University of Maryland, College Park
Dept. of Physics

PHYSICS 161 | Lubna Rana

Spring 2003 | FINAL EXAM | Wednesday, May 21, 2003

NAME: Solutions –

SECTION: ______

Special Notes:

1). This is a CLOSED BOOK examination. No Calculators, Notes or Electronic Devices are allowed.

2). There are TWO Sections on this test.

Section I contains THREE PROBLEMS.

Problem 1: 25 Points; (pgs 2 & 3)

Problem 2: 25 Points; (pgs 3 & 4)

Problem 3: 25 Points; (pg 5)

For Section I, you must show COMPLETE WORK for CREDIT.

3). **Section II contains 15 Multiple Choice Questions, 5 Points Each.**

Record answers to multiple choice questions on the sheet entitled "ANSWERS TO MULTIPLE CHOICE QUESTIONS" (Page 6)

ANY MARKS YOU MAKE ON PAGES 7-11 WILL NOT BE GRADED. ALL WORK MUST APPEAR ON PAGES 1-6

4). The FORMULA SHEETS are on the last three pages. You may tear them off for easy reference.

5). DO NOT UNSTAPLE OR SEPARATE ANY PAGES OTHER THAN THE FORMULA SHEET.

6). If a question is unclear, please ASK IMMEDIATELY.

Lots of Luck!

Section I: Problems

P1). [25 Points] The figure below shows a cart-spring system oscillating back and forth on a frictionless floor with maximum extension A. At the instant the spring is at $x = +A$, a girl running at a speed v_0 hops onto the cart and after that rides along with the cart. The surface of contact between the girl's feet and the cart has considerable friction. Let M be the mass of the cart, m the mass of the girl and let the spring's mass be negligible. The spring constant is k.

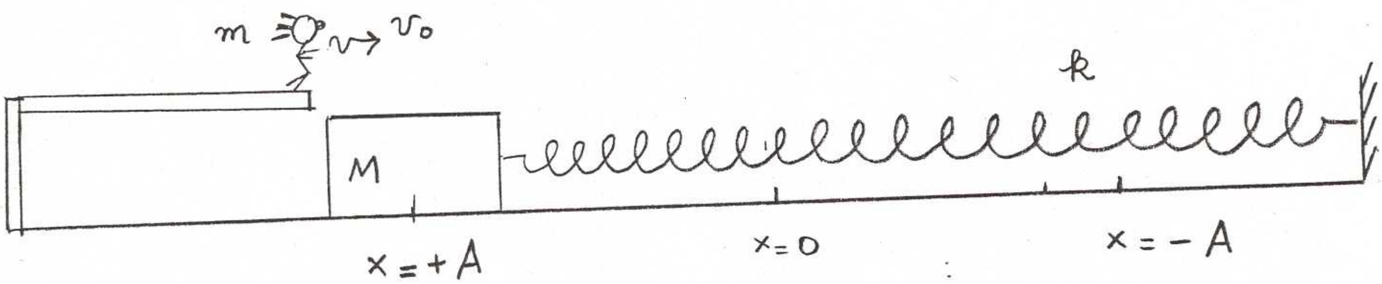


fig P1.

a).[5 Points] What is the total kinetic energy of the spring-cart-girl system **before** the girl jumps onto the cart?

$K.E_{Ti} = \frac{1}{2} m v_0^2$

b). [5 Points] Find the velocity v_f of the cart and the girl *immediately* after she jumps onto the cart in terms of m, M, and v_0.

$P_{ix} = P_{fx}$

$m v_0 = (m+M) v_f$

$\Rightarrow \boxed{v_f = \frac{m v_0}{m+M}}$

c). [5 Points] Find the kinetic energy of the spring-cart-girl system *immediately* after the collision. Is the total kinetic energy conserved in this collision?

