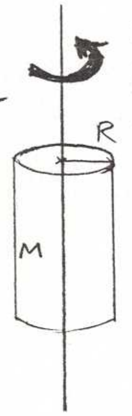
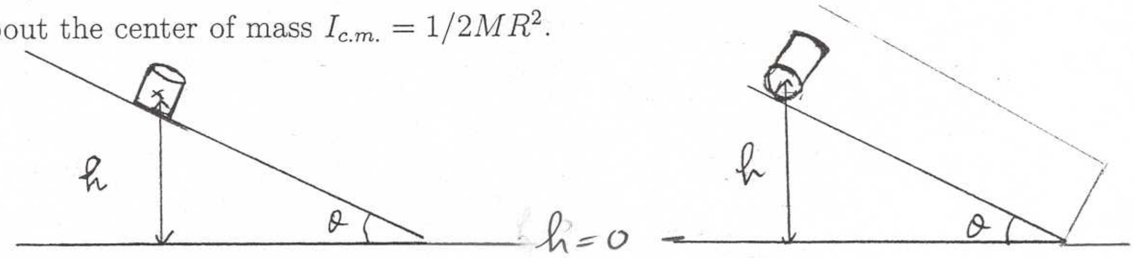
PHYSICS 161, Spring 2003

Discussion Quiz , Tuesday, May 6

$I_{cm} = \frac{1}{2} MR^2$



Q1). Consider a race between two **identical solid cylinders** down two identical ramps. One of the cylinders slides with negligible friction and the other one rolls down the ramp without slipping. The cylinders have radius R, mass M and moment of inertia about the center of mass $I_{c.m.} = 1/2MR^2$.



a). Write down the energy conservation equation for the slider. Take the initial point to be at the top of the ramp and the final point at the bottom of the ramp. Don't forget to indicate your $h = 0$ level.

$$E_i = E_f$$
$$mgh = \frac{1}{2} m v^2$$

b). Solve for the final velocity of the slider at the bottom of the ramp.

$$v^2 = 2gh$$
$$\Rightarrow v = \sqrt{2gh}.$$

c). Write down the energy conservation equation for the roller. Follow the same instructions as in part (a) above.

$$mgh = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

d). What is the relationship between the angular velocity *omega* of the roller and the velocity of its center of mass? Explain briefly.

$v_{cm} = R\omega$. As the cylinder rotates by an angle of 2π, the center of mass translates forward by $2\pi R$. In general $x_{cm} = R\theta \Rightarrow v_{cm} = R\omega$.

e). Solve for the final linear velocity of the roller. Who wins the race?

$$\cancel{m} gh = \frac{1}{2} \cancel{m} v_{cm}^2 + \frac{1}{4} \cancel{m} R^2 \cdot \frac{v_{cm}^2}{R^2} \Rightarrow \boxed{v_{cm}^2 = \frac{4}{3} gh}$$

The slider wins the race.

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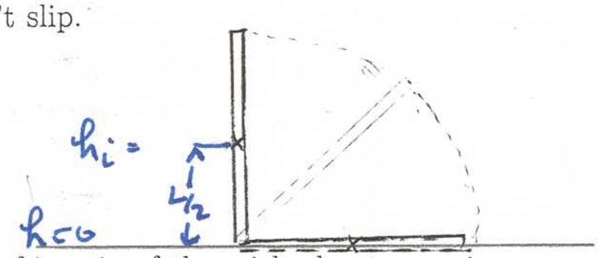
Discussion Quiz, Thursday, May 8

Q1). Consider a uniform stick of mass M and length L. When the stick is released, it topples. The end touching the floor doesn't slip.

$I_{cm} = \frac{1}{12} ML^2$



a). Suppose you are given that the moment of inertia of the stick about an axis through its center of mass (see figure above) is $I_{c.m.} = (1/12)ML^2$. Use the parallel axis theorem to find the moment of inertia about its end.

$I_{end} = I_{cm} + Mh^2$ $\quad$ $h = L/2 \equiv$ distance between the axis through the cm & the axis parallel to it.

$= \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2$

$\boxed{I_{end} = \frac{1}{3} ML^2}$

b). Write down the energy conservation equation for the stick. Let the initial point be when the stick is upright and the final point when the stick is *just about to* hit the floor. Don't forget to indicate the $h = 0$ level.

