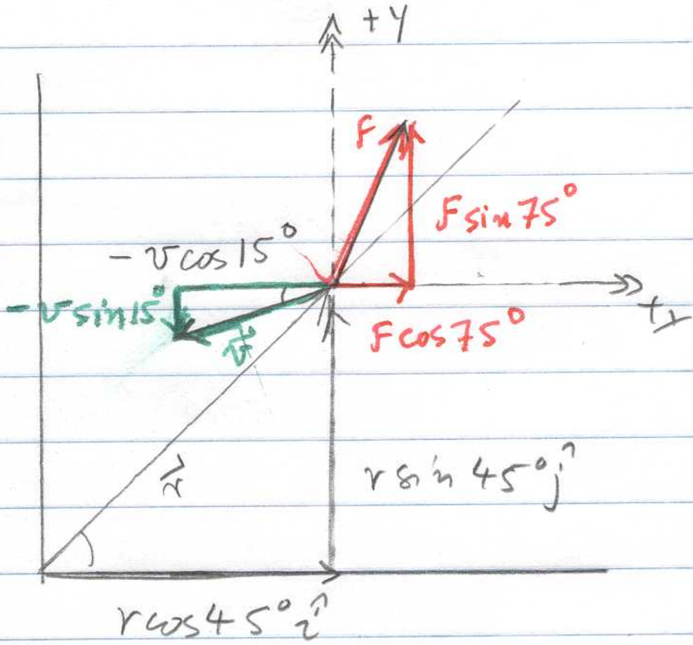
$$\boxed{\vec{r} = 2.12\,\text{m}\,\hat{i} + 2.12\,\text{m}\,\hat{j}}$$

$\vec{v} = -v\cos 15^\circ \hat{i} - v\sin 15^\circ \hat{j}$

$$\boxed{\vec{v} = -(3.86\,\text{m/s})\hat{i} - (1.035\,\text{m/s})\hat{j}}$$

$\vec{F} = F\cos 75^\circ \hat{i} + F\sin 75^\circ \hat{j}$

$$\Rightarrow \boxed{\vec{F} = +0.51\,\text{N}\,\hat{i} + 1.93\,\text{N}\,\hat{j}}$$



(b) $\vec{L} = \vec{r} \times \vec{p}$

$= (2.12\,\text{m}\,\hat{i} + 2.12\,\text{m}\,\hat{j}) \times (-3.86\,M\,\text{kg m/s}\,\hat{i} - 1.035\,M\,\text{kg m/s}\,\hat{j})$

$= (2.12\,\text{m})(-1.035\,M\,\text{kg m/s})(\hat{i} \times \hat{j})$

$+ (2.12\,\text{m})(-3.86\,M\,\text{kg m/s})(\hat{j} \times \hat{i})$

$= -2.19\,M\,\text{kg m}^2/\text{s}\,\hat{k} + 8.18\,M\,\text{kg m}^2/\text{s}\,\hat{k}$

$$\boxed{\vec{L} = +5.99\,M\,\text{kg m}^2/\text{s}\,\hat{k}.}$$

(ii) $\vec{\tau} = \vec{r} \times \vec{F}$

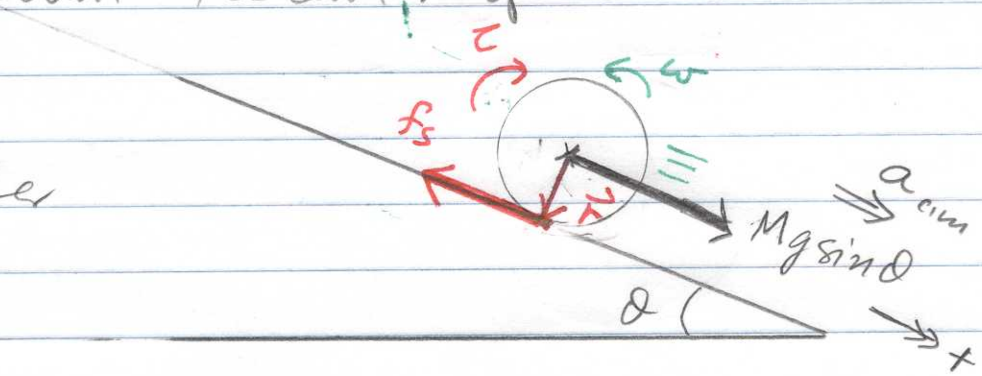
$$= (2.12\,m\,\hat{i} + 2.12\,m\,\hat{j}) \times (0.51\,N\,\hat{i} + 1.93\,N\,\hat{j})$$

