Physics for Scientists and Engineers

Chapter 8

Dynamics II: Motion in a Plane

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Dynamics in Two Dimensions

- Draw a free-body diagram
- Use Newton's second law in component form:

$$\sum F_x = ma_x, \ \sum F_y = ma_y$$

- Solve for the acceleration
  - If the acceleration is constant, use the two-dimensional kinematic equations:

$$x_f = x_i + v_{ix}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad y_f = y_i + v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x\Delta t \qquad v_{fy} = v_{iy} + a_y\Delta t$$

Projectile Motion

- In the absence of air resistance, a projectile has only the gravitational force acting on it:

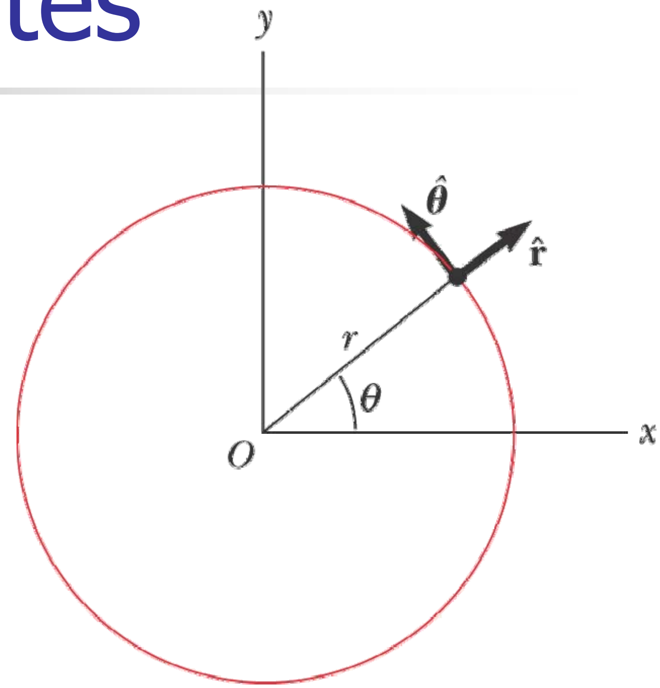
$$\vec{F}_{\mathrm{G}} = -mg\hat{j}$$

- From Newton's second law,

$$a_x = \frac{\left(F_G\right)_x}{m} = 0, \quad a_y = \frac{\left(F_G\right)_y}{m} = -g$$

- The vertical motion is free fall, while the horizontal motion is one of constant velocity

Polar vs *rtz* Coordinates



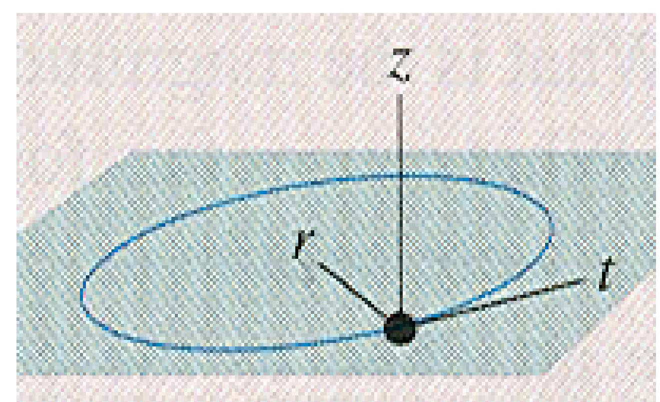
- Usually the radial unit vector points *outward*, so

$$\vec{a}_r = -\frac{v^2}{r}\hat{r}$$

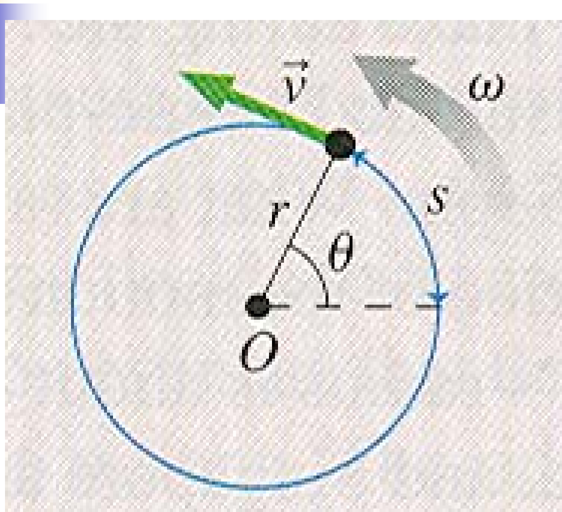
- The textbook uses an *inward* pointing radial unit vector, so

$$\vec{a}_r = +\frac{v^2}{r}\hat{r}$$

- Make sure you are consistent in the signs of the radial vectors



Circular Motion Kinematics



Angular position

$$\theta = s/r$$

Angular velocity

$$\omega = d\theta/dt$$

$$v_t = \omega r$$

Period $T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{\omega}$

Uniform circular motion

$v_t = \text{constant} \qquad \omega = \text{constant}$

$$\theta_f = \theta_i + \omega \Delta t$$

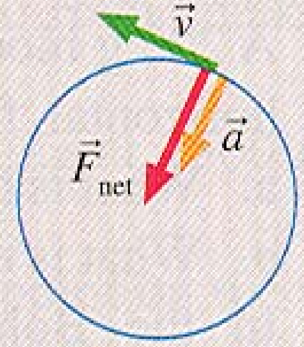
Nonuniform circular motion

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{a_t}{2r}(\Delta t)^2$$

$$\omega_f = \omega_i + \frac{a_t}{r}\Delta t$$

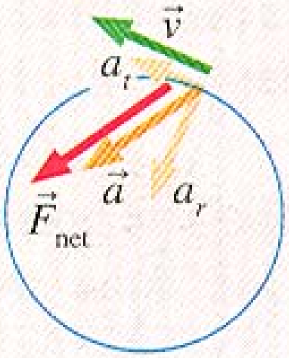
Circular Motion Dynamics

Uniform Circular Motion



- v is constant.
- $\vec{F}_{net}$ points toward the center of the circle.
- The **centripetal acceleration** $\vec{a}$ points toward the center of the circle. It changes the particle's direction but not its speed.