$E_i = E_f$

$mg h_i = \frac{1}{2} I \omega^2$ $\quad \Rightarrow \boxed{mgL = I_{end}\,\omega^2}$

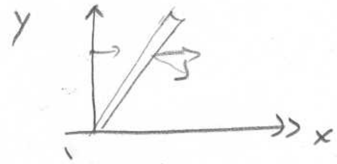
$mg L/2 = \frac{1}{2} I_{end} \omega^2$

c). Solve for the speed v with which the **center of mass** of the stick hits the floor. Is it bigger, smaller or the same as the speed of the tip of the stick a at that instant?

$\cancel{m} g \cancel{L} = \frac{1}{3} \cancel{m} L^{\cancel{2}} \cdot \omega^2$ $\Rightarrow \boxed{\omega^2 = \frac{3g}{L}}$

using $v_{cm} = \frac{L}{2}\omega$ we find $\left(\frac{2v_{cm}}{L}\right)^2 = \frac{3g}{L}$ $\Rightarrow v_{cm}^2 = \frac{3}{4} gL$ $\Rightarrow \boxed{v_{cm} = \sqrt{\frac{3}{4} gL}}$

d). Does the center of mass of the stick accelerate in the horizontal x-direction? Why or why not? Yes. First its at $x = 0$ and $v_{cm,x} = 0$. A moment later, $\boxed{v_{cm,x} \neq 0}$ as it moves from $x = 0$ to $x \neq 0$.



e) Do you suspect the acceleration of the center of mass of the stick in the y-direction to be larger smaller or equal to g? *Hint: If the $a_{y,c.m}$ were equal to g what would be its speed at $h = 0$? Compare this to the result you got in part (c) above.*

If $a_{y,cm} = g$, then expect $v_f^2 = 2 g h_i \Rightarrow v_{fy}^2 = \cancel{2} g \frac{L}{\cancel{2}} = gL$.
But here, found $v_{fy}^2 = \frac{3}{4} gL$. $\Rightarrow a_{ycm}$ less than g here.

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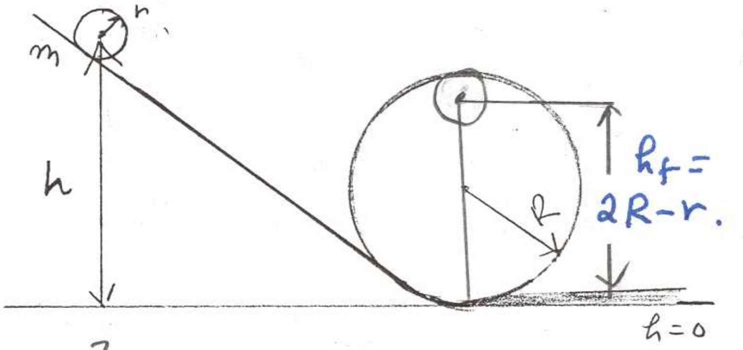
Discussion Quiz, Friday, May 9

Q1). A small solid marble of mass m and radius r rolls without slipping along the loop-the-loop track shown in the figure below. The moment of inertia of the marble about its center of mass is $I_{c.m.} = 2/5mr^2$. The marble is released from rest from some minimum height h such that it is able to go around the loop.

a). Write down the energy conservation equation for the marble. Let the initial point be when the marble is at $y = h$ and the final point be at the top of the loop.



$E_i = E_f$

$$mgh = mg(2R-r) + \frac{1}{2}mv_{c.m}^2 + \frac{1}{2}I_{c.m}\omega^2$$

c). Solve for the speed v of the marble at the **top of the loop** in terms of m, g, r and R. using $v_{c.m} = r\omega$

$$\cancel{m}gh = \cancel{m}g(2R-r) + \frac{1}{2}\cancel{m}v_{c.m}^2 + \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{5}\cancel{m}r^2\omega^2 \leftarrow v_{c.m}^2$$

$$\Rightarrow gh = g(2R-r) + \left(\frac{1}{2}+\frac{1}{5}\right)v_{c.m}^2 \Rightarrow \boxed{v_{c.m}^2 = \frac{10}{7}g[h-2R+r]}.$$

d). Now let's compare the motion of the rolling marble to the motion of a block of the same mass m that slides without friction along an identical loop-the-loop track. Does one of them have a larger **translational kinetic energy** at the top of the loop if both are released from the same height h? If so, then which one and why?

The slider has a larger translational kinetic energy. Since the roller has some its kinetic energy in the form of rotational kinetic energy & both have the same total energy.

e) Do you expect the minimum height h needed for the rolling marble to go around the loop to be less than, greater than or equal to that needed by the sliding block? Explain your answer briefly.

Minimum height h must be larger for the roller than the slider. To be able to complete the circular path, each object needs the same minimum $v_{c.m}$. And for each initial h, the roller's $v_{c.m}$ at the top is less than the slider's.