$$= (2.12\,m)(1.93\,N)(\hat{i} \times \hat{j}) + (2.12\,m)(0.51\,N)(\hat{j} \times \hat{i})$$

$$= 4.09\,N.m\,\hat{k} - 1.08\,N.m\,\hat{k}$$

$$\boxed{\vec{\tau} = 3.01\,N.m\,\hat{k}}$$

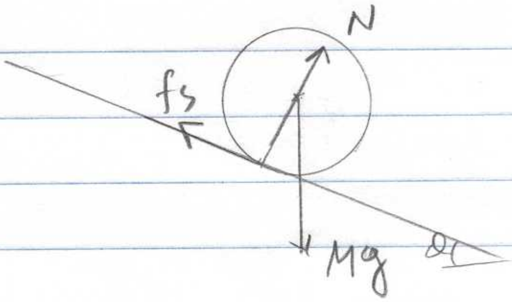
Let's calculate torques about the center of mass.



P3) Note that mg & N don't apply a torque about the center of mass.

$$\tau_{net,ext,cm} = I_{cm}\,\alpha$$

$$f_s R = I_{cm}\,\alpha$$



$$f_s \cancel{R} = \frac{2}{5} M R^{\cancel{2}} \cdot \frac{a_{cm}}{\cancel{R}}$$

$$\Rightarrow \boxed{f_s = \frac{2}{5} M a_{cm}}$$

Now let's use $F_{net,ext_x} = M a_{cm,x}$

$$Mg\sin\theta - f_s = M a_{cm}$$

$$Mg\sin\theta - \frac{2}{5} M a_{cm} = M a_{cm}$$

$$\Rightarrow \cancel{M} g\sin\theta = \left(\frac{2}{5} + 1\right) \cancel{M} a_{cm}$$

$$\Rightarrow \boxed{a_{cm} = \frac{5}{7} g\sin\theta}$$

[Note a_{cm} is the same as for a sphere rolling down hill!]

b/. a_{cm} doesn't depend on M so a_{cm} is the <u>same</u> for all spheres of <u>all</u> weights (and radii in fact.).

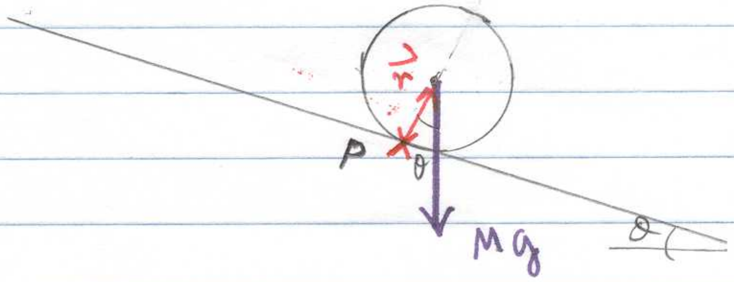
c/. $f_s = \frac{2}{5} M a_{cm}$

$$\Rightarrow f_s = \frac{2}{\cancel{5}} \cdot \frac{\cancel{5}}{7} M \cdot g\sin\theta \Rightarrow \boxed{f_s = \frac{2}{7} Mg\sin\theta}$$ it points <u>up</u> the ramp.