Nonuniform Circular Motion



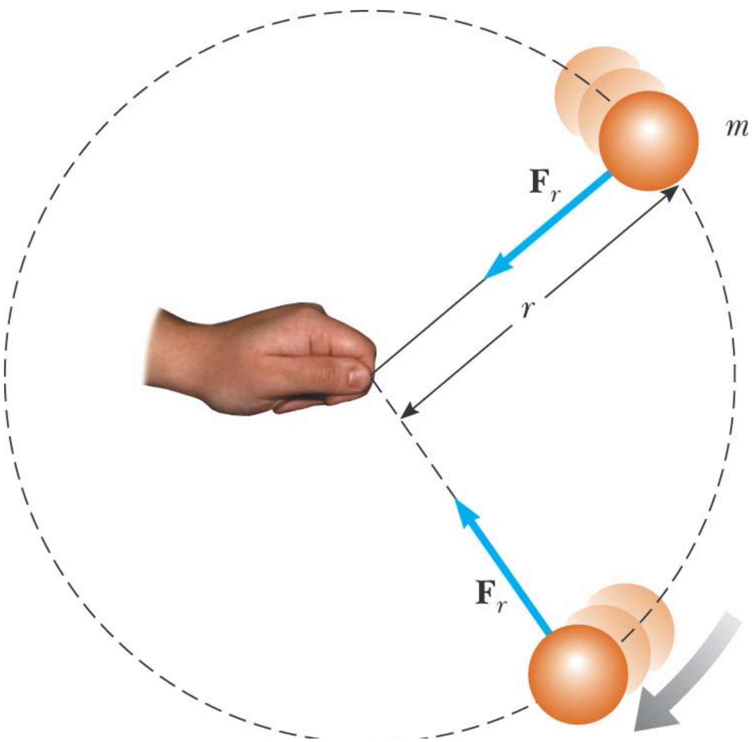
- v changes.
- $\vec{a}$ is parallel to $\vec{F}_{net}$.
- The radial component a_r changes the particle's direction.
- The tangential component a_t changes the particle's speed.

Uniform Circular Motion

- A force causing a centripetal acceleration acts *toward* the center of the circle

$$\sum F_r = ma_r, \quad -F_r = -m\frac{v^2}{r}$$

- It causes a change in the direction of velocity vector
  - If the force vanishes, the object would move in a straight line tangent to the circle



Example 1: Amusement Park Ride

An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough such that any person inside is held up against the wall when the floor drops away. Experiment shows that the angular rotation rate has to be large enough, or the person falls.

What is the necessary rotation rate?

The coefficient of static friction between person and wall is μ_s, and the radius of the cylinder is R.

Example 1, cont

The friction holds the person up!

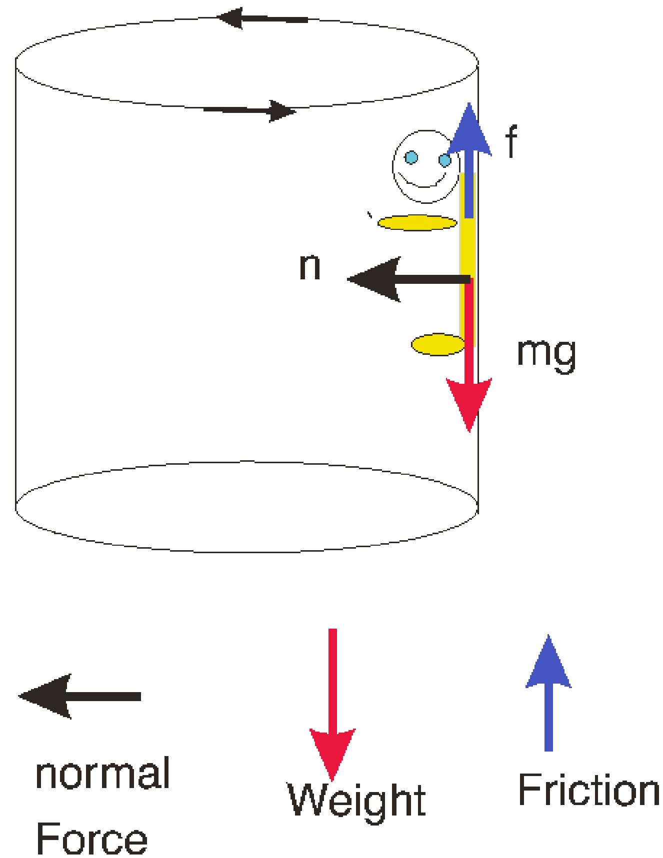
$$\text{Radial}: \sum F_r = ma_c, \quad -n = -m\frac{v^2}{R}$$

$$\text{Vertical}: \sum F_y = ma_y, \quad f_s - mg = ma_y$$

$$\text{For stability}, \quad a_y = 0$$

$$\text{Thus} \quad f_{s,\max} = \mu_s n \geq mg \Rightarrow \frac{mv^2}{R}\mu_s \geq mg$$

$$\Rightarrow \quad v \geq \sqrt{\frac{gR}{\mu_s}} \quad \Rightarrow \omega = \frac{v}{R} \geq \sqrt{\frac{g}{R\mu_s}}$$



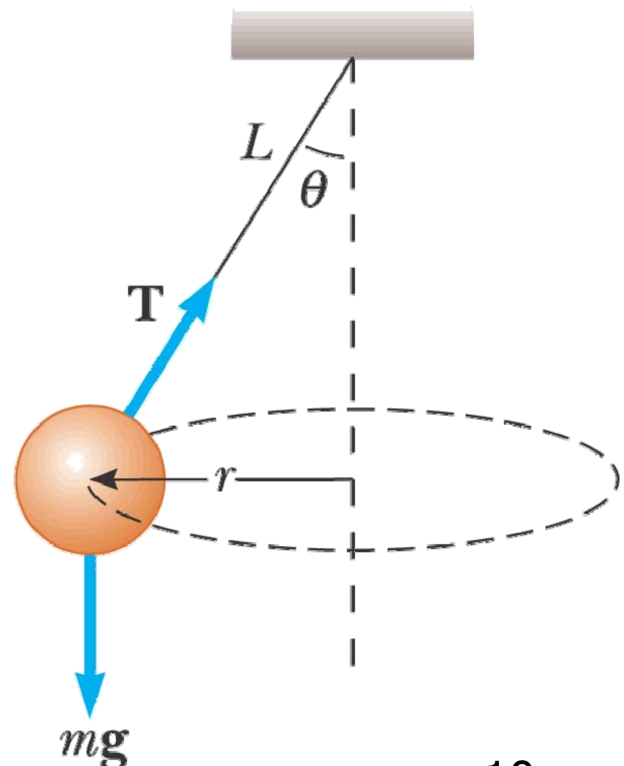
Example 2: Conical Pendulum

The object is in equilibrium in the vertical direction and undergoes uniform circular motion in the horizontal direction.

Find the tangential speed.

$$r: \sum F_r = -T_r = -\frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{rT_r}{m}}$$

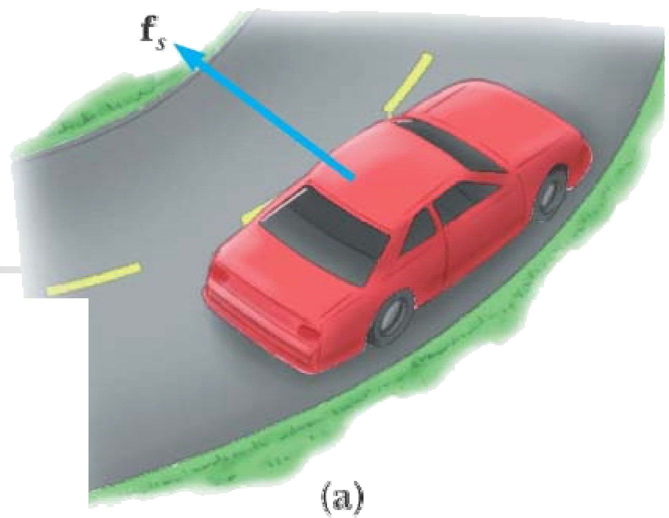
$$y: \sum F_y = T_y - mg = 0$$



Since $T_r = T\sin\theta,\ T_y = T\cos\theta,\ r = L\sin\theta$

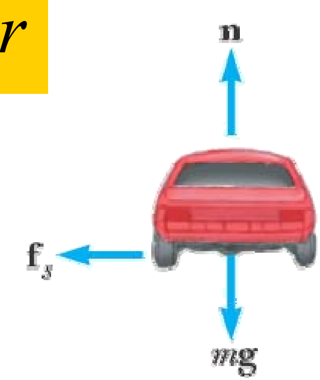
$$v = \sqrt{\frac{L\sin\theta\, T\sin\theta}{m}} = \sqrt{Lg\sin\theta\tan\theta}$$

