Physics for Scientists and Engineers

Chapter 6

Dynamics I: Motion Along a Line

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Applications of Newton's Law

- Objects can be modeled as particles
- Masses of strings or ropes are negligible
  - When a rope attached to an object is pulling it, the magnitude of that force, **T**, is the *tension* in the rope
- Interested only in the *external forces* acting on the object
  - Can neglect internal reaction forces

Objects in Equilibrium

- If the acceleration of an object is zero, the object is said to be *in equilibrium*
- Mathematically, the net force acting on the object is zero
- Apply Newton's first Law in component form:

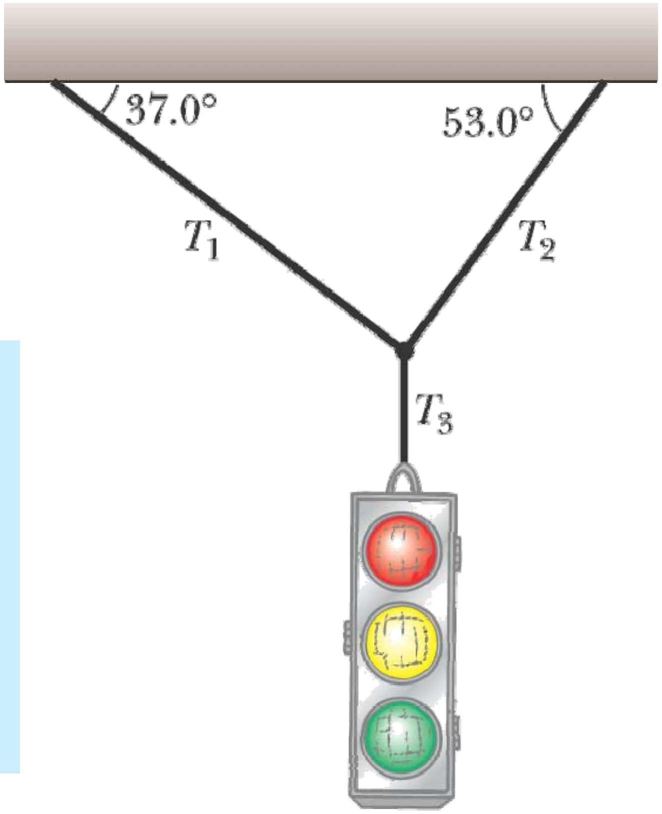
$$\sum \vec{F} = 0$$

$$\sum F_x = 0, \quad \sum F_y = 0$$

Static Equilibrium, Example 1

A traffic light is suspended with three ropes as shown in the figure.

Find the tension in the three ropes.

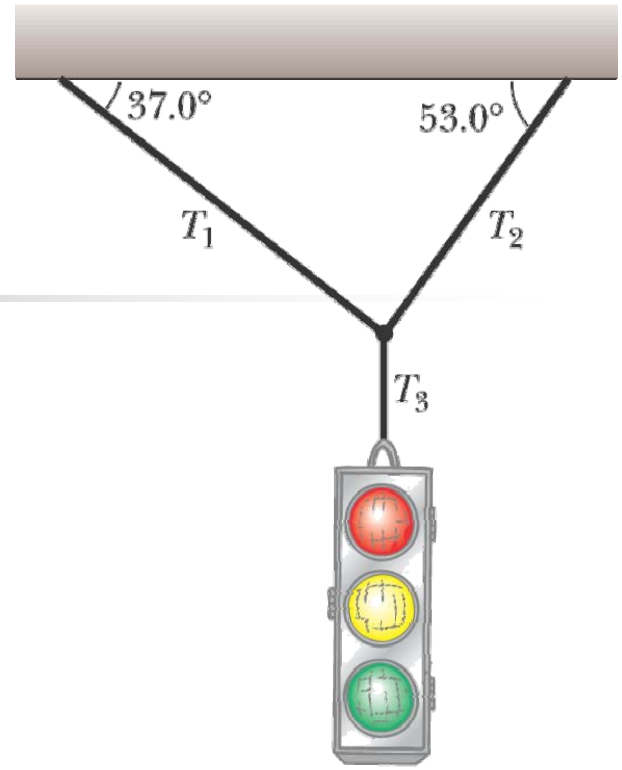


The traffic light does not move, so the net force must be zero.

$\Rightarrow$ Static equilibrium

Need two free-body diagrams: one for the light and the other for the knot.

Example 1, cont



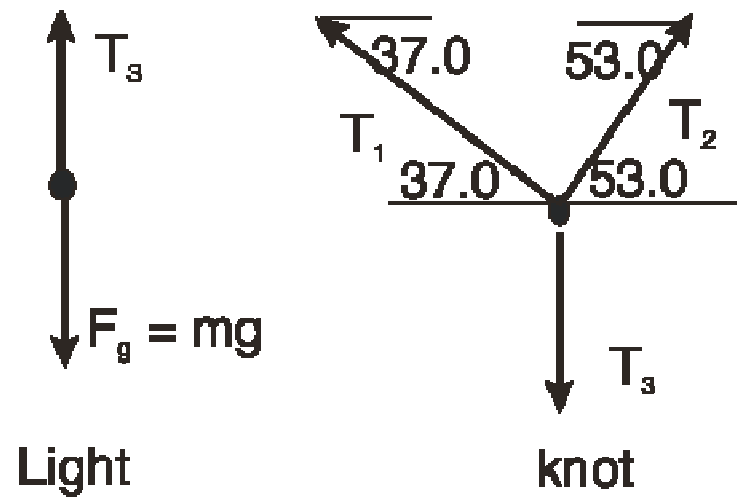
Light:

$$\Sigma F_y = 0: \quad T_3 - mg = 0$$
$$\Rightarrow \quad T_3 = mg$$

Knot :

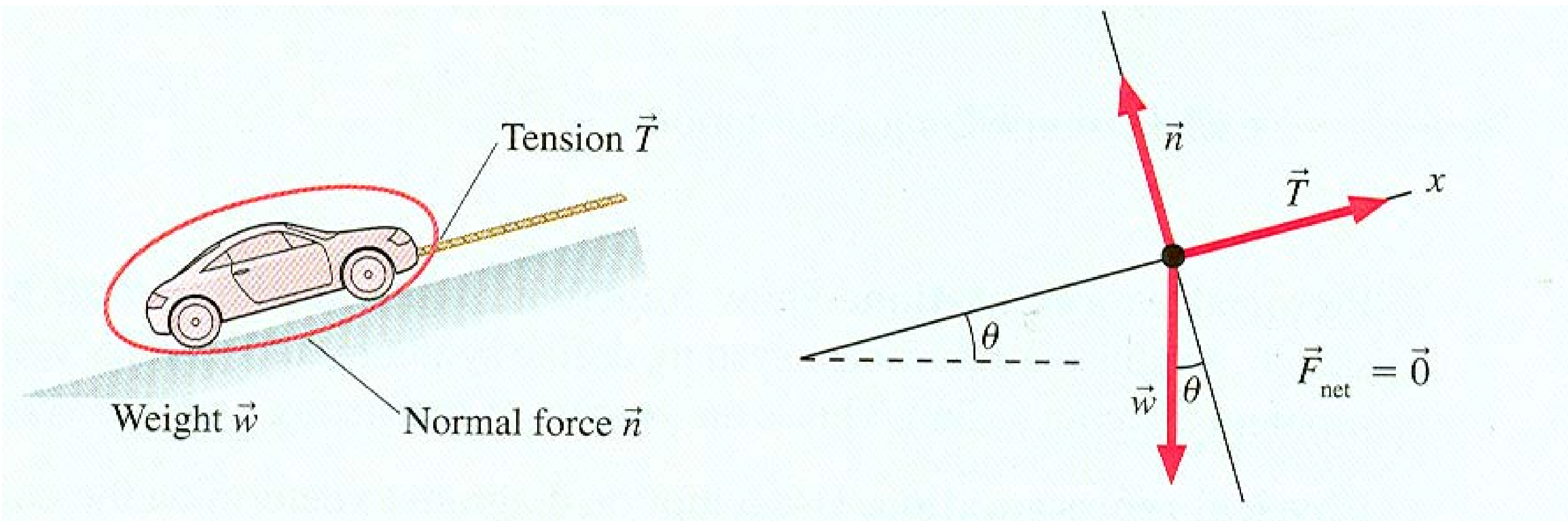
$$\Sigma F_x = 0: \quad -T_1 \cos 37^\circ + T_2 \cos 53^\circ = 0$$
$$\Rightarrow \quad T_1 = 0.754 \; T_2$$

$$\Sigma F_y = 0: \quad T_1 \sin 37^\circ + T_2 \sin 53^\circ - T_3 = 0$$
$$\Rightarrow \quad T_2 = 0.798 \; mg$$
$$\Rightarrow \quad T_1 = 0.602 \; mg$$



Dynamic Equilibrium, Example 2

A car with a weight of 15,000 N is being towed up a 20° slope at constant velocity. Friction is negligible. The tow rope is rated at 6000 N. Will it break?



Know: $\theta = 20°$, $w = 15{,}000$ N
Find: T

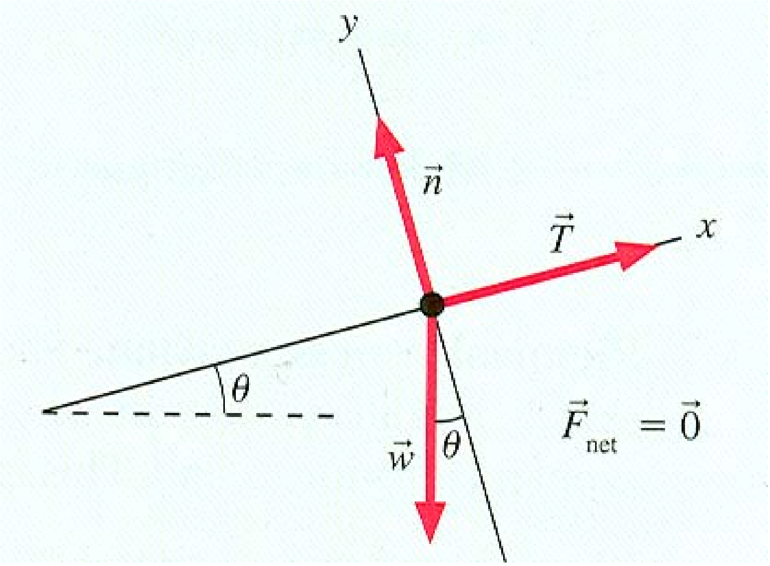
Example 2, cont

Apply Newton's 2nd Law (**a** = 0)

$$\sum \vec{F} = 0$$

$$\sum F_x = T - w\sin\theta = 0$$

$$\sum F_y = n - w\cos\theta = 0$$



From the x equation,

$$T = w\sin\theta = (15{,}000\ \text{N})\sin 20^0 = 5130\ \text{N}$$

Since T < 6,000 N, the rope won't break.