P3/ ALTERNATIVE METHOD — Calculate the torque about the instantaneous point of contact —

Note that now, neither f_s, nor N apply a torque since $\underline{f_s}$ and $\underline{N}$ act at the pivot point. Only Mg applies a torque. So,

$$\tau_{net,ext,P} = I_P \alpha$$



Let's find I_P first using the parallel axis theorem —

$$I_P = I_{cm} + MR^2$$
$$= \frac{2}{5} MR^2 + MR^2$$

$$\Rightarrow I_P = \frac{7}{5} MR^2$$

$$\Rightarrow \cancel{M} g \cancel{R} \sin\theta = \frac{7}{5} \cancel{M} R^{\cancel{2}} \alpha$$

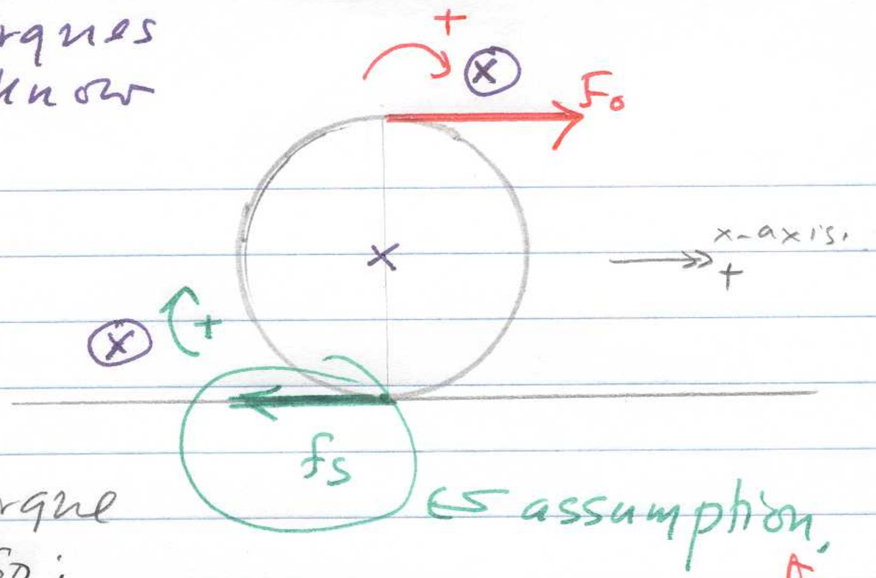
$$g\sin\theta = \frac{7}{5} \cancel{R} \cdot \frac{a_{cm}}{\cancel{R}}$$

$\Rightarrow \boxed{a_{cm} = \frac{5}{7} g\sin\theta}$. Now we can find f_s

same as before!

using $Mg\sin\theta - f_s = Ma_{cm}$ as before.

P5/ METHOD I: calculate torques about c.m. Since don't know direction of f_s, assume it acts one way or the other. I'm choosing to assume it acts to the left.



N & mg don't apply a torque about the center of mass. So:

$$\tau_{net,ext,cm} = I_{cm}\,\alpha$$

[Both torques into the page ⊗]

$$F_0 R + f_s R = \frac{1}{2} M R^{\cancel{2}} \cdot \frac{a_{cm}}{\cancel{R}}$$

$$\Rightarrow \boxed{F_0 + f_s = \frac{1}{2} M a_{cm}} \quad \text{eq 1}$$

$$F_{net,ext,x} = M a_{cm,x}$$

$$\boxed{F_0 - f_s = M a_{cm}} \quad \text{eq 2.}$$

adding eq 1 & 2 we get

$$2F_0 = \frac{3}{2} M a_{cm} \Rightarrow \boxed{a_{cm} = \frac{4}{3}\frac{F_0}{M}}$$

⇒ use eq 1 or eq 2 to find f_s — I'll use eq 2.