Flat Horizontal Curve



(a)

- Static friction supplies the centripetal force
- Maximum speed at which the car can negotiate the curve is $v_{\max} = \sqrt{\mu_s g r}$



(b)

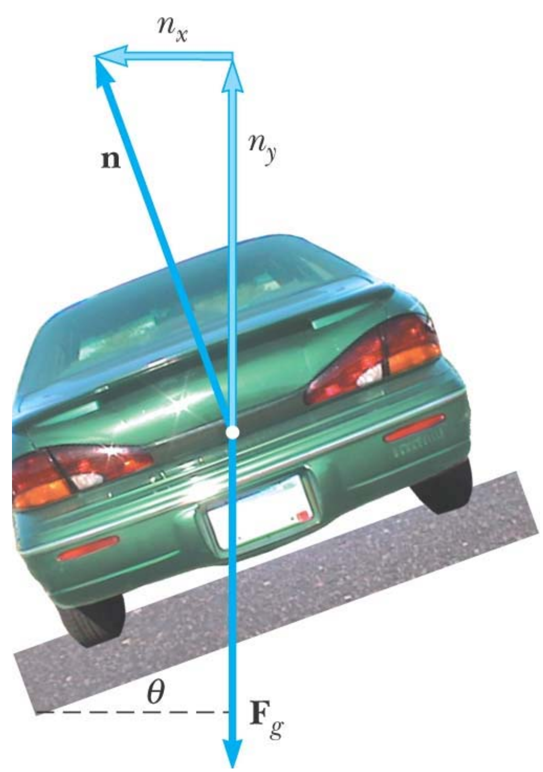
$$r: \sum F_r = m a_r, \quad -f_s = -m\frac{v^2}{r}$$

$$y: \sum F_y = 0, \quad n - mg = 0$$

$$f_s \le \mu_s n = \mu_s mg, \; m\frac{v^2}{r} \le \mu_s mg \Rightarrow v \le \sqrt{\mu_s g r}$$

- Note, this does not depend on the mass of the car

Banked Curve



- A component of the normal force supplies the centripetal force
- With zero friction, the car can negotiate the curve with speed

$$v = \sqrt{rg\tan\theta}$$

$$r:\ \sum F_r = -n_x = -\frac{mv^2}{r},\ \ y:\ \sum F_y = n_y - mg = 0$$

$$n_x = n\sin\theta,\ \ n_y = n\cos\theta = mg$$

$$v = \sqrt{\frac{rn_x}{m}} = \sqrt{\frac{r(mg/\cos\theta)\sin\theta}{m}} = \sqrt{rg\tan\theta}$$

Banked Curve, cont

- With zero friction, there is only one speed that a car can turn in a circle on a banked curve
  - If the car went slower than $v = (rg \tan\theta)^{1/2}$, it would slide to a smaller radius, i.e. downhill
  - If it went at a higher speed, it would slide to a larger radius
- In a real situation, friction would come into play preventing this sliding
  - The necessary frictional force would, however, be less than if the road were flat

Example 3: Flat Curved Road

The cornering performance of an automobile is evaluated on a skidpad, where the *maximum* speed that a car can maintain around a circular path on a dry, flat surface is measured. Then the centripetal acceleration, also called the *lateral acceleration*, is calculated as a multiple of the free-fall acceleration *g*. A Dodge Viper GTS can negotiate a skidpad of radius 61.0 m at 86.5 km/h (53.7 mph).

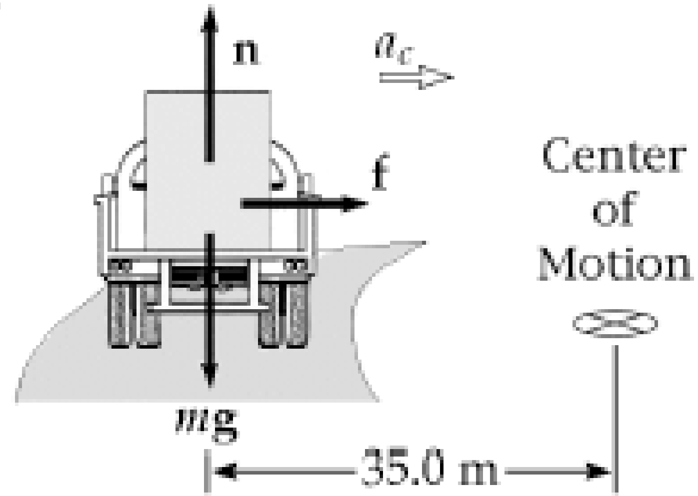
Calculate its maximum lateral acceleration.

$$a = \frac{v^2}{r} = \frac{\left[(86.5\ \mathrm{km/h})(1\ \mathrm{h}/3600\ \mathrm{s})(1000\ \mathrm{m}/1\ \mathrm{km})\right]^2}{61.0\ \mathrm{m}}\left(\frac{1g}{9.80\ \mathrm{m/s^2}}\right)$$

$$= 0.966g$$

Example 4: Egg Crate on Truck

A crate of eggs is located in the middle of the flat bed of a pickup truck as the truck negotiates an *unbanked* curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m.



If the coefficient of static friction between the crate and truck is 0.600, how fast can the truck move without the crate sliding?

The static friction supplies the centripetal force: $f_s = mv^2/r$. Since there is no vertical acceleration, $n - mg = 0$. Therefore,

$$f_s = mv^2/r \le \mu_s n = \mu_s mg, \quad v \le (\mu_s rg)^{1/2}.$$

$$v \le [(0.600)(35.0\text{ m})(9.80\text{ m/s}^2)]^{1/2} = 14.3\text{ m/s (32mph)}$$