$K.E_{Tf} = \frac{1}{2}(m+M) v_f^2$

$= \frac{1}{2} \frac{m^2 v_0^2}{m+M}$

No. $K.E_{Tf} < K.E_{Ti}$.
This is an inelastic collision.

d).(i) [2.5 Points] Without doing any calculations, state if you expect the kinetic energy of the center of mass of the system, $K.E_{c.m.}$, to be the same shortly before and shortly after the collision i.e., is $K.E_{c.m.,i} = K.E_{c.m.,f}$?

Yes. $K.E_{c.m,i} = K.E_{c.m,f}$

d). (ii) **[2.5 Points]** State any principles on which you are basing your answer to Part d (i). During the collision, $F_{net,ext} = 0 \Rightarrow$
$\vec{a}_{c.m} = 0 \Rightarrow \vec{v}_{c.m} = \text{Constant} \Rightarrow K.E_{c.m.i} = K.E_{cm f}$.

e). **[5 Points]** Is the maximum extension of the spring-cart-girl system the same, greater or less than A? Think carefully and express the maximum extension of the spring-cart-girl system, call it B, in terms of A, M, m, v_0 and k.

After the collision has taken place, there is no more any loss in mechanical energy to thermal energy - so

$$E_i = E_f$$

$$K.E_i + P.E_{sp\,i} = P.E_{sp,f}$$

$$\frac{1}{2}\frac{m^2 v_0^2}{m+M} + \frac{1}{2}kA^2 = \frac{1}{2}kB^2$$

$$\boxed{\pm\sqrt{\frac{m^2}{m+M}\frac{v_0^2}{k} + A^2} = B} \Rightarrow B > A.$$

P2). **[25 Points]** A block of mass m slides down the frictionless surface and collides with the uniform vertical rod, sticking to it. The rod pivots about O through the angle θ_{max} before momentarily coming to rest. Our final objective in this problem will be to find the final height h_{max} of the center of mass of the rod-block system. I will lead you through the problem. In the following, express your answers for each part in terms of m, M, h, l and g ONLY. ($I_{c.m.}(rod) = 1/12\, Ml^2$.)

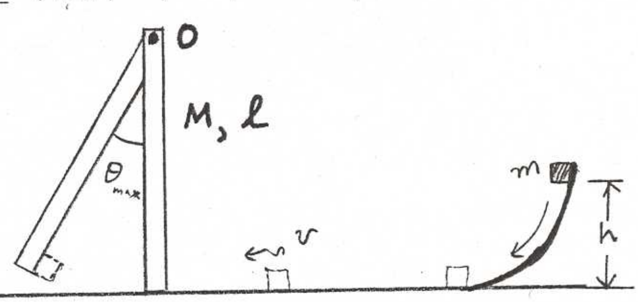


fig P2.

a). **[5 Points]** Find the magnitude and direction of the angular momentum of the rod-block system immediately before the collision. To indicate the direction, say "into the page" or "out of the page".

$$mgh = \frac{1}{2}mv_0^2 \Rightarrow v_0 = \sqrt{2gh}$$

$$\vec{L}_i = \vec{r} \times \vec{p}$$

$L_i = lmv_0$, into the page.

$$\Rightarrow \boxed{L_i = lm\sqrt{2gh}}$$

Calculate I about the pivot point.

$$I_0 = \frac{1}{12}Ml^2 + M\left(\frac{l}{2}\right)^2 + ml^2 = \frac{1}{3}Ml^2 + ml^2$$

b). **[5 Points]** Find the angular velocity ω of the rod-block system *immediately after* the collision.

$L_i = L_f$ since no net external torque –

$$mv_0 l = I\omega$$

$$\Rightarrow \omega = \frac{mv_0 l}{I} \Rightarrow \boxed{\omega = \frac{m\sqrt{2gh}}{\frac{1}{3}Ml + ml}}$$

c). **[5 Points]** What is the kinetic energy of the rod-block system *immediately after* the collision?

$$K.E = \frac{1}{2}I\omega^2$$

$$K.E = \frac{1}{2}\frac{m^2 v_0^2 l^2}{I^2}\cdot I = \frac{m^2 v_0^2 l^2}{2I} = \frac{2m^2 g h l^2}{2\left(\frac{1}{3}Ml^2 + ml^2\right)}$$

$$\boxed{K.E = \frac{m^2 g h}{\frac{1}{3}M + m}}$$

d). **[5 Points]** Use conservation of energy to find the final height h_{max} of the center of mass of the rod-block system.