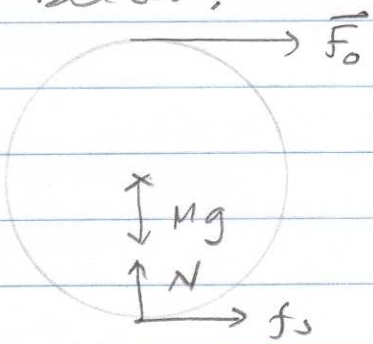
$$f_s = F_0 - M a_{cm}$$
$$= F_0 - \cancel{M} \cdot \frac{4}{3} \frac{F_0}{\cancel{M}}$$

$$f_s = -\frac{1}{3} F_0$$ — Note that this implies that f_s should've been in the opposite direction to what I assume.

our assumption is wrong.

Attention: possible source of confusion. Now I know what some of you may be thinking, that look, $f_s = -\frac{1}{3} F_o$ isn't that correct because f_s points to the left & right is positive? Actually not. The catch is that in eq 2 we have already used the fact that $\vec{f_s} = -f_s$ so for us f_s should have turned out + if we had assumed the correct direction.

so correct direction is as shown below:



Method 2: Calculate torques about the instantaneous point of contact –

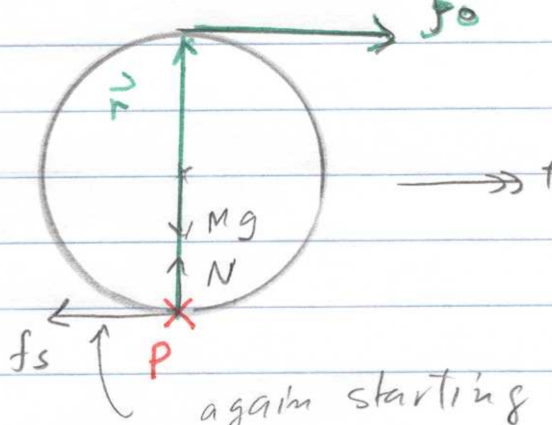
$$\tau_{net,ext,P} = J_P \alpha$$

$$F_o (2R) = \left(\frac{1}{2} MR^2 + MR^2\right) \frac{a_{cm}}{R}$$

used parallel axis theorem here –

$$\Rightarrow 2F_o = \frac{3}{2} M a_{cm}$$

$$\Rightarrow \boxed{a_{cm} = \frac{4}{3} \frac{F_o}{M}}$$ ✓ same as before.



again starting with wrong assumption on purpose – could do it either way.

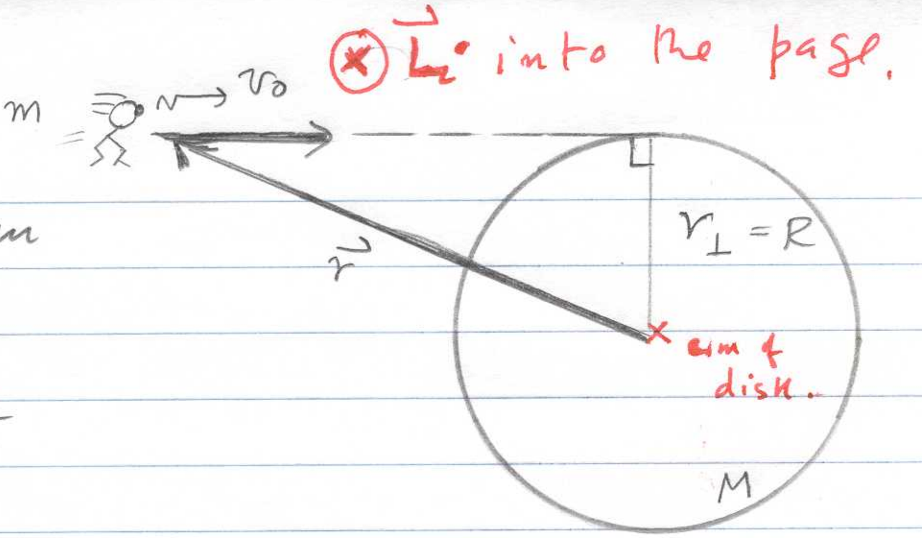
To find f_s: use

$$F_{net,ext,x} = M a_{cm,x}$$

$$F_o - f_s = M a_{cm} \Rightarrow f_s = -\frac{1}{3} F_o$$ – same as before.

