Key

PHYSICS 161, Spring 2003

Discussion Quiz , Tuesday, Feb 25

Q1). A ball is thrown with an initial velocity v_0 at an angle θ with the vertical. On its way down, the ball gets stuck in a tree at height H. A y vs x plot of its motion is given in Fig 1 below.

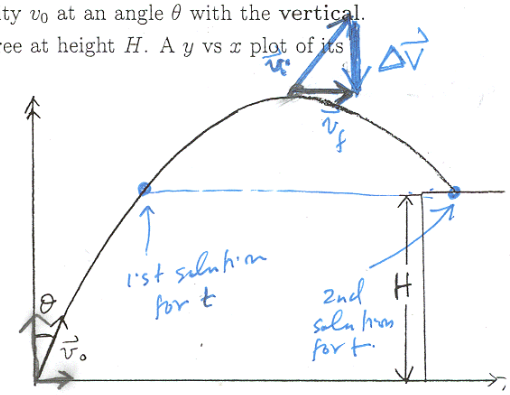

Fig 1

a). In term of unit vectors $\hat{i}$ and $\hat{j}$, write down the initial velocity vector $\vec{v}_i$.

$\vec{V}_i = V_0 \sin\theta\,\hat{i} + V_0 \cos\theta\,\hat{j}$

b). Write down the velocity vector $\vec{v}_{top}$ at the instant the ball reaches the peak of its trajectory. Indicate this velocity vector on the plot above.

$\vec{V}_{top} = V_0 \sin\theta\,\hat{i} + 0\,\hat{j}$

c). Figure out $\Delta\vec{v}$ between $t = 0$ and $t = t_{max}$ graphically i.e., by using the head-to-tail rule of vector addition. Here, $t = t_{max}$ is the instant the ball reaches its maximum height. How is the direction of the average acceleration vector related to $\Delta\vec{v}$? What about its magnitude?

$\Delta\vec{V} = \vec{V}_i - \vec{V}_{top} = V_0 \cos\theta\,\hat{j}$ / $\vec{a}_{avg} = \frac{\Delta\vec{V}}{\Delta t}$ see Fig 1

The direction of the average acceleration is the same as $\Delta\vec{V}$. The magnitude $|\vec{a}_{avg}| = \frac{|\Delta\vec{V}|}{\Delta t}$.

d). Using $y(t) = y_0 + v_{0y}t + 1/2\,a_y t^2$, set up a quadratic equation to find the time it takes the ball to reach the tree at height H.

$Y = V_0 \cos\theta\, t - \frac{1}{2} g t^2$

$\boxed{\frac{1}{2} g t^2 - V_0 \cos\theta\, t + H = 0} \Rightarrow \boxed{t^2 - \frac{2V_0\cos\theta}{g} t + \frac{2H}{g} = 0}$

e). Look at Fig 1 again carefully. How many valid solutions do you expect to find for t from this quadratic equation? What do each of these solutions correspond to?

2 Solutions, the first corresponds to $Y = H$ with $\frac{dY}{dt} < 0$ the second $Y = H$; $\frac{dY}{dt} > 0$ ✓

see Fig 1.

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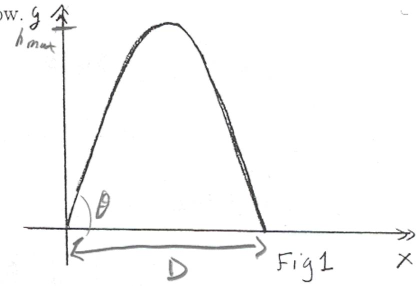
Discussion Quiz 3, Thursday, Feb 27

Q1). A ball is thrown with an initial velocity v_0 at an angle of θ above the horizontal. A y vs x plot of its motion is given below.



a). Find the time t_{up} for the ball to reach its maximum height h_{max} in terms of v_0, θ and g.

$\vec{V} = V_0 \cos\theta \hat{i} + V_0 \sin\theta \hat{j}$

$V_{fy} = V_{oy} + a_y t$

$0 = V_0 \sin\theta - gt$

$\underline{t = \dfrac{V_0 \sin\theta}{g}}$

b). Find the time t_{down} for the ball to travel from $y = h_{max}$ back down to $y = 0$. What seems to be the relationship between t_{up} and t_{down}?

$Y_f = Y_0 + V_0 t + \frac{1}{2} a t^2$

$h = 0 + V_0 \sin\theta t - \frac{1}{2} g t^2$

$Y_f = Y_0 + V_0 t + \frac{1}{2} a t^2$

$0 = (V_0 \sin\theta t - \frac{1}{2} g t^2) - \frac{1}{2} g t^2$

$0 = V_0 \sin\theta t - g t^2$

$0 = V_0 \sin\theta - gt$

$\underline{t = \dfrac{V_0 \sin\theta}{g}}$

c). Find the total distance x_{total} travelled by the ball in the x-direction in terms of v_0, θ and g.

$D = V_{0x} t$

$= (V_0 \cos\theta) t$

$D = (V_0 \cos\theta)\left(\dfrac{V_0 \sin\theta}{g}\right)$

$\boxed{D = \dfrac{V_0^2 \sin\theta \cos\theta}{g}}$

d & e &f). For a fixed v_0, suppose we **increase** θ. Let's call the smaller angle θ_1 and the larger angle θ_2.

d). Compare the total time of flight $t^{(1)}_{flight}$ with $t^{(2)}_{flight}$.