Circular Orbits

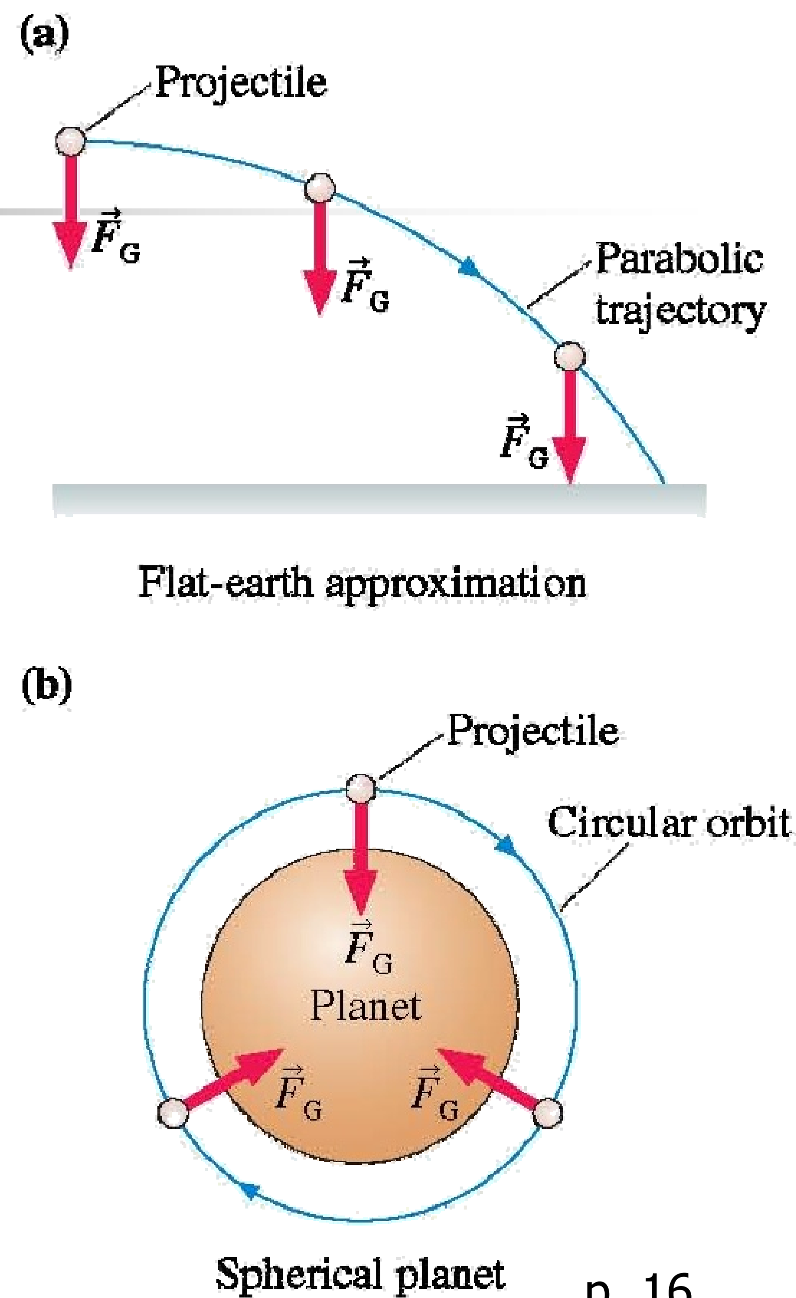
- Gravitational force provides the centripetal acceleration needed for a circular orbit:

$$a_r = \frac{(v_{\text{orbit}})^2}{r} = g$$

  - An object moves parallel to the surface with speed

$$v_{\text{orbit}} = \sqrt{rg}$$

  - An object with any other speed will not follow a circular orbit



Loop-the-Loop

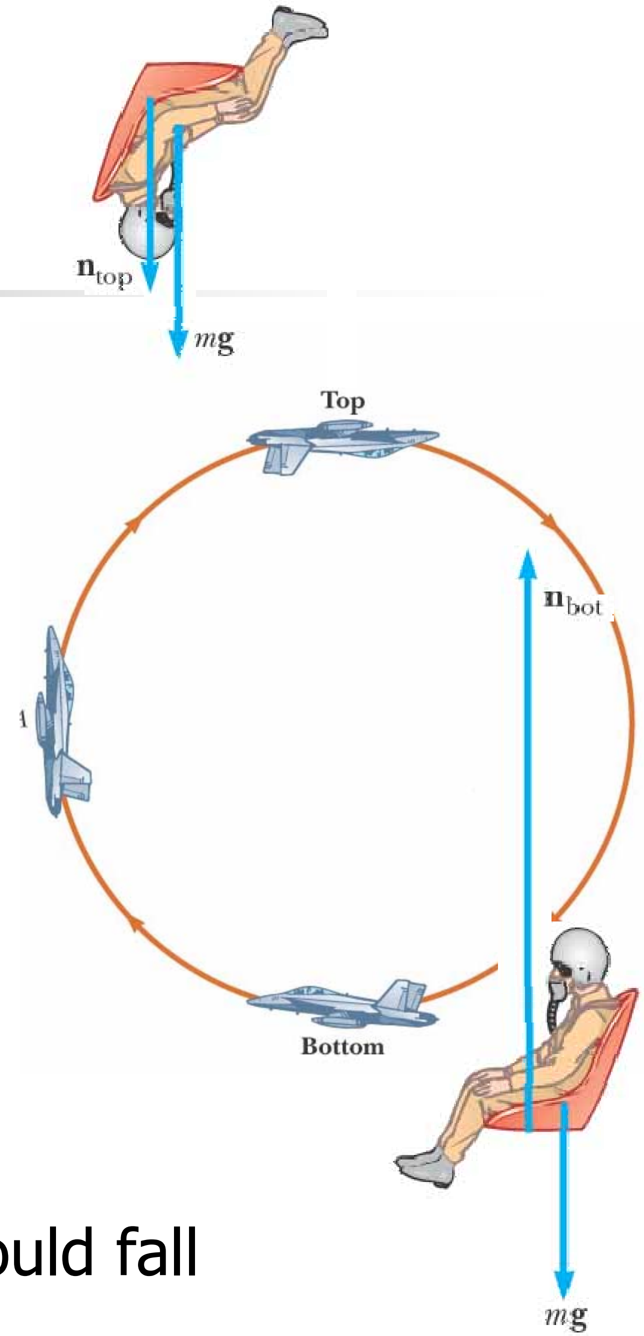
- At the bottom of the loop, the upward force experienced by the pilot is greater than his weight

$$n_{bot} - mg = m\frac{v^2}{r}, \;\; n_{bot} = mg\left(1 + \frac{v^2}{rg}\right)$$

- At the top of the circle, the force exerted on the pilot is less

$$-n_{top} - mg = -m\frac{v^2}{r}, \;\; n_{top} = mg\left(\frac{v^2}{rg} - 1\right)$$

  - n_{top} must be positive lest the pilot should fall



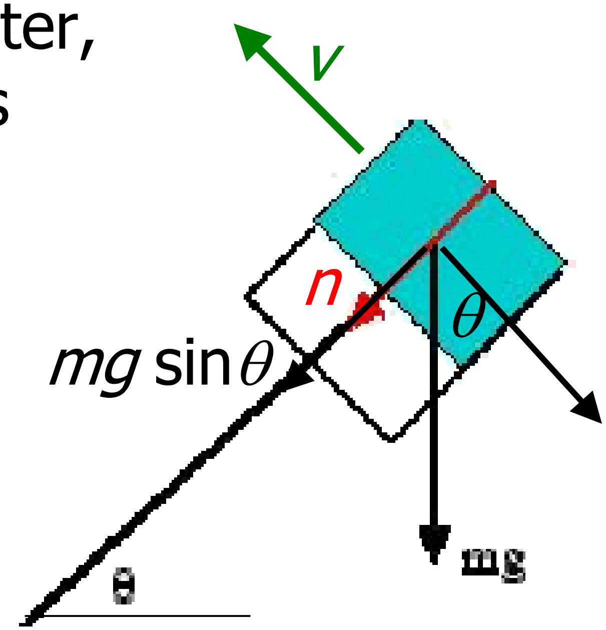
Example 5: Water Pail

A pail of water is rotated in a vertical circle of radius 1.00 m. What is the minimum speed of the pail at the top of the circle if no water is to spill out?