- Note that y equation is not needed.
- Check solution at θ = 0° and 90°.

Objects Experiencing a Net Force

- If an object experiences an acceleration, there must be a nonzero net force acting on it
- Draw a free-body diagram
- Apply Newton's Second Law in component form:

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x, \quad \sum F_y = ma_y$$

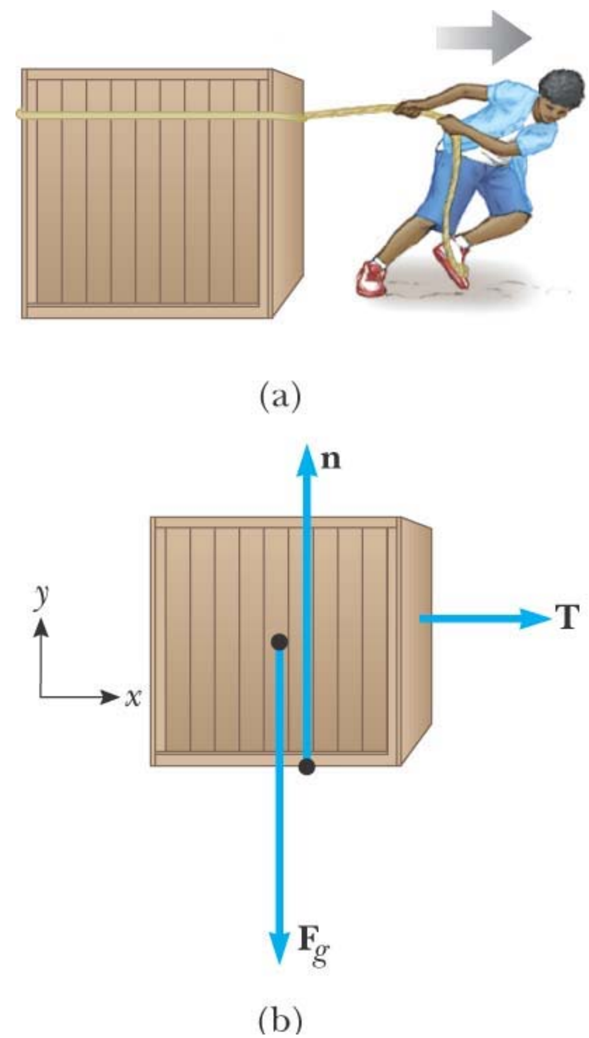
Crate Pulled on Floor, Example 3

A crate weighing 490 N is pulled with a rope on a *frictionless* floor. The tension in the rope is 20 N.

What is the resulting acceleration of the crate?

Forces acting on the crate:

- Tension: $T = 20$ N
- Gravitational force: $F_g = 490$ N
- Normal force **n** exerted by the floor



Example 3, cont

- Apply Newton's 2nd Law in component form:

$$\sum F_x = T = ma_x$$

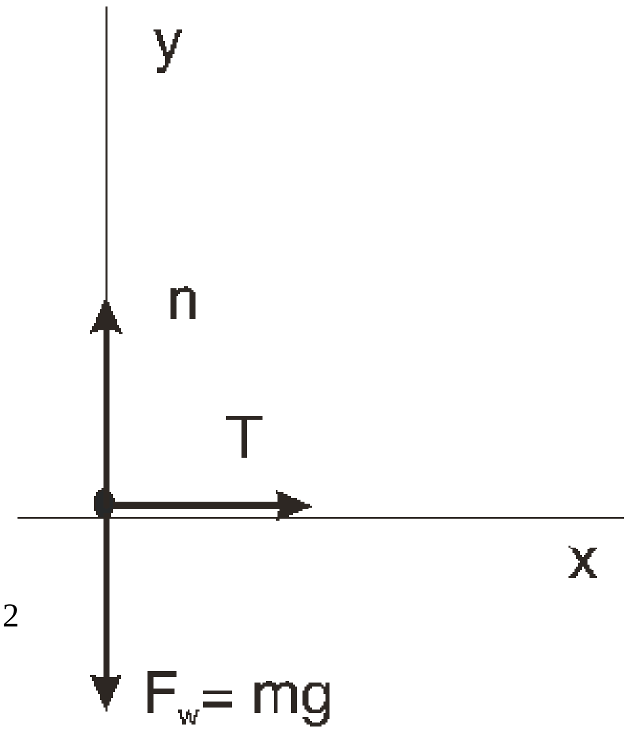
$$\sum F_y = n - F_g = 0 \rightarrow n = F_g$$

- Solve for the unknown a_x:

$$a_x = \frac{T}{m} = \frac{20\ \text{N}}{490\ \text{N}/9.80\ \text{m/s}^2} = 0.400\ \text{m/s}^2$$

- If a = constant and v_{x0} = 0, then

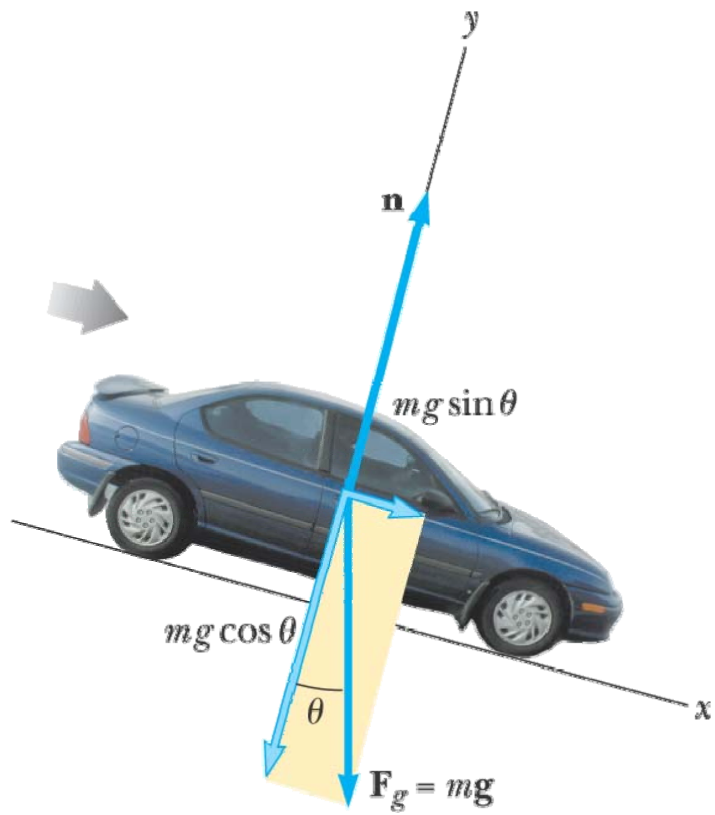
$$v_{xt} = a_x t = (0.400\ \text{m/s}^2)t$$



Car on Incline, Example 4

A car is at rest on top of a hill. Find the car's velocity at the bottom of the 30.0 m hill with an incline angle of 30.0°.

- Forces acting on the object:
  - Normal force **n** $\perp$ the plane
  - Gravitational force $\mathbf{F}_g$ straight down
- Choose the coordinate system with *x* // the incline, *y* $\perp$ the incline
- Replace the force of gravity with its components

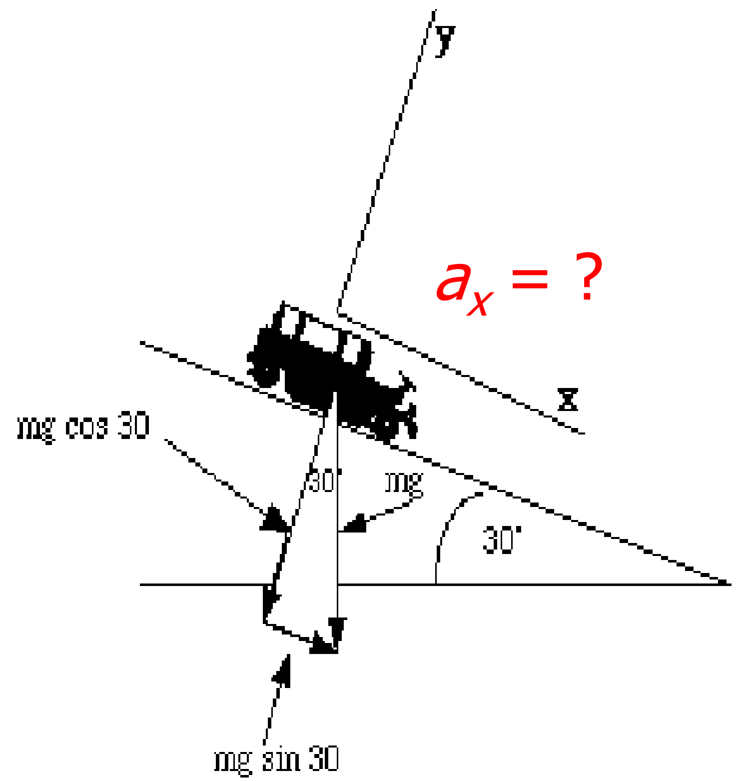


Example 4, cont

- Apply Newton's 2nd Law in component form:

$$\sum F_x = mg\sin 30^0 = ma_x$$

$$\sum F_y = n - mg\cos 30° = 0$$



- Solve for the unknown a_x:

$$a_x = g\sin 30^0 = 4.90\ \text{m/s}^2$$

- The velocity at the bottom of the slope is obtained from

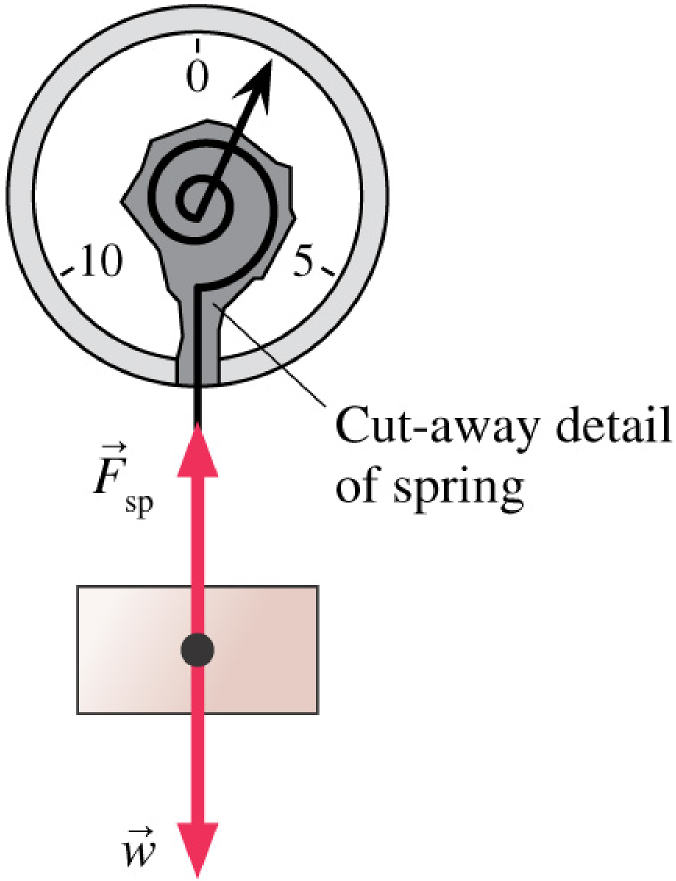
$$v_{xf}^2 = v_{xi}^2 + 2a_x \Delta x,\ v_{xf} = \sqrt{0 + 2(4.90\ \text{m/s}^2)30.0\ \text{m}} = 17.1\ \text{m/s}$$