So f_s mus point in opposite direction than assumed.

P6/ use conservation of angular momentum



$$L_i = L_f$$

$$m v_0 r_\perp = I_f \omega_f$$

$$\Rightarrow \frac{m v_0 R}{I_f} = \omega_f .$$

$$I_f = I_{disk,\ about\ its\ c.m} + I_{kid,\ about\ disk's\ c.m.}$$

$$I_f = \frac{1}{2} M R^2 + m R^2$$

$$\Rightarrow \boxed{\omega_f = \frac{m v_0 R}{\frac{1}{2} M R^2 + m R^2}} \Rightarrow \boxed{\omega_f = \frac{v_0 R}{R^2\left(\frac{1}{2}\frac{M}{m} + 1\right)}}$$

(b). No, $K.E_f$ would be less than $K.E_i$ because the kid will slide first and therefore $\Delta E_{thermal} \neq 0$

$$\Rightarrow \Delta K.E + \Delta E_{thermal} = 0$$

$$\Rightarrow \Delta K.E = -\Delta E_{thermal}.$$

so $K.E_f < K.E_i$.

(c). $P_i = m v_0 + M \cancel{v}_{disk}^{\,0} \Rightarrow \boxed{P_i = m v_0}$

$P_f \cong 0$ since the center of mass of of the kid + disk is roughly at the c.m of the disk. assuming $m/M \ll 1$. For an exact calculation of $P_{f,cm}$ see end of this problem.

The axle must've applied a force to the <u>left</u>

since $\Delta \vec{P} = \vec{P}_f - \vec{P}_i = -P_i$ is to the left,
$\boxed{P_i > 0}$.

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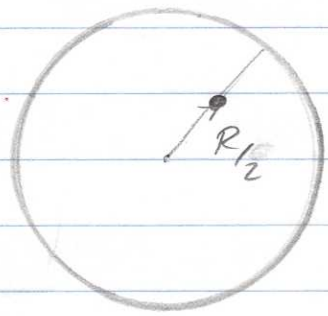
d). $\mathcal{L}_i = \mathcal{L}_f$ again because $\tau_{net,ext} = 0$



$$I_i \omega_i = I_f \omega_f$$

$$\frac{I_i \omega_i}{I_f} = \omega_f \Rightarrow \omega_f = \omega_i \frac{I_i}{I_f}$$

$$I_f = \frac{1}{2} M R^2 + m (R/2)^2, \text{ note } I_f < I_i$$

$\Rightarrow$ so $\boxed{\omega_f > \omega_i}$

e). $K.E_i = \dfrac{\mathcal{L}_i^2}{2I_i}$; $K.E_f = \dfrac{\mathcal{L}_f^2}{2I_f}$

$$\mathcal{L}_i = \mathcal{L}_f, \Rightarrow K.E_f = \frac{\mathcal{L}_i^2}{2I_f}$$

but $I_f < I_i$ so $K.E_f > K.E_i$.
The extra energy comes from the kid burning up the candy that he/she ate.

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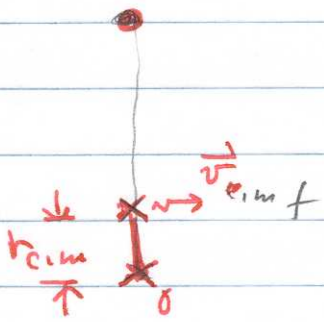
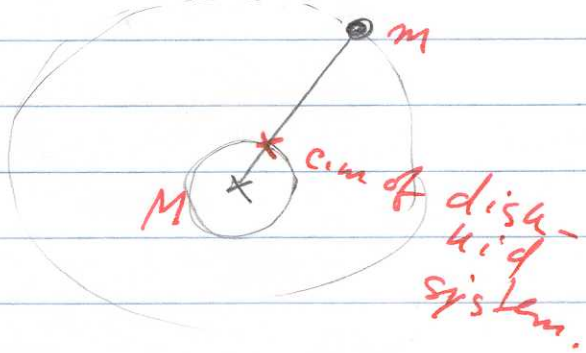
supplement to P6(c).