Write down Newton's 2nd Law for the water, assuming the water, like the bucket, goes around in circular motion.

$$\sum F_r = -n - mg\sin\theta = ma_r = -m\frac{v^2}{r}$$

$$\Rightarrow \; n = m\frac{v^2}{r} - mg\sin\theta = mg\left(\frac{v^2}{gr} - \sin\theta\right)$$



Example 5, cont

Suppose $v \le (gr)^{1/2} = \left(9.80\ \text{m/s}^2 \times 1.00\ \text{m}\right)^{1/2} = 3.13\ \text{m/s}.$

Then, at some angle θ, $n = mg\left(\dfrac{v^2}{gr} - \sin\theta\right) \le 0.$

For example, if $v = 2.63\ \text{m/s}$, $n = 0$ for $\theta = 45°$.

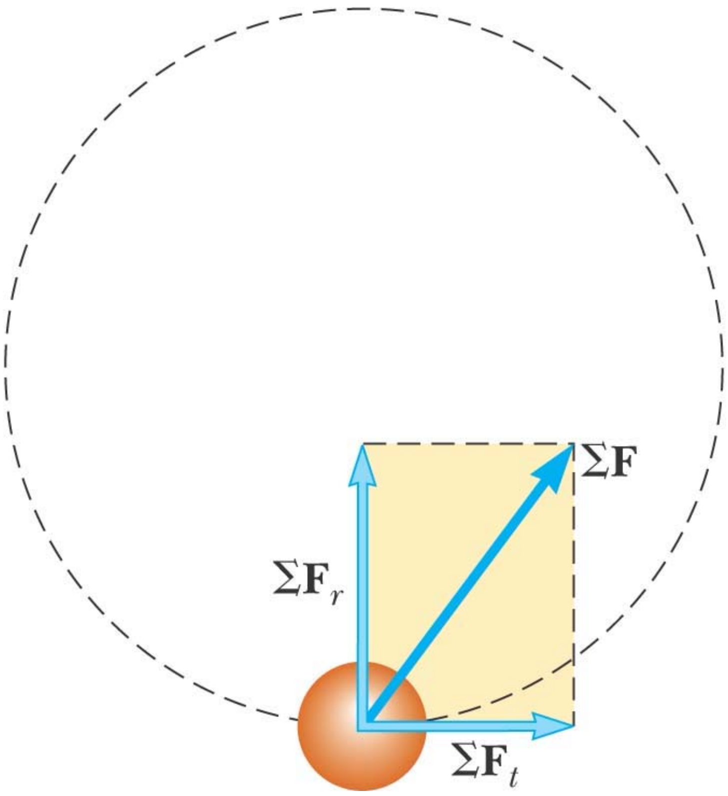
Once $n = 0$ is reached, the bucket no longer pushes on the water to make it go in a circle with the bucket. At this point, the radial acceleration of the water becomes $a_r = -g\sin\theta$ and the water falls away from the bucket.

In order for the water to stay in the bucket at $\theta = 90°$,

$$v \ge (rg)^{1/2}$$

Non-uniform Circular Motion

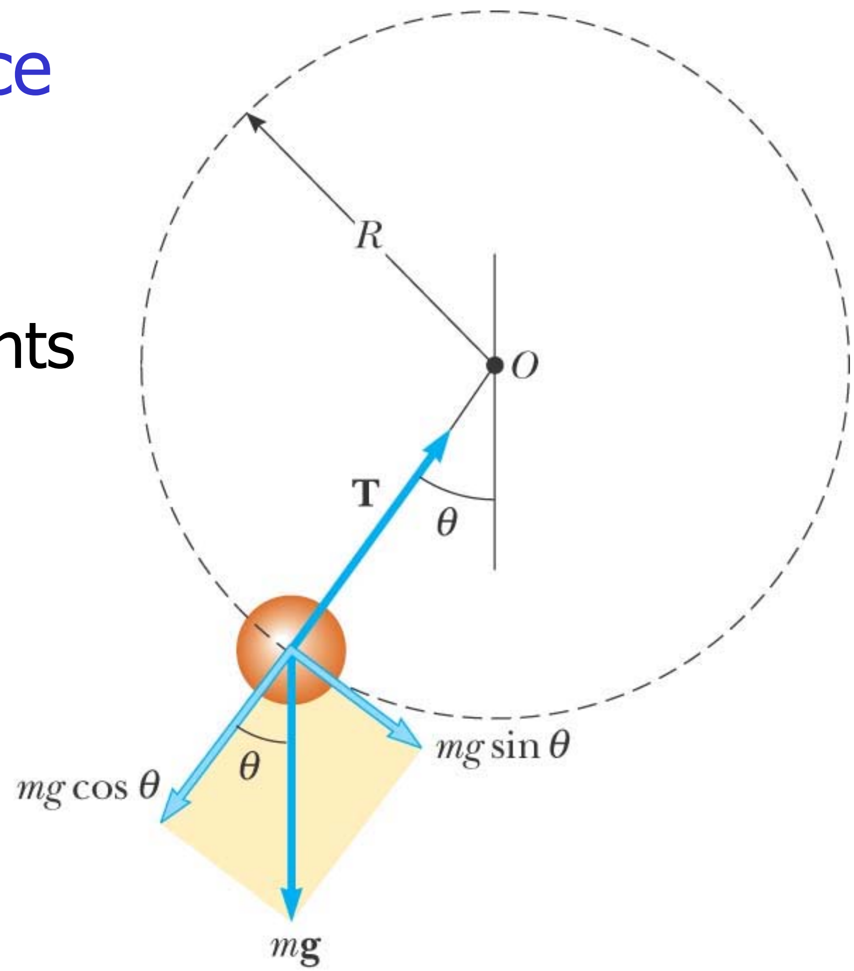
- The acceleration and force have *tangential* components
- $\mathbf{F}_r$ produces the centripetal acceleration
- $\mathbf{F}_t$ produces the tangential acceleration
- $\Sigma\mathbf{F} = \Sigma\mathbf{F}_r + \Sigma\mathbf{F}_t$



Vertical Circle Under Gravity

- The gravitational force exerts a *tangential* force on the object
  - Look at the components of F_g
  - Example: pendulum
- The tension at any point can be found:

$$T = m\left(\frac{v^2}{R} + g\cos\theta\right)$$



Example 6: Pendulum

A 2.00-kg ball is tied to a string which is 0.500 m long. The ball swings in a vertical circle under the influence of gravity. When the ball makes an angle of 20.0° with the vertical, its speed is 1.50 m/s.