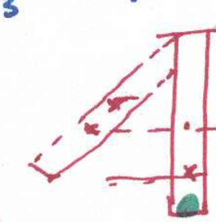
$$\Rightarrow h_{max} = \frac{m^2 h}{(m+M)\left(\frac{1}{3}M+m\right)} + \frac{Ml}{2(m+M)}$$

$$E_i = E_f$$

$$K.E_i + P.E_i = P.E_f$$

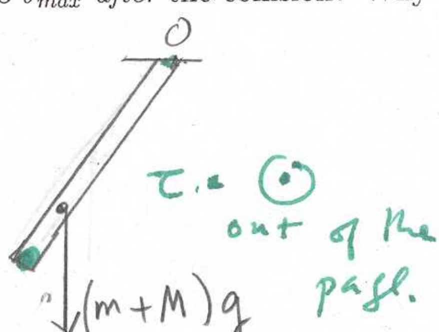
$$\frac{1}{2}I\omega^2 + (m+M)gh_i = (m+M)gh_f$$

$$\frac{m^2 g h}{\frac{1}{3}M+m} + \frac{Mgl}{2} = (m+M)gh_f$$



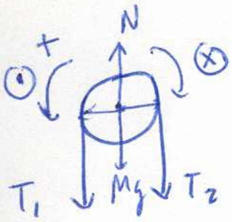
e). **[5 Points]** Is the angular momentum of the rod-block system conserved as it swings from $\theta = 0$ to θ_{max} *after* the collision? Why or why not?

e/ Gravity applies a torque about point O so $\vec{L}_f$ is not constant. You can also see that because ω decreases to ZERO as the rod + block swing up to hmax..



* A lot of you forgot that the center of mass is initially not at what you were considering $h=0$.

P3).[25 Points] Two blocks, of mass m_1 and m_2 are connected by a massless cord that is wrapped around a pulley that has a radius R and mass M. $m_1 > m_2$. The cord causes the pulley to rotate *without slipping.* The pulley rotates without friction. ($I_{c.m.}(pulley) = 1/2\,MR^2$.) In solving the problem, make sure to draw **a separate free body diagram for each object.**

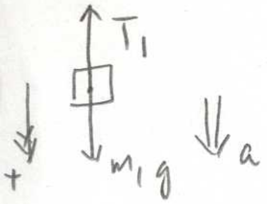


a). [5 Points] Write down the $\tau_{net,ext} = I\alpha$ equation for the pulley. On the diagram, make sure to indicate the direction of all the torques on the pulley.

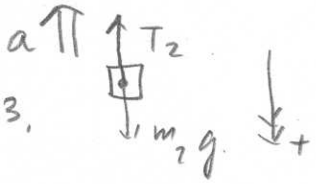
eq 1. $\boxed{T_1 R - T_2 R = I\alpha}$ ← this was sufficient for this part.

$\cancel{R}(T_1 - T_2) = \frac{1}{2} M R^{\cancel{2}} \cdot \frac{a}{\cancel{R}} \Rightarrow \boxed{T_1 - T_2 = \frac{1}{2} M a}$ eq 1'

b).[10 Points] Write down the $F_{net,y} = ma_y$ equations for object 1 and object 2 consistent with the coordinate system indicated in the diagram.



$\boxed{m_1 g - T_1 = m_1 a}$ eq 2 $\quad \boxed{T_2 - m_2 g = m_2 a}$ eq 3.



c). [10 Points] Solve the above three equations to find the acceleration of object 1 and object 2 in terms of m_1, m_2, M, R and g ONLY.