$\theta_2 > \theta_1 > \frac{\pi}{4} \Rightarrow V^{(2)}_{0y} > V^{(1)}_{0y} \Rightarrow t^{(2)}_{flight} > t^{(2)}_{flight}$

$V_{0y} = V_0 \sin\theta$

e). What would happen to h_{max}? Why?

$h^{(2)}_{max} > h^{(1)}_{max}$ because $V^{(1)}_{0y} > V^{(2)}_{0y}$

$h = V^{(1)}_{0y} t - \frac{1}{2} g t^2$

f). What would happen to the range x_{total}? Is it possible for h_{max} to change but x_{total} to remain the same? If so, find the condition that must be satisfied between θ_1 and θ_2 if $x^{(1)}_{total} = x^{(2)}_{total}$.

if $\theta_2 > \theta_1 > \frac{\pi}{4}$ then $X^{(1)}_{tot} > X^{(2)}_{tot}$

if $\theta_1 < \frac{\pi}{4}$ and $\theta_2 > \frac{\pi}{4}$ it is possible

$V_{fy} = V_{oy} - at$

$0 = V_0 \sin\theta - gt$

$t = \dfrac{V_0 \sin\theta}{g}$

$\Rightarrow D = \dfrac{2}{g} V_0^2 \sin\theta \cos\theta$

$D_1 = D_2 = \dfrac{2V_0^2}{g} \sin\theta_1 \cos\theta_1 = \dfrac{2V_0^2}{g} \sin\theta_2 \cos\theta_2$

$\Rightarrow \sin\theta_1 \cos\theta_1 = \sin\theta_2 \cos\theta_2$

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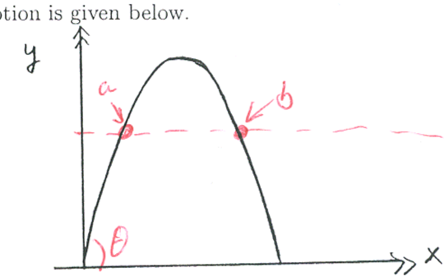
Discussion Quiz, Friday, Feb 28

$\vec{V}_i = V_0\cos\theta\,\hat{i} + V_0\sin\theta\,\hat{j}$

Q1). A ball is thrown with an initial velocity $\vec{v}_i$ with magnitude v_0 at an angle θ above the horizontal. A y vs x plot of its motion is given below.



a). Find the time t_{up} for the ball to reach its maximum height h_{max} in terms of v_0, θ and g.

$V_{fy} = V_{oy} + at$

$0 = V_0\sin\theta - gt$

$t = \frac{V_0\sin\theta}{g}$

b). Find the time t_{down} for the ball to travel from $y = h_{max}$ back down to $y = 0$. What seems to be the relationship between t_{up} and t_{down}?

$X_f = Y_0 + V_{oy}t - \frac{1}{2}gt^2$

$h = 0 + V_0\sin\theta\, t - \frac{1}{2}gt^2$

$Y_f = Y_0 + V_0 t - \frac{1}{2}gt^2$

$0 = h + 0(t) - \frac{1}{2}gt^2$

$0 = V_0\sin\theta\, t - \frac{1}{2}gt^2 - \frac{1}{2}gt^2$

$0 = (V_0\sin\theta - gt)\,t$

$V_0\sin\theta - gt = 0$

$t = \frac{V_0\sin\theta}{g}$

c). Find the total distance x_{total} traveled by the ball in the x-direction in terms of v_0, θ and g.

$t_{up} = \frac{V_0\sin\theta}{g} \Rightarrow t_{tot} = \frac{2V_0\sin\theta}{g}$

$\Rightarrow x_{total} = V_x t = \frac{2V_0\cos\theta\, V_0\sin\theta}{g} = \frac{2V_0^2\sin\theta\cos\theta}{g}$

d). Find the final vertical velocity v_{fy} and the final horizontal velocity v_{fx} of the ball right before it hits the ground. What angle does the final velocity vector make with the horizontal?

$V_{fy} = -V_{iy} \Rightarrow V_{fy} = -V_0\sin\theta$

V_{fy} makes an angle of θ below the horizontal

e). How does the magnitude of initial velocity $\vec{v}_i$ compare with the magnitude of the final velocity $\vec{v}_f$?

$\vec{V}_i = V_0\cos\theta\,\hat{i} + V_0\sin\theta\,\hat{j}$

$\vec{V}_f = V_0\cos\theta\,\hat{i} - V_0\sin\theta\,\hat{j}$

$|\vec{V}_i| = |\vec{V}_f| = \sqrt{(V_0^2\cos^2\theta) + (V_0^2\sin^2\theta)}$

f). Mark a pair of points (of your choice) on the trajectory of the ball that the **same** vertical position y. What implications dos your result for part (e) have for the ball's vertical velocity at any such pair of points.

$|V_a| = |V_b|$, $V_{y_a} = -V_{y_b}$, $V_{x_a} = V_{x_b}$