$$r_{c.m} = \frac{mR}{m+M}$$





$$(m+M)\vec{v}_{c.m,f} = \vec{P}_{c.m\,f}$$

$$(m+M) r_{cm} \omega_f = P_{c.m,f}$$

$$\cancel{(m+M)}\left(\frac{mR}{\cancel{m+M}}\right)\omega_f = P_{c.m\,f}$$

$$\Rightarrow mR\omega_f = P_{c.m\,f}$$

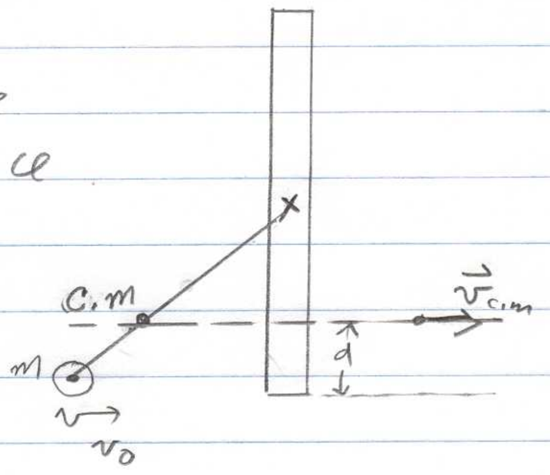
$$\frac{mR(v_0 R)}{R^2(\frac{1}{2}\frac{M}{m}+1)} = \frac{m v_0}{\frac{1}{2}\frac{M}{m}+1}$$

$$\Rightarrow P_{c.m\,f} \cong 0 \text{ if } M/m \gg 1.$$

P7| Since there are no net external forces acting on the system, the linear momentum is conserved.

$$P_i = m v_0$$

$$P_f = (m + M) v$$ where v must be the velocity of the center of mass since they stick together.



$$P_i = P_f$$

$$\Rightarrow m v_0 = (m + M) v_{c.m}$$

$$\Rightarrow v_{c.m} = \frac{m v_0}{m + M}$$

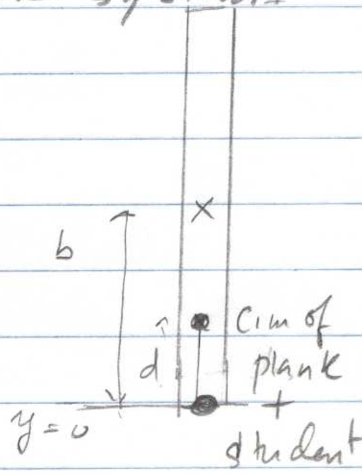
$$= \frac{(70kg)(3m/s)}{(70 + 50)kg} \Rightarrow \boxed{v_{c.m} = 1.75 m/s}$$

Let's find the c.m of the student + plank system.

$$y_{c.m} = \frac{m(0) + Mb}{m + M}$$

$$y_{c.m} = \frac{Mb}{m + M}$$

$$= \frac{(50kg)(2.5m)}{120kg} = 1.04m.$$



Let's calculate the angular momentum about the center mass of the disk-plank system. Since there are no torques acting on the system, angular momentum is conserved.