Add eq 2 & eq 3

$$
\begin{aligned}
T_2 - m_2 g &= m_2 a \\
+\quad -T_1 + m_1 g &= m_1 a \\
\hline
T_2 - T_1 + (m_1 - m_2) g &= (m_1 + m_2) a
\end{aligned}
$$

$\Rightarrow \quad T_2 - T_1 = (m_1 + m_2) a - (m_1 - m_2) g$

from eq 1', $\quad T_2 - T_1 = -\frac{1}{2} M a$

$\Rightarrow \quad -\frac{1}{2} M a = (m_1 + m_2) a - (m_1 - m_2) g.$

$\Rightarrow \quad \boxed{\frac{(m_1 - m_2) g}{(m_1 + m_2 + \frac{1}{2} M)} = a}$

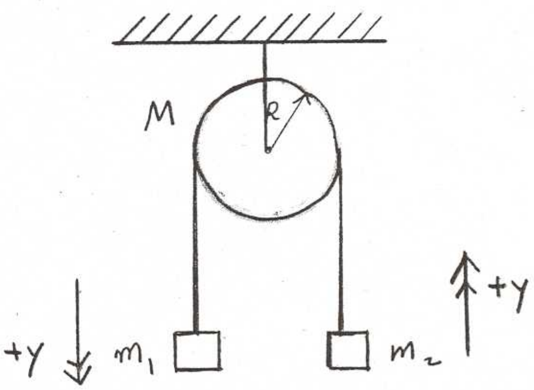


fig P3

If $m_1 = m_2$, get $a = 0$. Makes sense!

Section II: Multiple Choice Questions: 5 Points Each

S1). A bug is squashed when it hits the windshield of a car traveling along a highway. Are the magnitudes of the force exerted on the bug by the windshield F_{BW} and the force exerted on the windshield by the bug F_{WB} such that:

A). $F_{BW} < F_{WB}$

B). $F_{BW} = F_{WB}$ Newton's 3rd Law.

C). $F_{BW} > F_{WB}$

D). Not enough information, need to know the mass of the car and the bug.

S2-S3). While ringing a doorbell with one hand, a delivery person holds a package of mass m in place by using the other hand. He does this by pressing the package perpendicularly against a vertical wall. Assume $\mu_s < 1$. If the package does not slide, which of the following statements are true:

S2). The normal force applied on the package by the wall is equal to mg.
(A) True (B) False

$F_s = mg$

$\Rightarrow mg = F_s \le \mu_s N$

$mg \le \mu_s N$

$\Rightarrow \boxed{N > mg}$

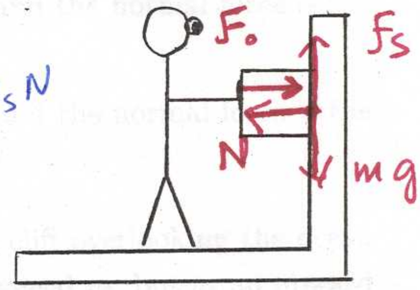


fig S2 - S3

S3). The force of friction on the package due to the wall is equal and opposite to mg.
(A) True (B) False

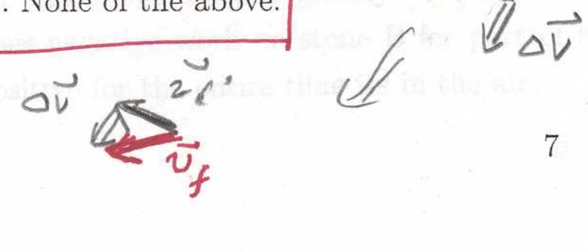
S4). The earth does not fall into the sun, but keeps moving around it with constant speed (assume a circular orbit) even though the force of gravitational attraction on the earth is always pointed directly toward the sun, because:

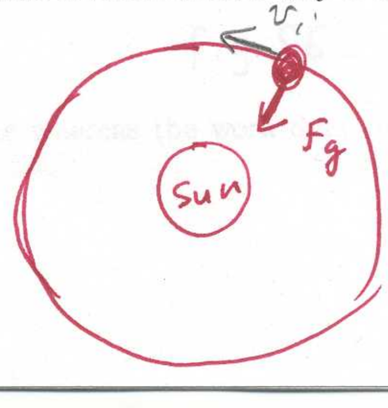
A). There is an *outward* centrifugal force acting on the earth that exactly cancels out the force of gravity.