Weight & Apparent Weight, 1

- Weight is the force exerted by the earth on mass *m*
  - $w = mg$, where $g = 9.80$ m/s^2
- Consider a scale and weight at rest
  - When the weight comes to rest with respect to the spring,

$$F_{sp} = w$$

  - The pointer is calibrated to read $F_{sp} = w$ at equilibrium ($a = 0$)



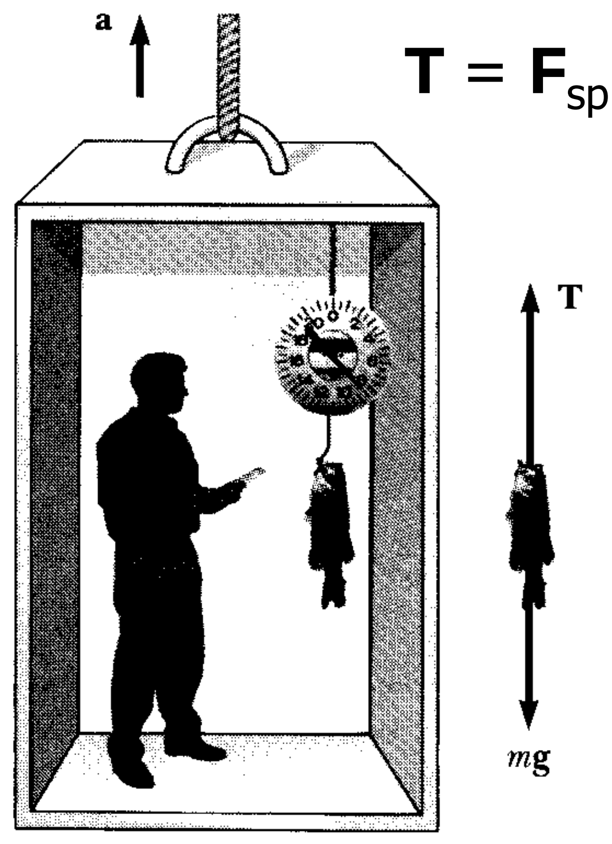
Weight & Apparent Weight, 2

- Consider an object hanging from a scale in an elevator moving up with acceleration *a*
- The *y* component of 2nd Law:

$$\sum F_y = F_{sp} - mg = ma$$

$$\therefore F_{sp} = m(g+a)$$

  - The spring dial comes to rest with the force $F_{sp} = m(g+a) > w$
- $m(g+a)$ is the apparent weight



Weight & Apparent Weight, 3

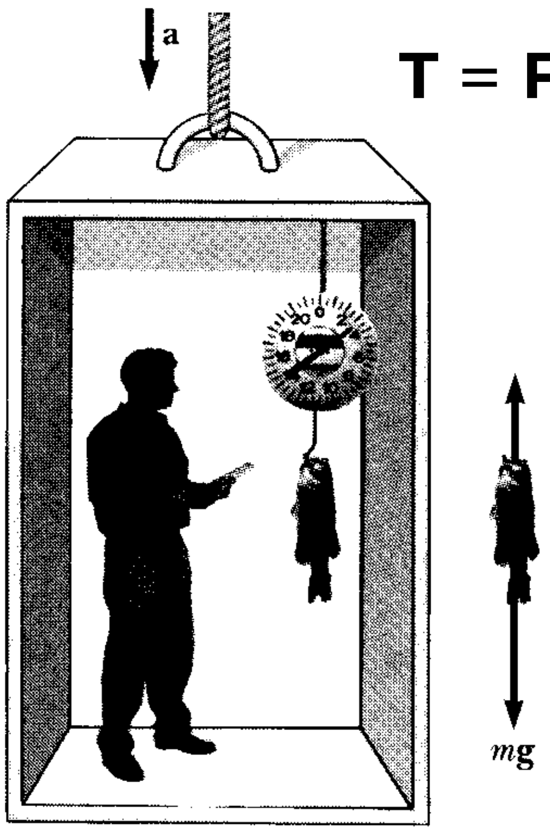
- Now consider the elevator accelerating downward
- The y component of 2nd Law:

$$\sum F_y = F_{sp} - mg = m(-a)$$

$$\therefore F_{sp} = m(g - a)$$

  - The scale reads $F_{sp} = m(g - a) < w$
- If $a = g$ (freefall), the apparent weight is 0 $\Rightarrow$ "weightless"

$\mathbf{T} = \mathbf{F}_{sp}$



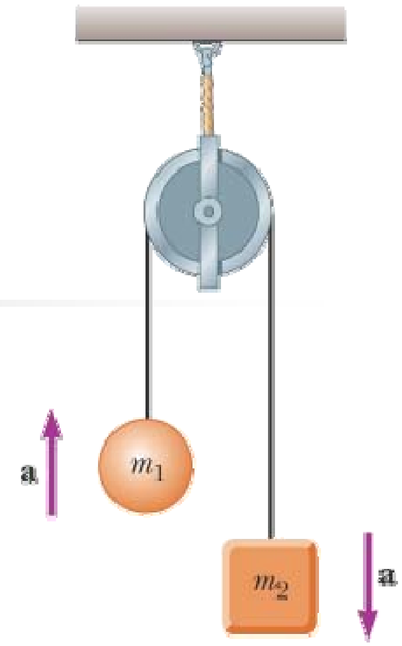
Multiple Weights/Objects

- Forces acting on the objects:
  - Tension (same along the same string)
  - Gravitational force
  - Normal force and friction
- Objects connected by a (non-stretch) string have the same magnitude of acceleration
- Draw the free-body diagrams
- Apply Newton's Laws

Example 5

An object of mass m_1 is connected to a light string that passes over a frictionless pulley and is fastened to another object of mass m_2.

Find the acceleration of the objects.

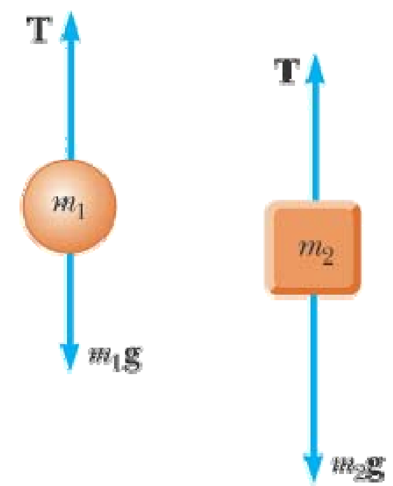


(a)

$$m_1 : \sum F_y = T - m_1 g = m_1 a \Rightarrow T = m_1(g+a)$$

$$m_2 : \sum F_y = T - m_2 g = m_2(-a) \Rightarrow T = m_2(g-a)$$

$$\Rightarrow m_1(g+a) = m_2(g-a) \Rightarrow a = \frac{m_2 - m_1}{m_2 + m_1} g$$



(b)

Check if the answer makes sense.

- $m_1 = m_2 \Rightarrow a = 0$
- $m_1 << m_2 \Rightarrow a \approx g$, $m_1 >> m_2 \Rightarrow a \approx -g$

Multiple Pulleys, Example 6

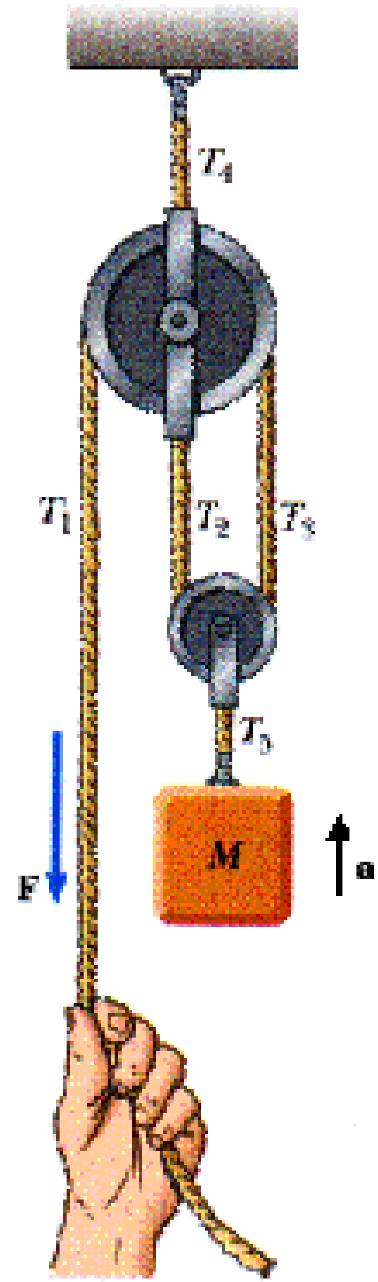
Assume the block is accelerated upward, i.e. $a > 0$. Let $F = 20.0$ N and $M = 5.00$ kg.

Find the acceleration and all the T's.

- All the pulleys and strings are considered massless and there is no friction in the pulleys

$$\Rightarrow F = T_1 = T_3 = T_2$$

- Note that top pulley doesn't move. The bottom pulley and mass M have the same acceleration.