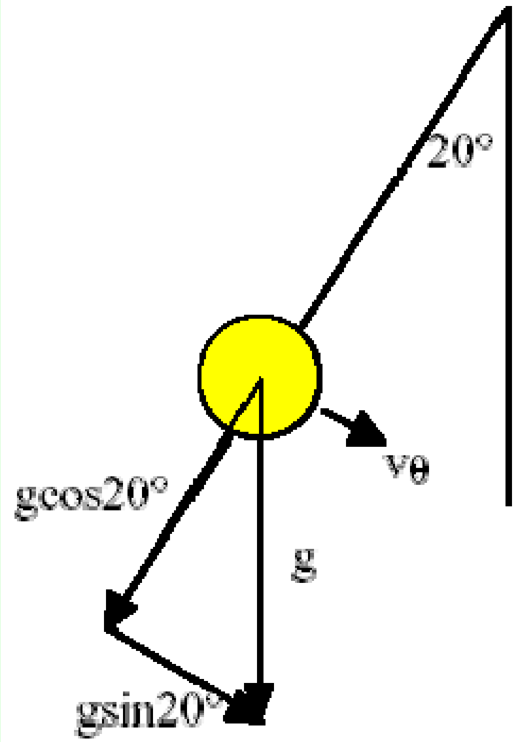
a) What is the tangential acceleration, a_θ?

b) What is the radial acceleration, a_r?

c) What is the total acceleration?

d) What is the tension in the string?

e) What is a_θ when the ball passes through the vertical?

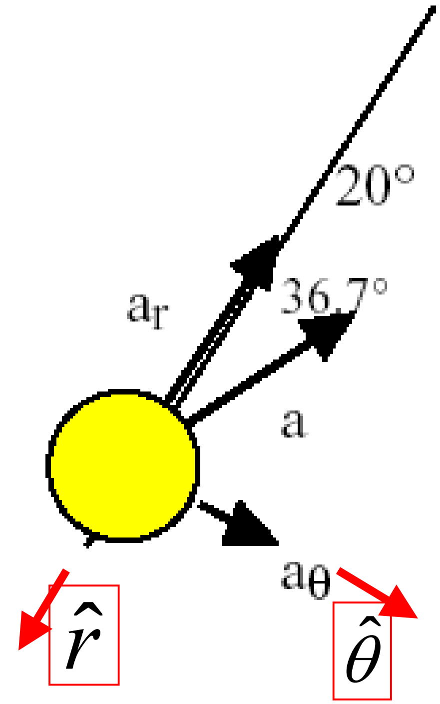


Example 6, cont

(a) Tangential acceleration

$$\sum \vec{F}_\theta = mg\sin\theta\hat{\theta} = m\vec{a}_\theta,\ \ \vec{a}_\theta = +g\sin\theta\hat{\theta}$$

$$\left|\vec{a}_\theta\right| = 9.80\ \mathrm{m/s^2}\sin 20.0° = +3.35\ \mathrm{m/s^2}$$



(b) Radial acceleration

$$\vec{a}_r = -\frac{v_\theta^2}{r}\hat{r} = -\frac{(1.50\ \mathrm{m/s})^2}{0.500\ \mathrm{m}}\hat{r} = \left(-4.50\ \mathrm{m/s^2}\right)\hat{r}$$

(c) Total acceleration

$$a_{total} = \sqrt{a_r^2 + a_\theta^2} = \sqrt{3.35^2 + 4.50^2} = 5.61\ \mathrm{m/s^2}$$

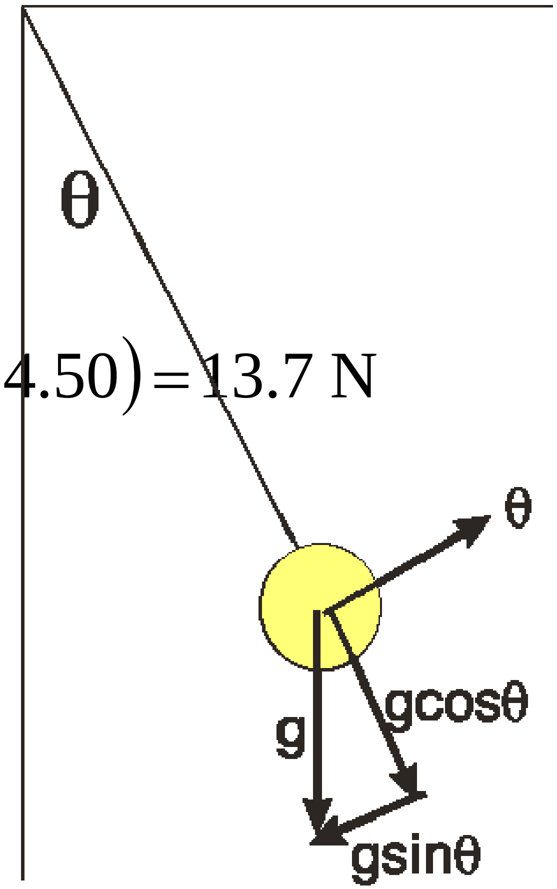
$$\phi = \tan^{-1}\left(a_\theta / a_r\right) = \tan^{-1}\left(3.35/4.50\right) = 36.7°$$

Example 6, cont

(d) Tension in the string

$$\sum F_r = ma_r, \; -T + mg\cos\theta = -m\frac{v^2}{r}$$

$$T = m\left(g\cos\theta + \frac{v^2}{r}\right) = 2.00\,(9.80\cos 20.0° + 4.50) = 13.7\ \mathrm{N}$$



(e) Angular acceleration at $\theta = 0$

$$\sum \vec{F}_\theta = -mg\sin\theta\hat{\theta} = m\vec{a}_\theta, \quad \vec{a}_\theta = -g\sin\theta\hat{\theta}$$

$$\Rightarrow \; \text{At } \theta = 0°, \; a_\theta = 0\ \mathrm{m/s^2}$$

Fictitious Forces

- A *fictitious force* results from an accelerated frame of reference
  - A force appears to act on an object, but you cannot identify the agent for this force
  - Although fictitious forces are not real forces, they have real effects
- Examples:
  - Centrifugal force: Objects in the car turning a corner do slide and you feel pushed to the outside
  - The Coriolis force is responsible for the rotation of weather systems and ocean currents