B). There is a sideways force in addition to the force of gravity that keeps giving it a tangential velocity.

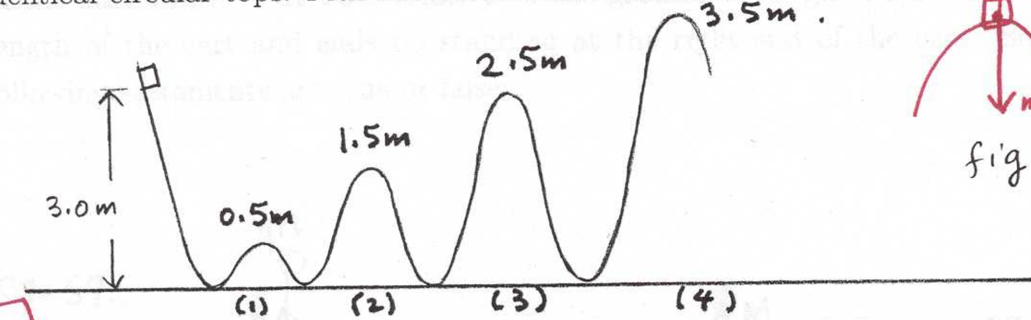
C). The force of gravity on the earth by the sun cancels out with the force of gravity on the sun by the earth. So the net force on the earth is zero. That's why it moves with constant speed.

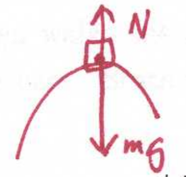
D). None of the above.





S5). In the figure below, a small initially stationary block is released on a frictionless ramp at a height of $3.0m$. Hill heights along the ramp are shown. The hills have identical circular tops. Pick out the correct statement.





$mg - N = \frac{mv^2}{R}$

fig S5

* $\boxed{\text{A).}}$ On hill (1) the centripetal acceleration is the largest and the normal force is the least. The larger the speed v, the large a_r, the smaller the normal force.

B). On hill (1) the centripetal acceleration is the largest and the normal force is the greatest.

C). On hill (3) the centripetal acceleration is the largest and the normal force is the least.

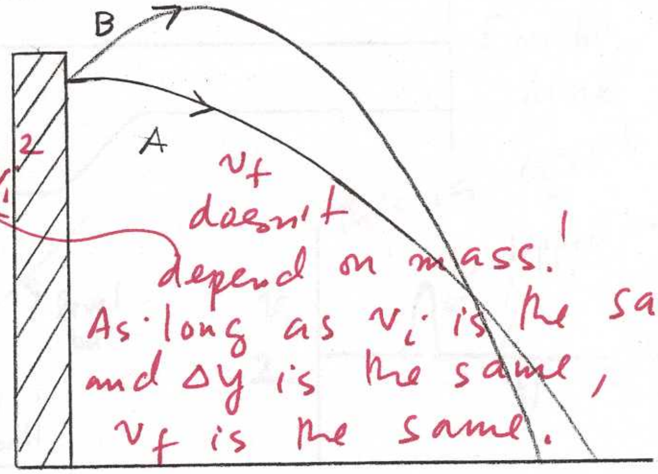
D). On hill (3) the centripetal acceleration is the largest and the normal force is the greatest.

S6). A stone A is thrown horizontally from the top of a cliff overlooking the ocean with speed v_0. Another stone B is thrown with the *same* speed v_0, but at an upward angle from the horizontal. Assume air resistance to be negligible. The speeds v_{Af} and v_{Bf} with which the stones hit the water are such that

A). $\boxed{v_{Af} = v_{Bf}}$ because the work done by gravity on both stones is the same. $W_{net} = \Delta K.E$

$\cancel{m} gh = \frac{1}{2} \cancel{m} v_f^2 - \frac{1}{2} \cancel{m} v_i^2$

B). Can't say: the final speeds depend on the mass of the stones.



v_f doesn't depend on mass! As long as v_i is the same, and Δy is the same, v_f is the same.