$$L_i = L_f$$

$$m v d = I_c \omega$$

$$m v d = \left[\frac{1}{12} M (2b)^2 + M (b - d)^2 + m d^2\right] \omega$$

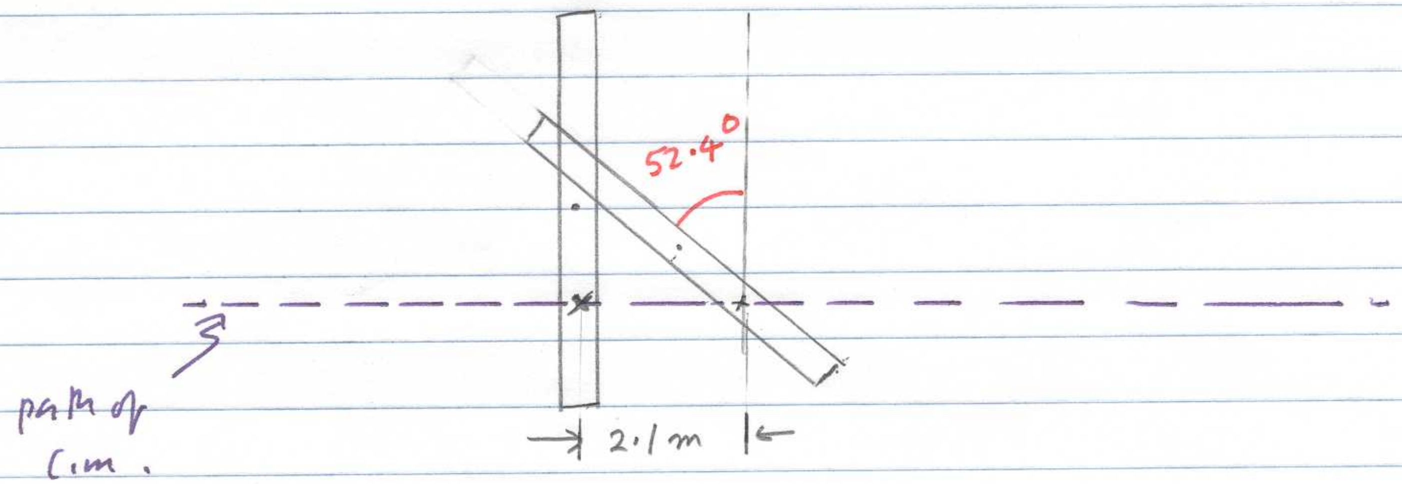
$$\Rightarrow \boxed{\omega = 0.762 \, rad/s.}$$

Now that we know v_{cm} & ω we can find x_{cm}, & the angle θ.

$$x_{cm} = v_{cm}\,\Delta t$$
$$= (1.75\,\tfrac{m}{s})(1.2s)$$

$$\Rightarrow \boxed{x_{cm} = 2.1\,m}$$

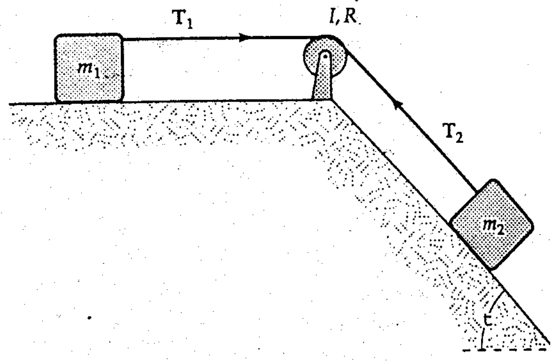
$$\Delta\theta = \omega t \Rightarrow \Delta\theta = (0.762\,rad/s)(1.2s)$$
$$\Rightarrow \Delta\theta = 0.91\,rad =$$
$$= 52.4°$$



Since there are no external forces on the system, the center of mass just keeps on trucking in a straight line.

P1.