Example 6, cont

$$M:\ \sum F_y = T_5 - Mg = Ma \ \Rightarrow\ T_5 = M(g+a)$$

$$P_{lower}: \sum F_y = T_2 + T_3 - T_5 = 0a$$

$$\Rightarrow\ T_2 + T_3 = T_5 = M(g+a) \ \Rightarrow\ T_2 = T_3 = \tfrac{1}{2}M(g+a)$$

$$P_{upper}: \sum F_y = T_4 - T_1 - T_2 - T_3 = 0$$

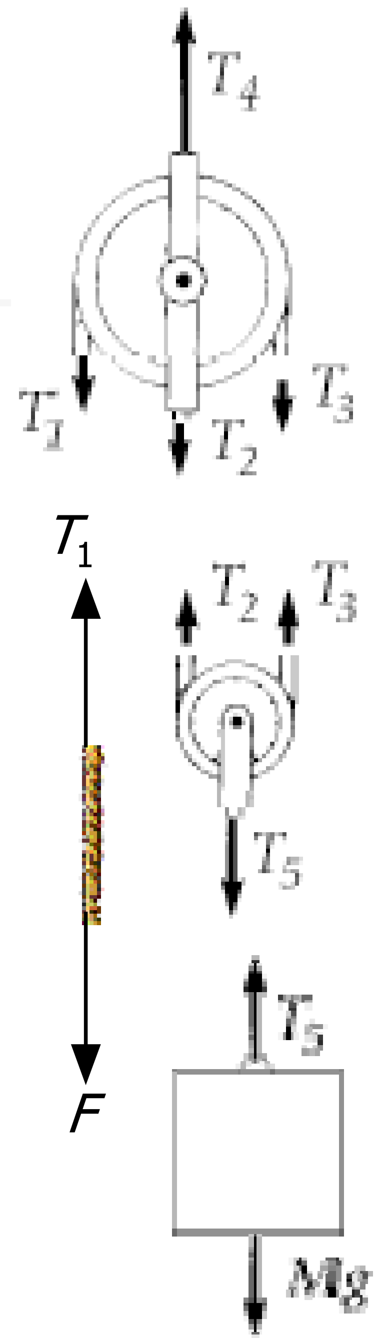
$$\Rightarrow\ T_1 = T_3 = \tfrac{1}{2}M(g+a),\ \ T_4 = \tfrac{3}{2}M(g+a)$$

$$S: \sum F_y = T_1 - F = 0 \ \Rightarrow\ F = T_1 = \tfrac{1}{2}M(g+a)$$

Therefore,

$$a = \frac{2F}{M} - g = \frac{2(20\text{ N})}{5\text{ kg}} - 9.8\text{ m/s}^2 = -1.8\text{ m/s}^2,$$ $a < 0!$

$$T_1 = T_2 = T_3 = F = 20\text{ N},\ T_4 = 3F = 60\text{ N},\ T_5 = 2F = 40\text{ N}$$



Example 7

A 5.00-kg object placed on a frictionless, horizontal table is connected to a cable that passes over a pulley and then is fastened to a hanging 9.00-kg object.

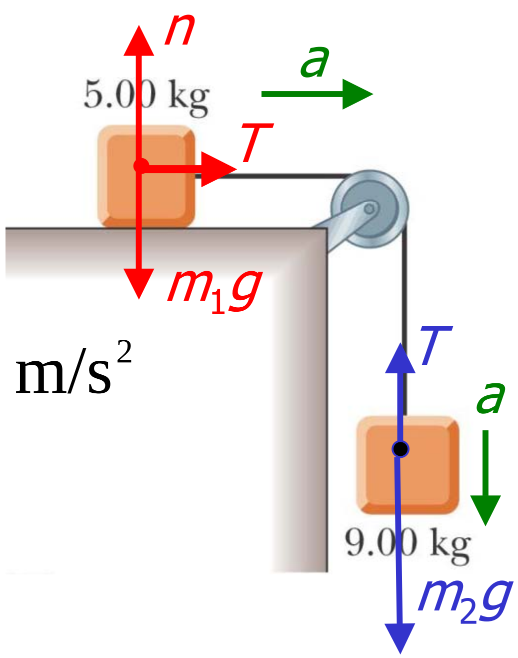
Find the acceleration of the objects and the tension in the string.

$$m_1 = 5\,\text{kg}: \sum F_x = T = m_1 a,\ \sum F_y = n - m_1 g = 0$$

$$m_2 = 9\,\text{kg}: \sum F_y = T - m_2 g = m_2(-a)$$

$$\Rightarrow a = \frac{m_2}{m_1 + m_2} g = \frac{9.00}{5.00 + 9.00} \times 9.80\,\text{m/s}^2 = 6.30\,\text{m/s}^2$$

$$T = 5.00\,\text{kg} \times 6.30\,\text{m/s}^2 = 31.5\,\text{N}$$

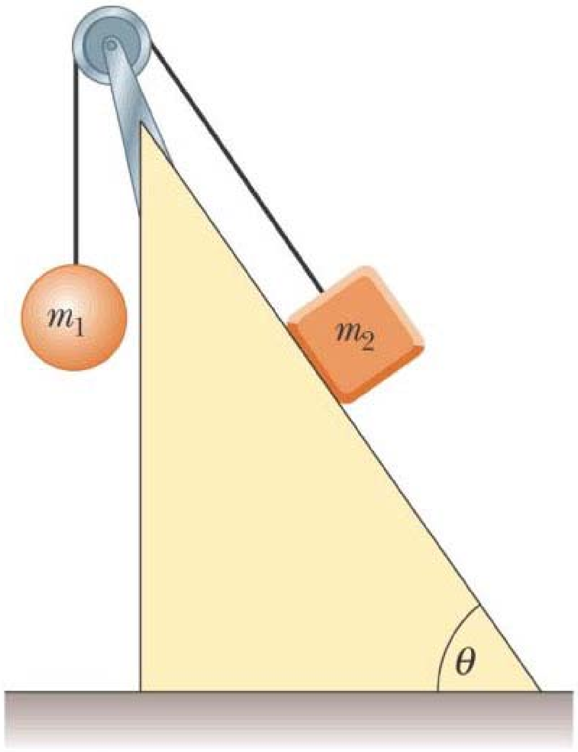


Ball and Cube, Example 8

Two objects are connected by a light string that passes over a frictionless pulley. The incline is frictionless, and $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, and $\theta = 55.0°$.

Draw free-body diagrams of both objects.

Find (a) the acceleration of the objects, (b) the tension in the string, and (c) the speed of each object 2.00 s after being released from rest.



Example 8, cont

(a) $m_1 = 2.00\ \text{kg}:$

$$\sum F_y = T - m_1 g = m_1 a$$

$m_2 = 6.00\ \text{kg}:$

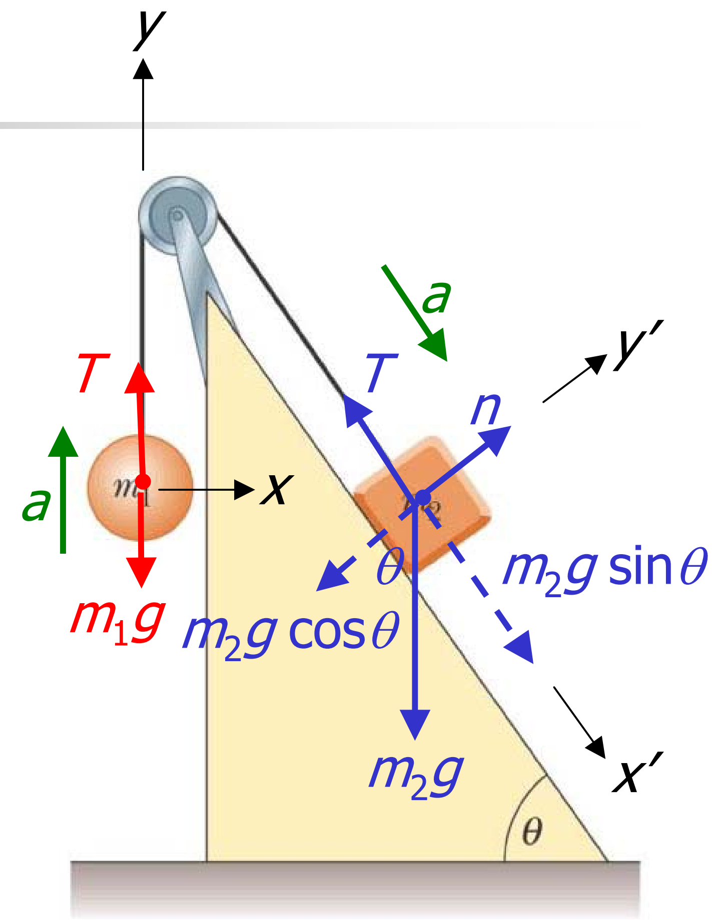
$$\sum F_x = m_2 g \sin\theta - T = m_2 a$$

$$\Rightarrow\ a = \frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2} = 3.57\ \text{m/s}^2$$

(b) $T = m_1(a+g) = 26.7\ \text{N}$

(c) $v_f = v_i + at = 0\ \text{m/s} + (3.57\ \text{m/s}^2)(2.00\ \text{s})$

$$= 7.14\ \text{m/s}$$

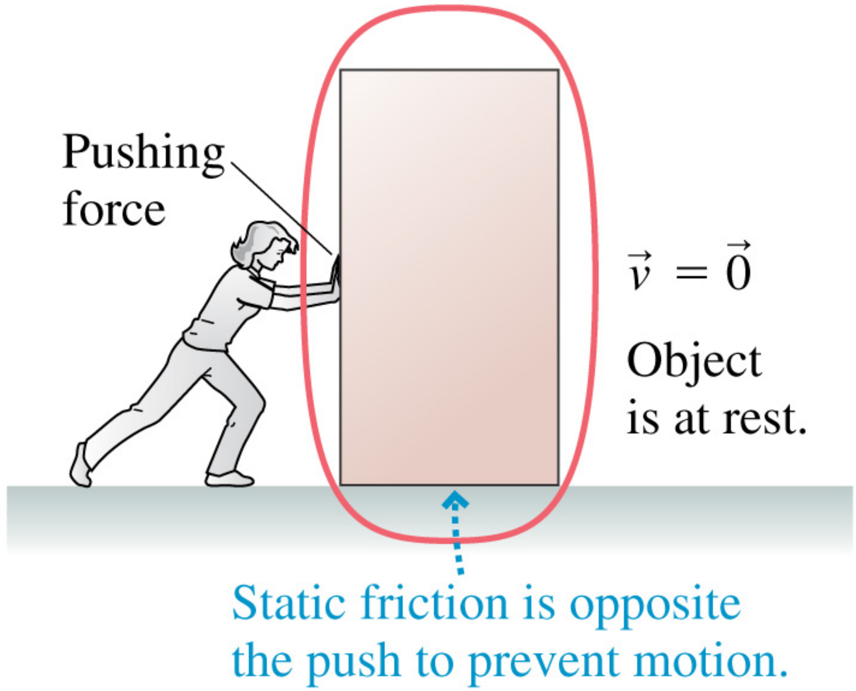


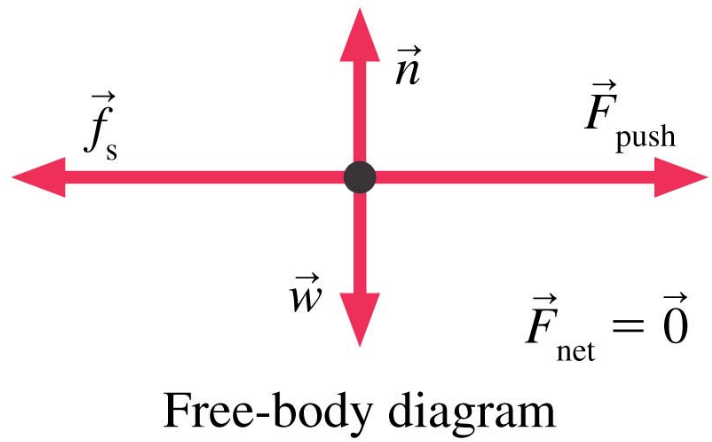
$\theta = 55.0°$

Forces of Friction

- When an object is in contact with a surface, there is a resistance to its motion, the *force of friction*
  - This is due to the interaction between the molecules on the mating surfaces
- Friction is proportional to the normal force
  - *Static* friction: $f_s \leq \mu_s n$ and *kinetic* friction: $f_k = \mu_k n$
  - These equations relate the magnitudes of the forces only
  - Generally, $\mu_s > \mu_k$
  - The coefficient of friction (μ) depends on the surfaces in contact but is nearly independent of the area of contact