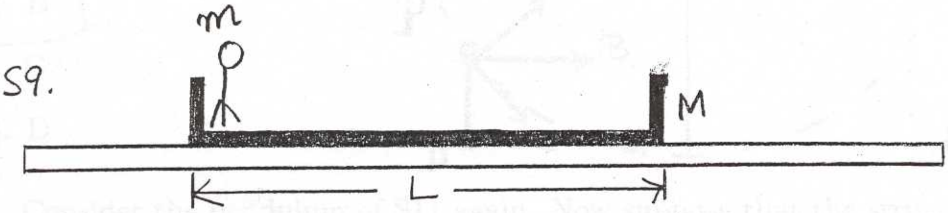
C). $v_{Af} < v_{Bf}$ because stone B accelerates for a longer time.

fig S6.

D). $v_{Af} > v_{Bf}$ because gravity does negative work on stone B for part of the time whereas the work done on A is positive for the entire time its in the air.

S7-S9). A girl of mass m stands on a cart of mass M which is free to slide with negligible friction along the horizontal floor. Initially, the girl stands at the left end of the cart which is **at rest** relative to the ground. The girl then walks the entire length of the cart and ends up standing at the right end of the cart. State if the following statements are true or false:

fig S7- S9.



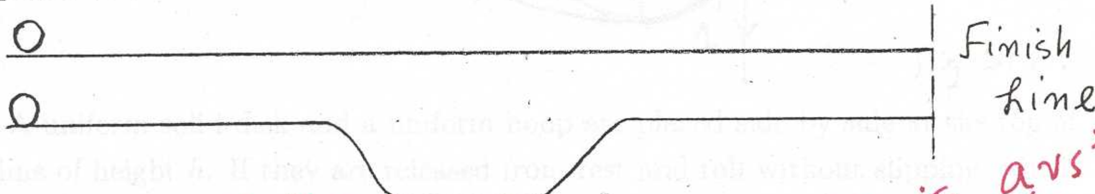
S7). At any instant while the girl is walking, the total linear momentum of the girl-cart system remains zero. (A) $\boxed{\text{True}}$ (B) False

$\vec{F}_{net,ext} = \frac{d\vec{P}}{dt} = 0 \Rightarrow \vec{P} = \text{const.} = 0!$ ∴ $F_{net,ext} = 0 \Rightarrow \vec{a}_{cm} = 0$

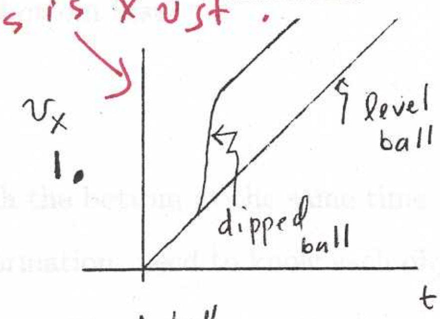
S8). The center of mass of the cart remains at rest relative to the ground at any instant while the girl is walking. (A) True (B) $\boxed{\text{False}}$

S9). The center of mass of the girl-cart system remains at rest relative to the ground at any instant while the girl is walking. (A) $\boxed{\text{True}}$ (B) False

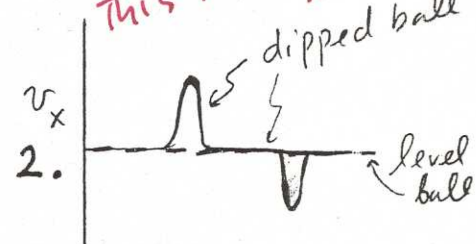
S10). Two balls race along parallel tracks, starting with the same initial speed v_0. One track is level. The second track has a dip in it as shown. Which v_x vs. t graph corresponds to this situation.



This is x vs t.



This is a_x vs t.