Linear Accelerating System

- Inertial observer sees

$$\sum F_x = T\sin\theta = ma$$ ← acceleration

$$\sum F_y = T\cos\theta - mg = 0$$

$$\tan\theta = a/g$$

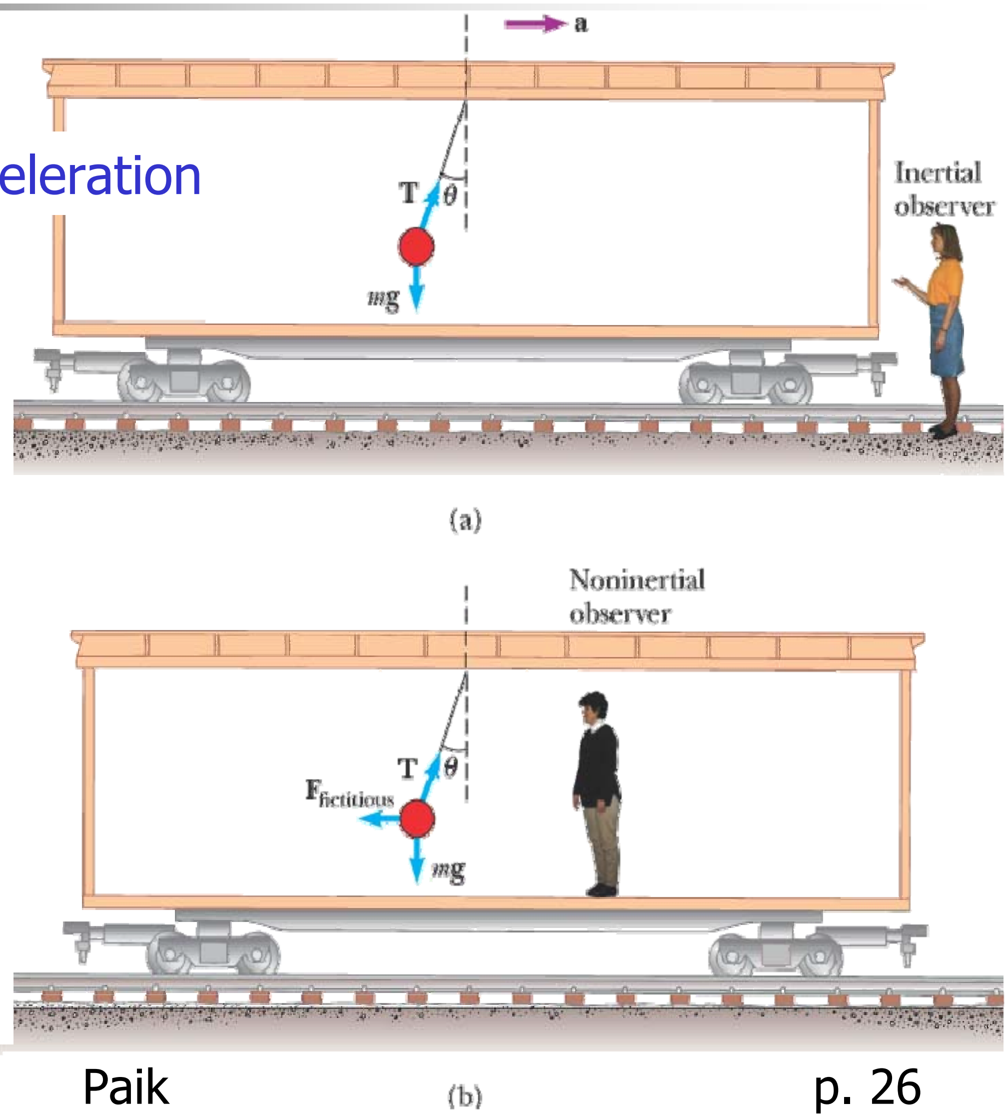
- Non-inertial observer sees

fictitious force → $-ma$

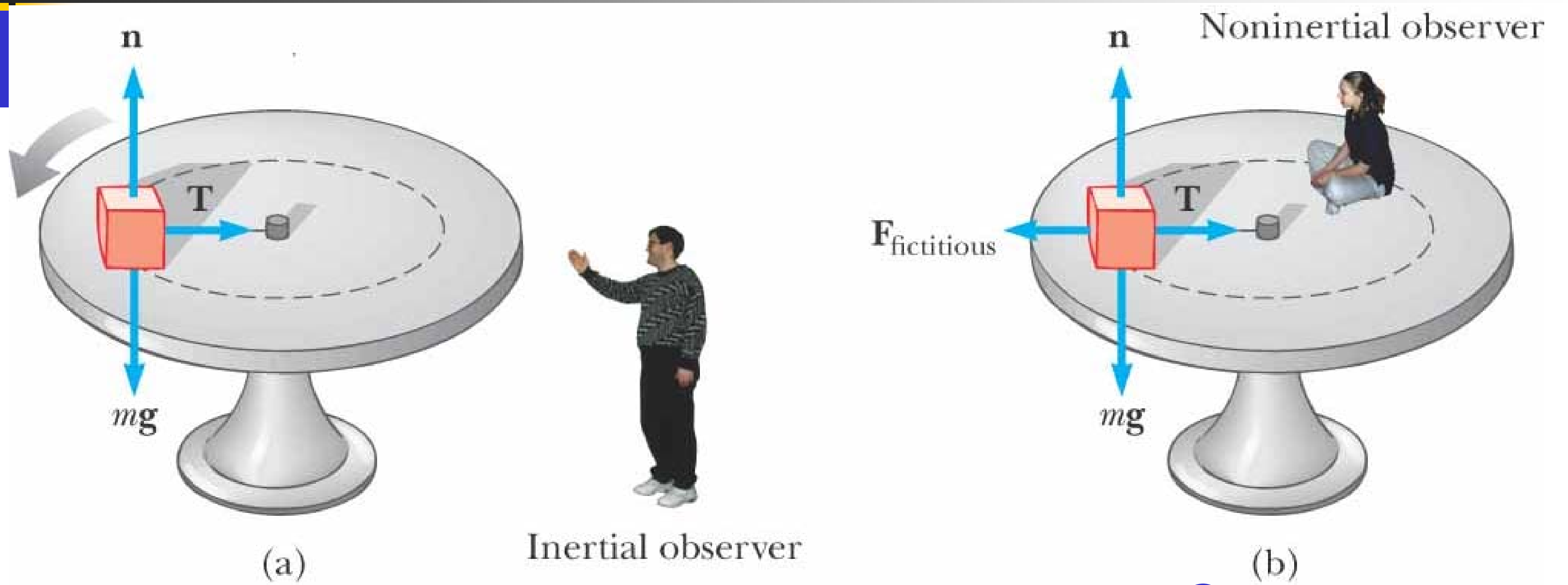
$$\sum F'_x = T\sin\theta - ma = 0$$

$$\sum F'_y = T\cos\theta - mg = 0$$

$$\tan\theta = ma/mg$$



Rotating System

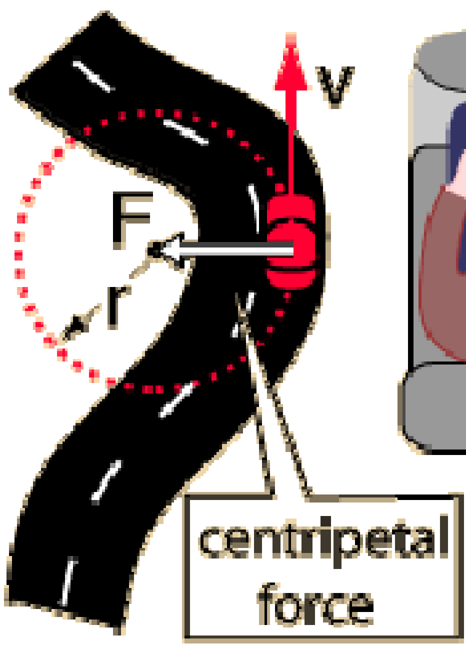


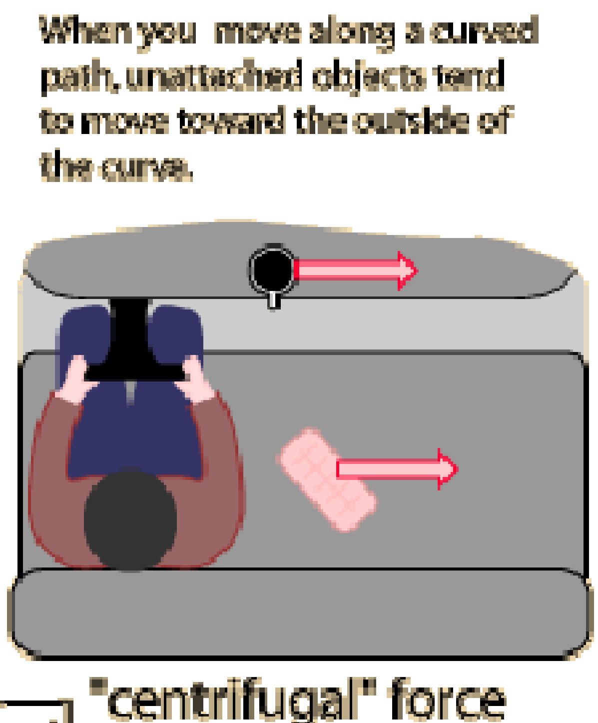
- Inertial observer sees $\sum F_r = -T = -m\dfrac{v^2}{r}$ ← centripetal acceleration