10.39 For m_1: $\sum F_y = ma_y$ $\qquad +n - m_1 g = 0$



$$n = m_1 g = 19.6 \text{ N}$$

$$f_k = \mu_k n = 7.06 \text{ N}$$

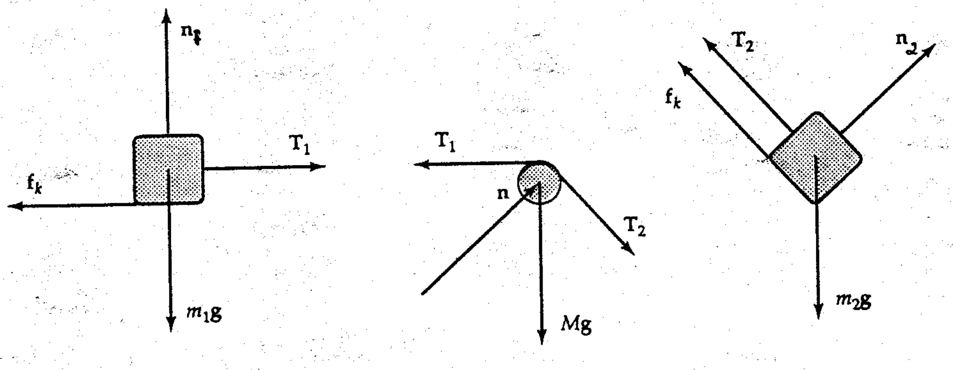
$$\sum F_x = ma_x \qquad -7.06 \text{ N} + T_1 = (2.00 \text{ kg})a \quad (1)$$

For the pulley

$$\sum \tau = I\alpha$$

$$-T_1 R + T_2 R = \frac{1}{2} MR^2 \left(\frac{a}{R}\right)$$

$$-T_1 + T_2 = \frac{1}{2}(10.0 \text{ kg})a \qquad -T_1 + T_2 = (5.00 \text{ kg})a \quad (2)$$



For m_2: $+n_2 - m_2 g \cos\theta = 0$

$$n_2 = 6.00 \text{ kg}(9.80 \text{ m/s}^2) \cos 30.0° = 50.9 \text{ N}$$

$$f_k = \mu_k n_2 = 18.3 \text{ N}$$

$$-18.3 \text{ N} - T_2 + m_2 g \sin\theta = m_2 a$$

$$-18.3 \text{ N} - T_2 + 29.4 \text{ N} = (6.00 \text{ kg})a \quad (3)$$

(a) Add equations (1) (2) and (3):

$$-7.06 \text{ N} - 18.3 \text{ N} + 29.4 \text{ N} = (13.0 \text{ kg})a$$

$$a = \frac{4.01 \text{ N}}{13.0 \text{ kg}} = \boxed{0.309 \text{ m/s}^2}$$

(b) $T_1 = 2.00 \text{ kg}(0.309 \text{ m/s}^2) + 7.06 \text{ N} = \boxed{7.67 \text{ N}}$

$T_2 = 7.67 \text{ N} + 5.00 \text{ kg}(0.309 \text{ m/s}^2) = \boxed{9.22 \text{ N}}$

p2.

11.4 $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping.

Also, $U_i = mgh$, $U_f = 0$, and $v_i = 0$

Therefore, $\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \frac{2gh}{[1 + (I/mR^2)]}$

For a disk, $I = \frac{1}{2}mR^2$, so

$v^2 = \frac{2gh}{[1 + (1/2)]}$ or $\boxed{v_{\text{disk}} = \sqrt{\frac{4gh}{3}}}$

For a ring, $I = mR^2$ so $v^2 = \frac{2gh}{2}$ or $\boxed{v_{\text{ring}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

P4.

11.60 (a) $L_i = 2\left[Mv\left(\frac{d}{2}\right)\right] = \boxed{Mvd}$

(b) $K = 2\left[\frac{1}{2}Mv^2\right] = \boxed{Mv^2}$

(c) $L_f = L_i = \boxed{Mvd}$

(d) $v_f = \frac{L_f}{2(Mr_f)} = \frac{Mvd}{2M(d/4)} = \boxed{2v}$

(e) $K_f = 2\left(\frac{1}{2}Mv_f^2\right) = M(2v)^2 = \boxed{4Mv^2}$

(f) $W = K_f - K_i = \boxed{3Mv^2}$