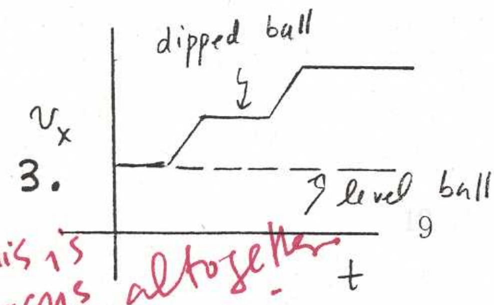


A). 1

B). 2

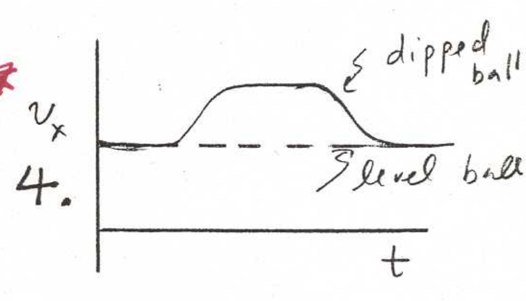
C). 3

$\boxed{\text{D). 4}}$



This is bogus altogether.

★



S11). A pendulum comprising a string of length L and a bob of mass m is released at rest from an angle of 90^0 with respect to the vertical. At the instant the pendulum bob is at point P, the **total acceleration** vector of the bob is directed along one of the arrows indicated in the figure. Which of these arrows best indicates the direction of the total acceleration?

a). A

b). B

c). C

d). D

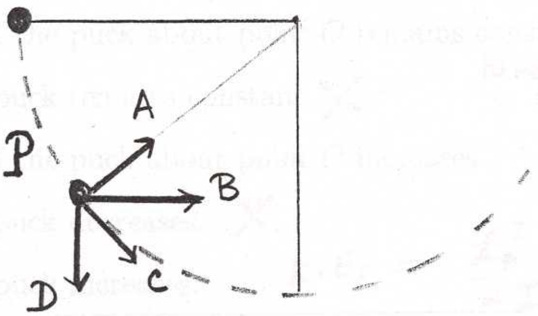


fig S11

S12). Consider the pendulum of S11 again. Now suppose that the string hits a peg located a distance d below the point of suspension such that the bob now swings in a complete circle centered on the peg. If the string is cut at the precise instant the pendulum bob reaches point Q, the bob's trajectory will most closely resemble

A). Path 1

B). Path 2

C). Path 3

D). Path 4

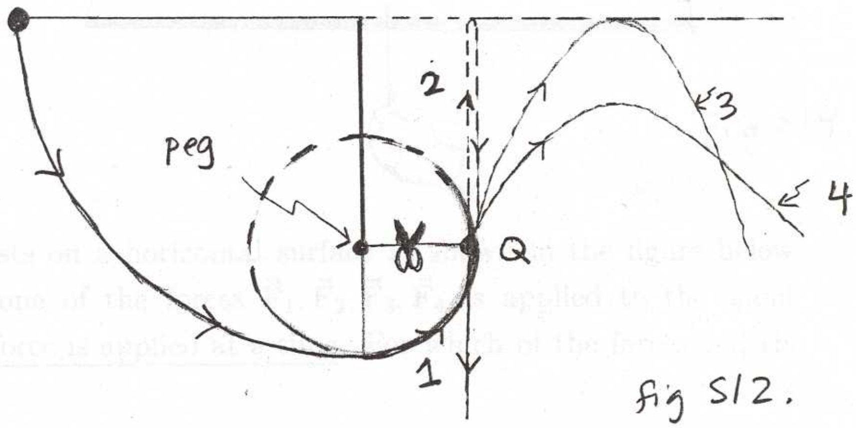


fig S12.

S13). A uniform solid disk and a uniform hoop are placed side by side at the top of an incline of height h. If they are released from rest and roll without slipping, which object reaches the bottom first?

a). The hoop

b). The disk

c). They both reach the bottom at the same time

d). Insufficient information: need to know each object's mass and radius.