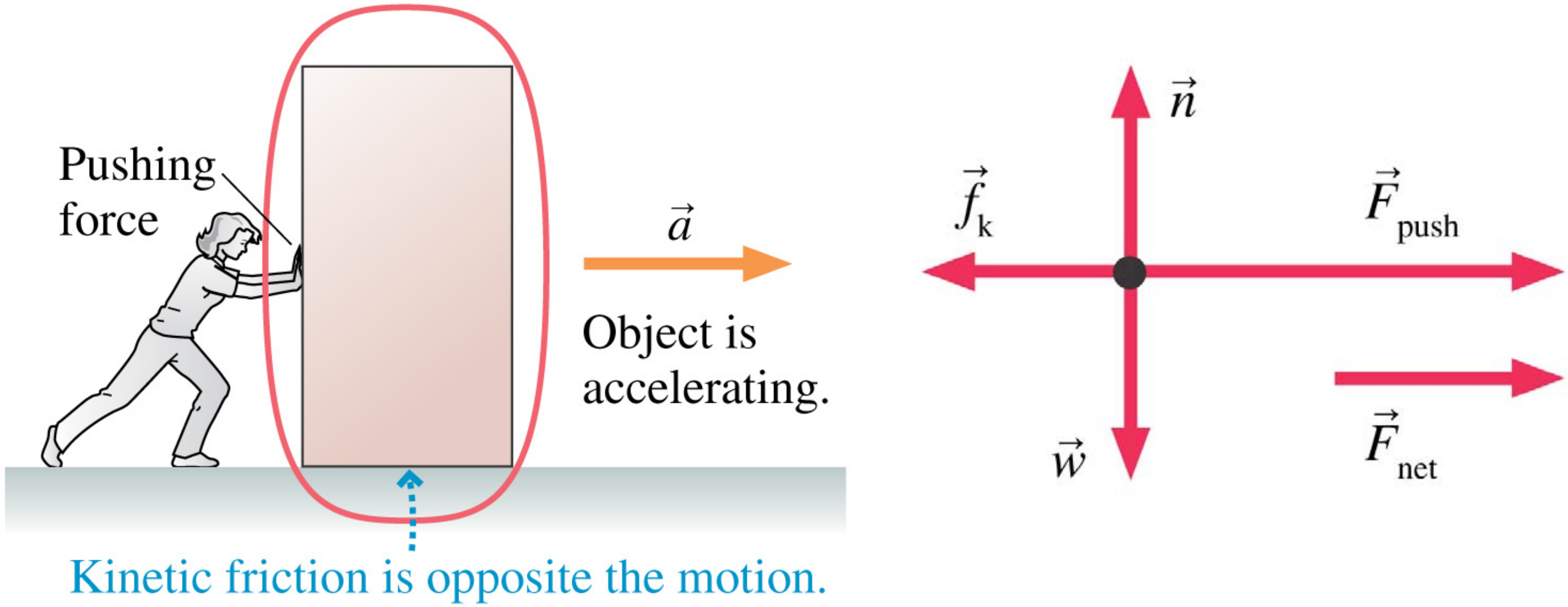
Static Friction





There is a maximum static frictional force, $f_{s,max} = \mu_s n$. If the applied force is greater, then the object slips.

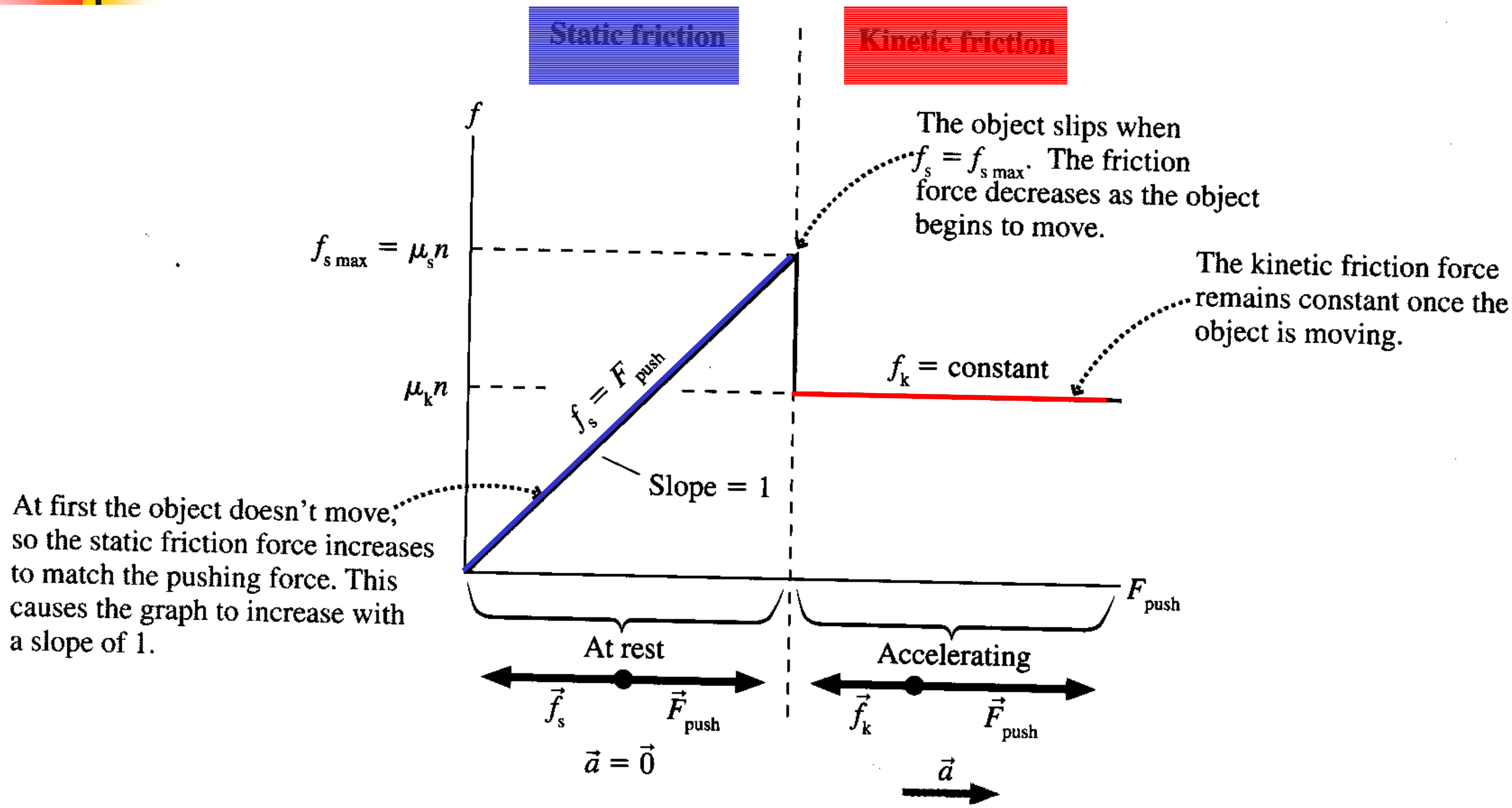
Kinetic Friction



The kinetic friction coefficient is less than the static friction coefficient, $\mu_k < \mu_s$.

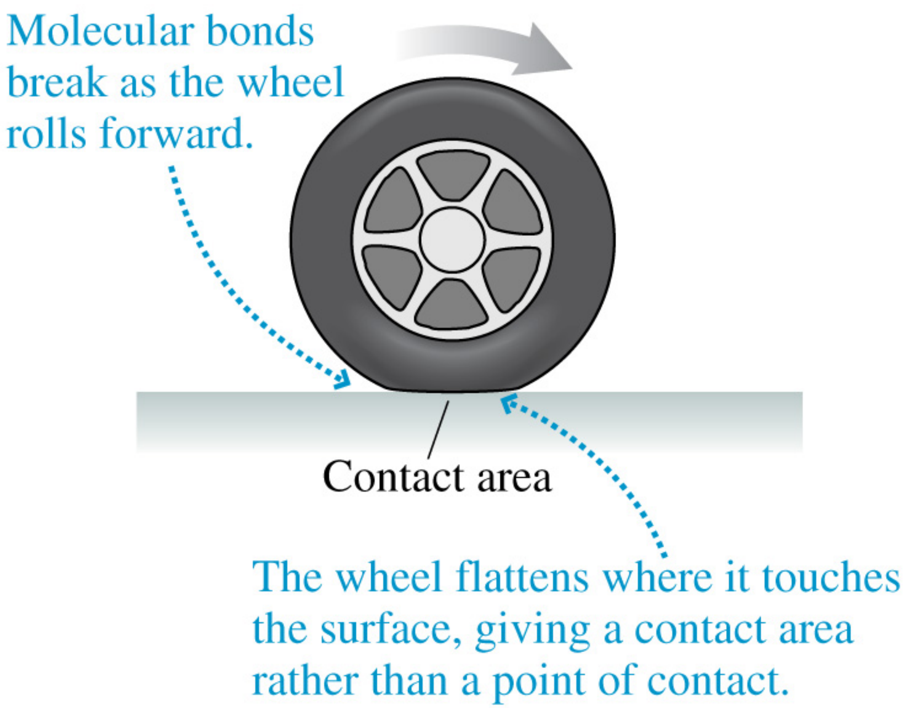
Therefore, $f_k = \mu_k n = \text{constant} < f_{s,max}$.