- Non-inertial observer sees $\sum F_r' = -T + m\dfrac{v^2}{r} = 0$

centrifugal force → $m\dfrac{v^2}{r}$

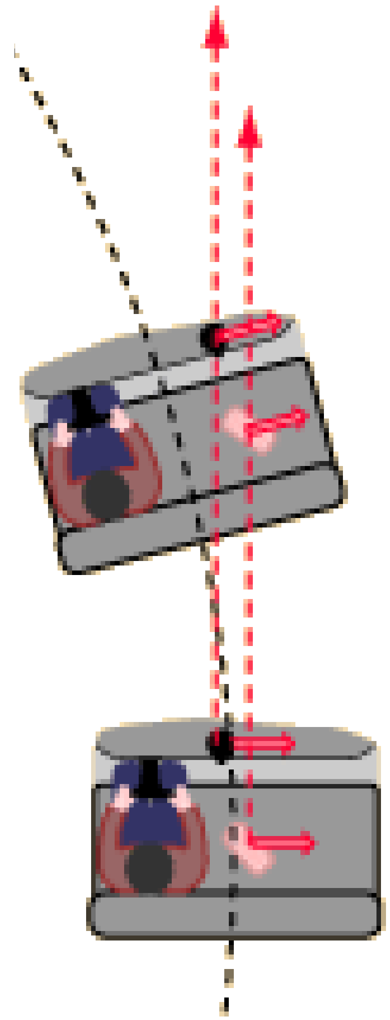
"Centrifugal Force"

As seen from the car





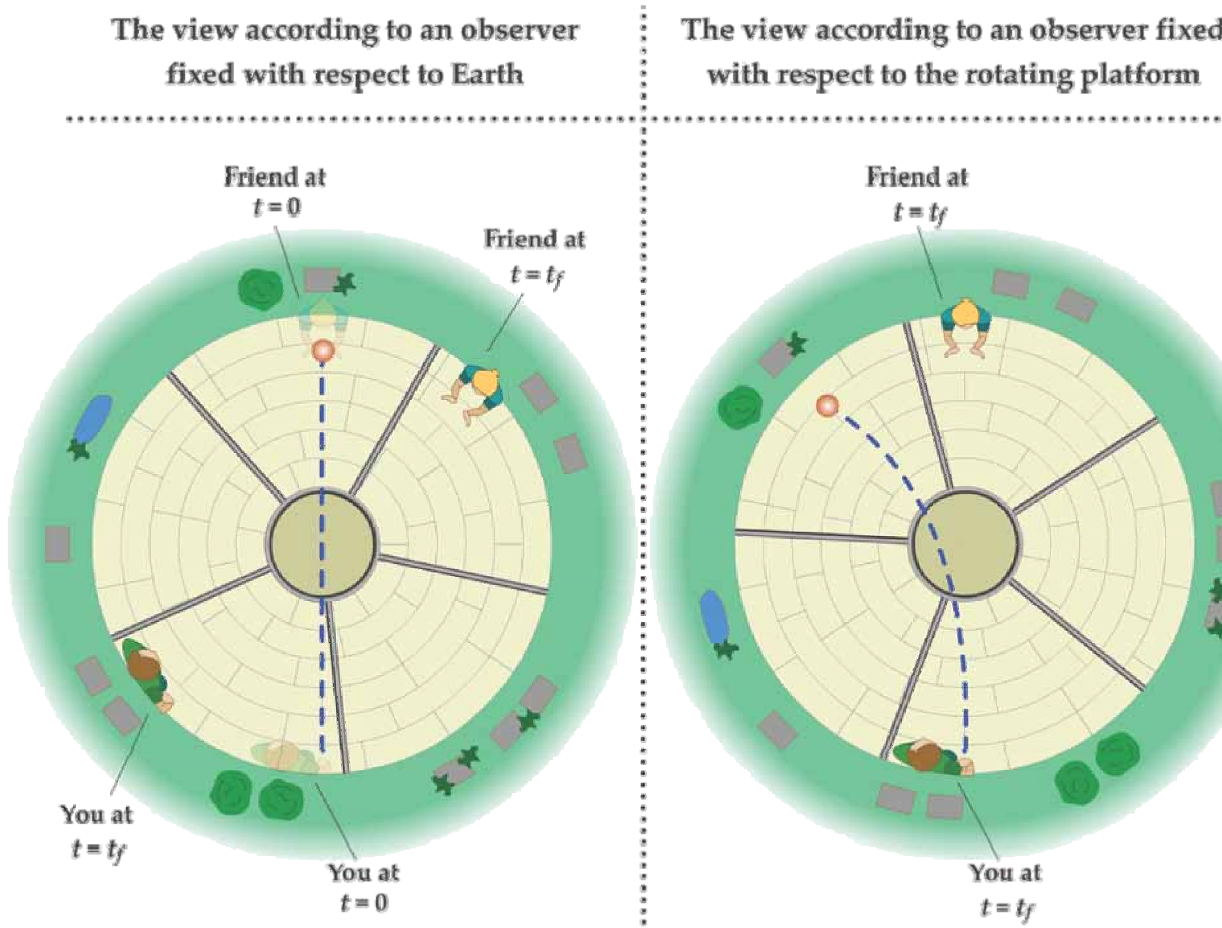
As seen from a helicopter



A person in a hovering helicopter above the car could describe the movement of the objects as just going straight while the car travels in a curved path

"Coriolis Force"

- An apparent force caused by changing the radial position of an object in a *rotating frame*
- The result is the *curved path* of the object



Air and Ocean Currents