Friction Forces vs. Applied Force



Rolling Friction

- Kinetic friction is operable for a wheel sliding on a surface
- *Rolling* friction is for a non-sliding, rolling wheel
- $f_r = \mu_r n$ and points opposite to the motion
- $\mu_r < \mu_k$



Some Coefficients of Friction

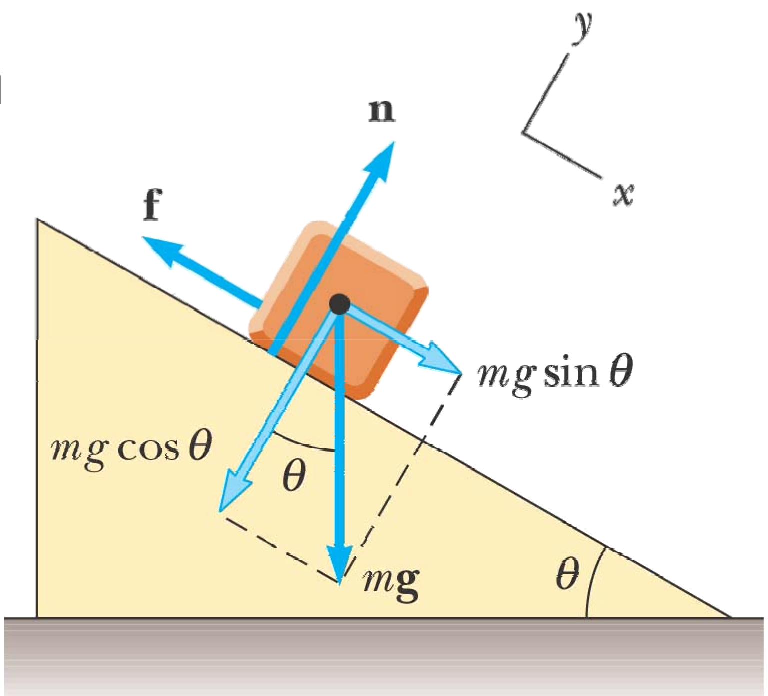
	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

Friction in Newton's Laws

- Draw the free-body diagram, including the force of kinetic friction
  - Opposes the motion
  - Is parallel to the surfaces in contact
- Friction is a force, so it is simply included in the $\Sigma \mathbf{F}$ in Newton's Laws
- Continue with the solution as with any Newton's Law problem

Static Friction - Measuring μ_s

- The block tends to slide down the plane, so the friction acts up the plane
- Increase θ up to the instant slipping starts
- Apply Newton's Law:



$$\sum F_x = mg\sin\theta - f_{s,\max} = 0,\ \sum F_y = n - mg\cos\theta = 0$$

$$\Rightarrow f_{s,\max} = mg\sin\theta = \mu_s n = \mu_s mg\cos\theta \Rightarrow \mu_s = \tan\theta$$

Sliding Down a Plane, Example 9

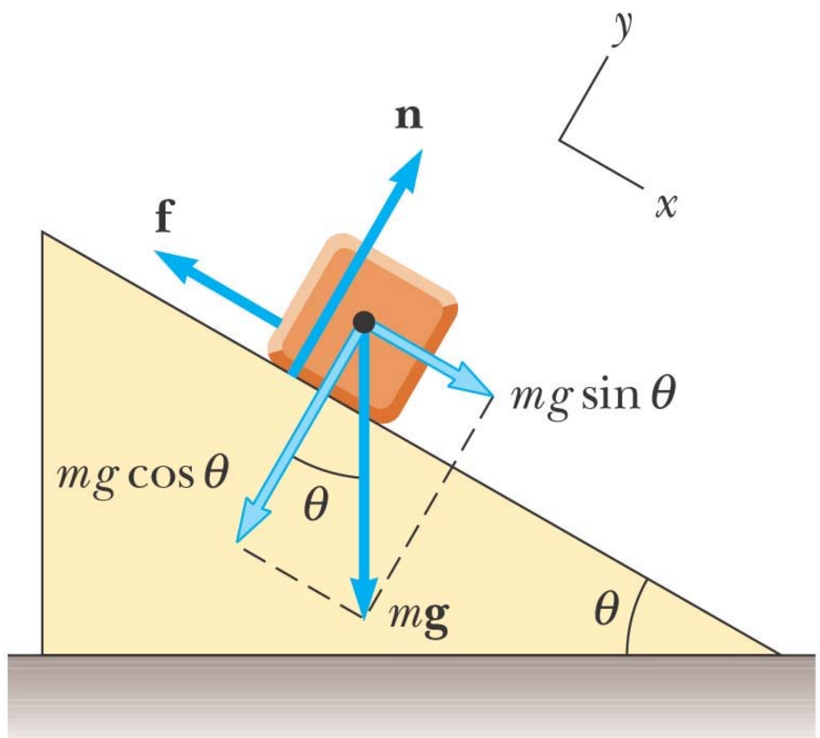
If μ_s = 0.30 and θ = 30°, will the cube slide?

Will not slide
if $f_s \geq mg \sin\theta = 0.5mg$

Since $f_s \leq \mu_s n$ and $n = mg \cos\theta$,

$$f_s \leq \mu_s mg \cos 30°$$
$$= 0.30 \times 0.866mg$$
$$= 0.26mg$$

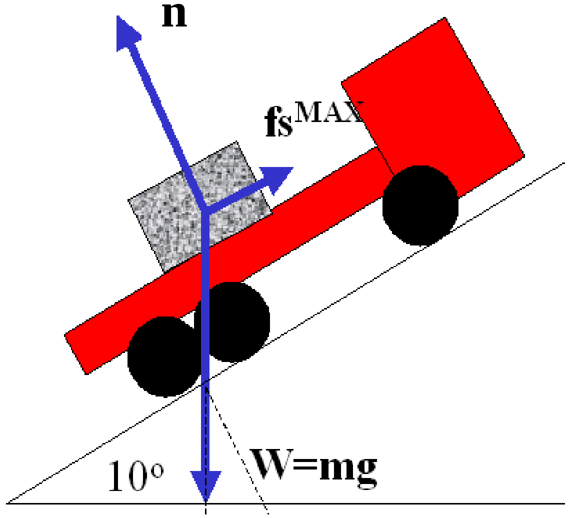
$\Rightarrow$ Will slide



Truck with a Crate, Example 10

A truck is carrying a crate up a 10° hill. The coefficient of static friction between the truck bed and the crate is $\mu_s = 0.35$.

Find the maximum acceleration that the truck can reach before the crate begins to slip backward.



- The crate will not slip if its acceleration is equal to that of the truck.
- For it to be accelerated, some force must act on it.
- One of these is the static friction force f_s. This force must be directed upward to keep the crate from sliding backwards.
- As the acceleration of the truck increases, so must f_s.
- Once $f_{s,\,max} = \mu_s n$ is reached, the crate will start sliding.

Example 10, cont

Applying Newton's law to the crate:

$$\Sigma F_x = -mg\sin\theta + \mu_s n = ma_{\max}$$

$$\Sigma F_y = -mg\cos\theta + n = 0$$

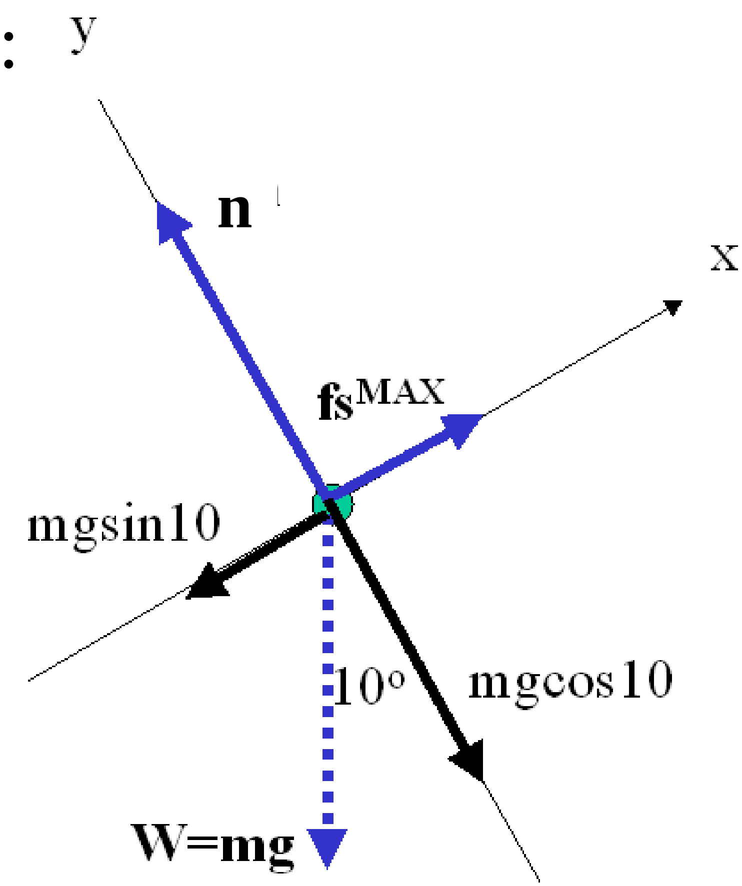
$$\Rightarrow \quad n = mg\cos\theta$$

$$-mg\sin\theta + \mu_s mg\cos\theta = ma_{\max}$$

$$\Rightarrow \quad a_{\max} = g(\mu_s\cos\theta - \sin\theta)$$

Substituting $\theta = 10°$ and $\mu_s = 0.35$,

$$a_{\max} = 1.68\ \mathrm{m/s^2}$$



Hockey Puck, Example 11

Consider a puck is hit and given a speed $v = 20.0$ m/s. How far will it go if $\mu_k = 0.1$?

1) $x: \; -f_k = ma$

   $y: \; n - mg = 0$

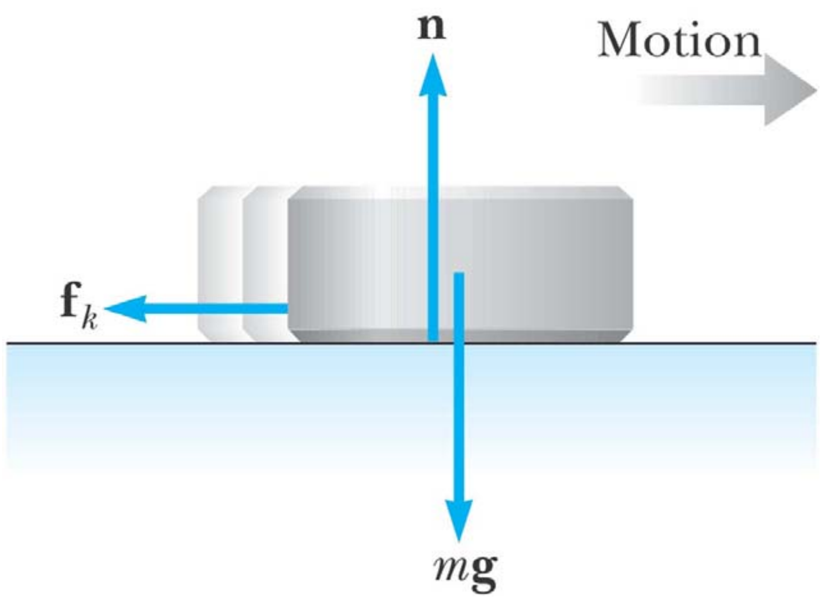
   $a = -f_k/m = -\mu_k n/m = -\mu_k g$

2) Use $v_f^2 = v_i^2 + 2a\Delta x$

   $v_i = 20.0$ m/s, $v_f = 0$, $\Delta x = ?$

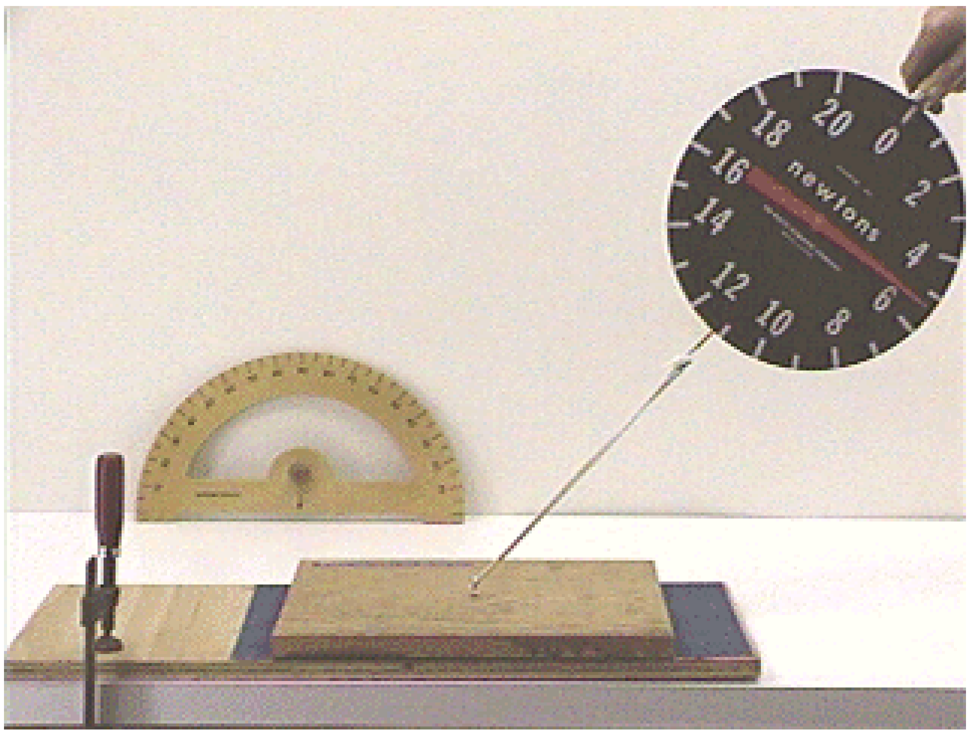
   $\Delta x = -v_i^2/2a$

   $= -400/(-2 \times 0.1 \times 9.80) = 204$ m



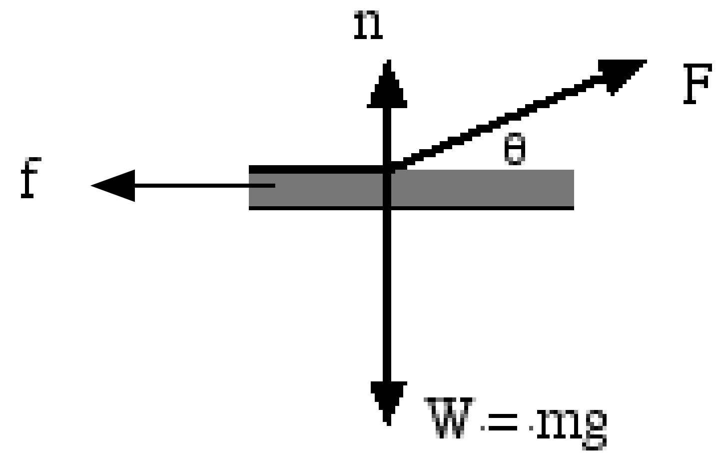
Pulling at an Angle, Example 12

What is the optimum angle for minimizing the pulling force that moves the block horizontally at constant velocity?



- Pulling straight up minimizes the normal force and hence the frictional force. However, it does not move the block horizontally.
- Pulling parallel to the table maximizes the normal force and hence the frictional force.
- The optimum angle must be somewhere in between.

Example 12, cont



$$x: -f + F\cos\theta = ma_x = 0$$

$$y: \quad n + F\sin\theta - mg = 0$$

$$n = mg - F\sin\theta$$

$$f = \mu_k(mg - F\sin\theta)$$

Substituting for f in the x equation: $F = \dfrac{\mu_k mg}{\mu_k \sin\theta + \cos\theta}$

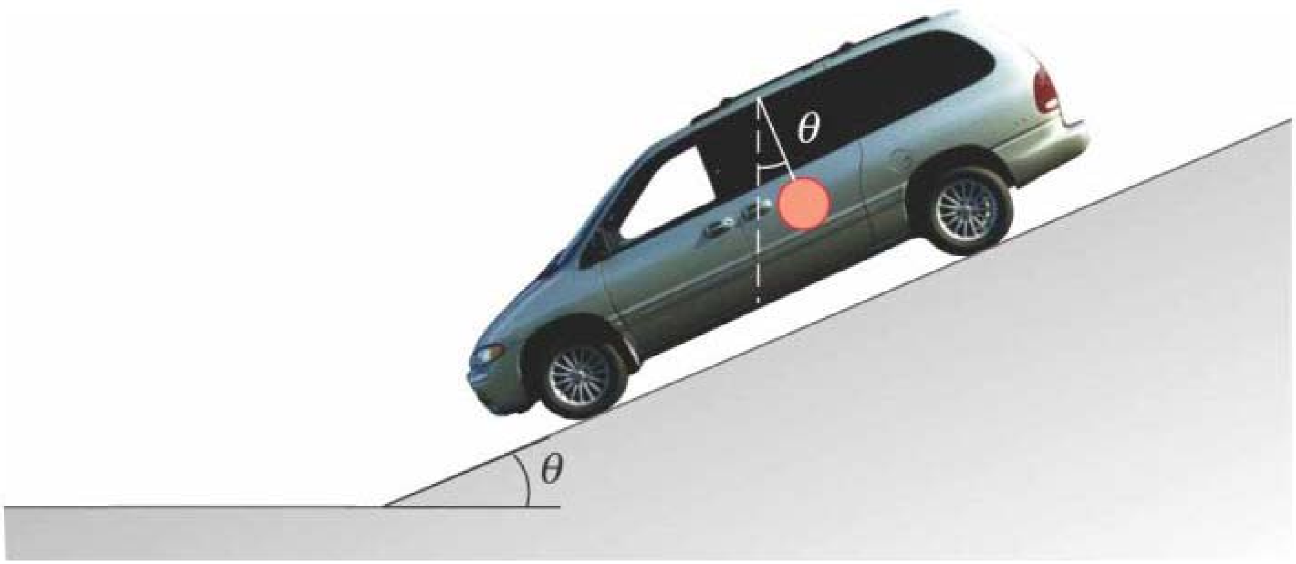
Minimum occurs when $\dfrac{dF}{d\theta} = -\mu_k mg \dfrac{\mu_k \cos\theta - \sin\theta}{(\mu_k \sin\theta + \cos\theta)^2} = 0$

Therefore, $\theta = \tan^{-1}\mu_k$ minimizes the pulling force.

Example 13

A van accelerates down a hill, going from rest to 30.0 m/s in 6.00 s. During the acceleration, a toy (m = 0.100 kg) hangs by a string from the van's ceiling. The acceleration is such that the string remains perpendicular to the ceiling.

Determine: (a) the angle and (b) the tension in the string.



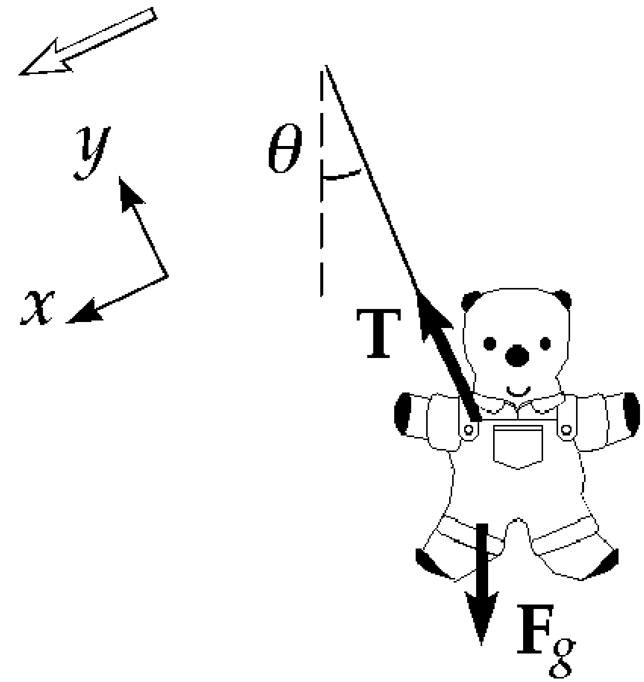
Example 13, cont

Choose *x*-axis pointing down the slope.

$x: \quad v_f = v_i + at$

$30.0 \text{ m/s} = 0 + a(6.00 \text{ s})$

$\Rightarrow a = 5.00 \text{ m/s}^2$



Consider the forces on the toy.

$x: \quad mg\sin\theta = m\,(5.00 \text{ m/s}^2)$

$\Rightarrow \theta = 30.7°$

$y: \quad T - mg\cos\theta = 0$

$T = 0.100 \text{ kg} \times 9.80 \text{ m/s}^2 \times \cos 30.7°$

$\Rightarrow T = 0.843 \text{ N}$

Example 14

Pat wants to reach an apple in a tree without climbing it. Sitting in a chair connected to a rope that passes over a frictionless pulley, Pat pulls on the loose end of the rope with a force that the spring scale reads 250 N. Pat's true weight is 320 N, and the chair weighs 160 N.

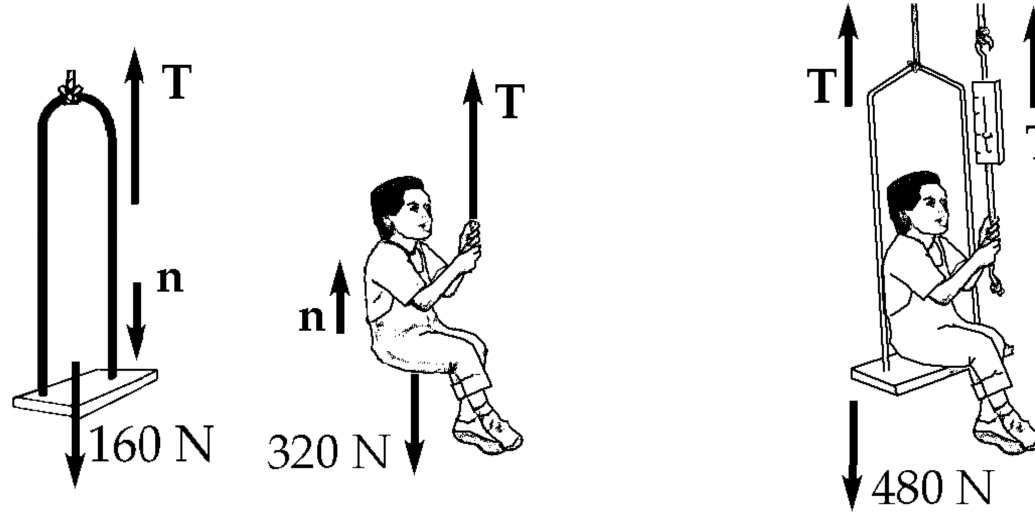
(a) Draw free-body diagrams for Pat and the chair considered as separate systems, and another diagram for Pat and the chair considered as one system.

(b) Show that the acceleration of the system is *upward* and find its magnitude.

(c) Find the force Pat exerts on the chair.

Ex 14, cont

(a) Free-body diagrams:



(b) Consider Pat and the chair as the system. Note that two ropes support the system and each rope has $T = 250$ N.

$\sum F_y = ma_y \Rightarrow 2 \times 250 \text{ N} - 480 \text{ N} = (480 \text{ N}/9.80 \text{ m/s}^2)a$

$\Rightarrow \; a = 0.408 \text{ m/s}^2 > 0$. Therefore, *upward*.

(c) Now consider Pat as the system:

$\sum F_y = n + T - 320 \text{ N} = (320 \text{ N}/9.80 \text{ m/s}^2)a$

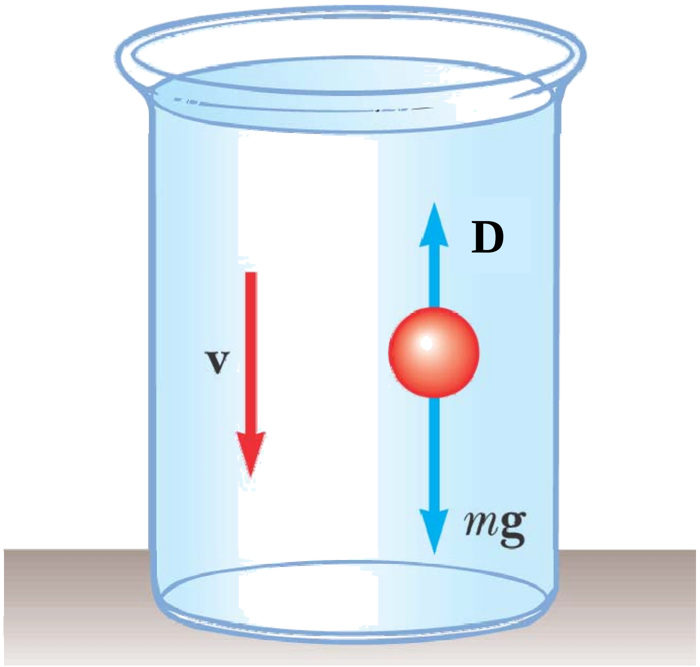
$\Rightarrow \; n = -250 \text{ N} + 320 \text{ N} + 32.7 \text{ kg} \times 0.408 \text{ m/s}^2 = 83.3 \text{ N}$

Motion with Resistive Forces

- Motion can be through a medium
  - Either a liquid or a gas
- Medium exerts a *resistive drag force* **D** on an object moving through the medium
  - Magnitude of **D** depends on the medium
  - Direction of **D** is opposite to the direction of motion
- Magnitude of **D** can depend on the speed in complex ways
  - We will discuss only two cases: $D \propto v$, $D \propto v^2$

Drag Proportional to *v*

- For objects moving at low speeds, the drag is approximately proportional to *v*: **D** = −*b***v**
  - *b* depends on the property of the medium, and on the shape and dimensions of the object
  - The negative sign indicates that **D** is opposite in direction to **v**
- The equation of motion of the suspended mass:



$$D - mg = m(-a)$$

$$bv - mg = -m\frac{dv}{dt}$$

Terminal Speed for $D \propto v$

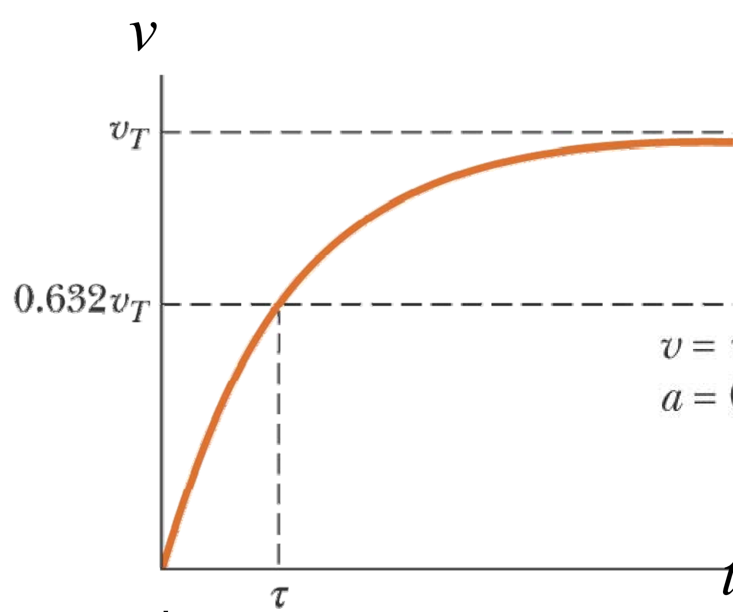
- Initially, $v = 0 \Rightarrow D = 0$ and $dv/dt = -g$
- As t increases, $D \to mg$, $a \to 0$ and $v \to v_T$: the *terminal speed* of the object:

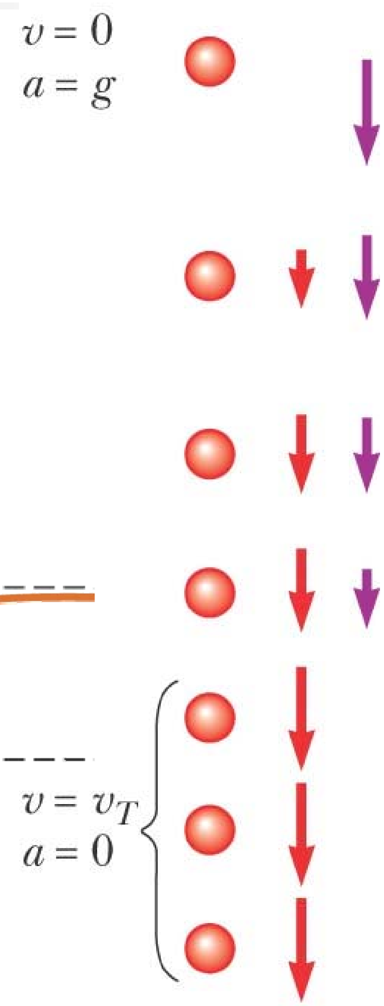
$$bv_T - mg = -m\frac{dv}{dt} = 0$$

$$\Rightarrow \boxed{v_T = \frac{mg}{b}}$$

- $v(t)$ is found by solving the differential equation:

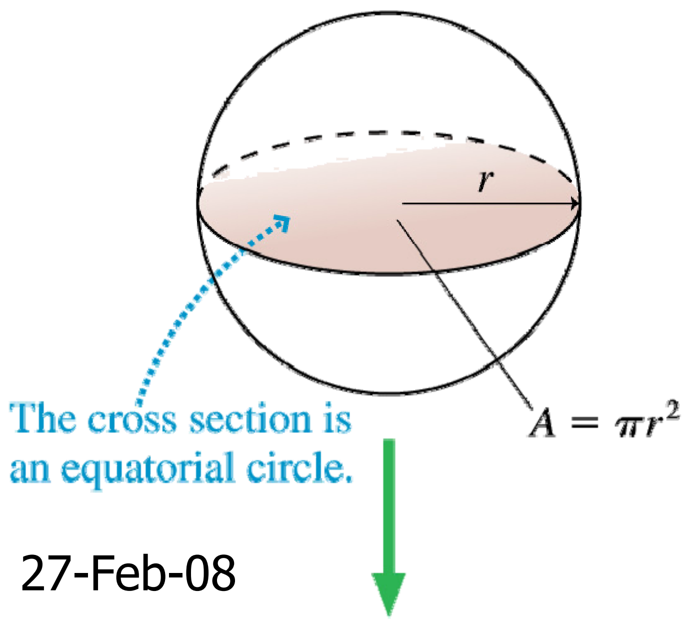
$$v = v_T\left(1 - e^{-t/\tau}\right),\ \ \tau = m/b$$

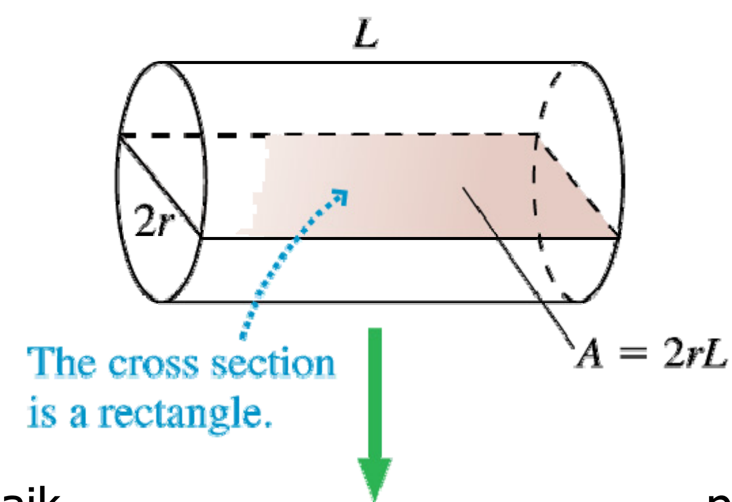




Drag Proportional to v^2

- For objects moving at high speeds, the drag is approximately proportional to v^2: $D = (1/2)c\rho A v^2$
  - c is a dimensionless empirical "drag coefficient"
  - ρ is the density of the fluid
  - A is the cross-sectional area of the object



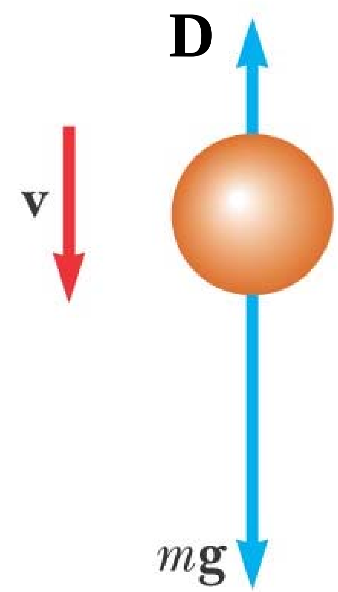


Terminal Speed for $D \propto v^2$

- Analysis of an object falling through a medium accounting for air resistance

$$D - mg = m(-a), \ \tfrac{1}{2}c\rho A v^2 - mg = -ma$$

$$\Rightarrow \ a = g - \left(\frac{c\rho A}{2m}\right)v^2$$



- The terminal speed will occur when the acceleration goes to zero
- Solving the equation gives $v_T = \sqrt{\dfrac{2mg}{c\rho A}}$

Some Terminal Speeds

Terminal Speed for Various Objects Falling Through Air			
Object	**Mass (kg)**	**Cross-Sectional Area (m^2)**	**v_T (m/s)**
Sky diver	75	0.70	60
Baseball (radius 3.7 cm)	0.145	4.2×10^{-3}	43
Golf ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hailstone (radius 0.50 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Raindrop (radius 0.20 cm)	3.4×10^{-5}	1.3×10^{-5}	9.0